Adjoint-state Method Based Strategy for Non-linear Seismic AVO Inversion

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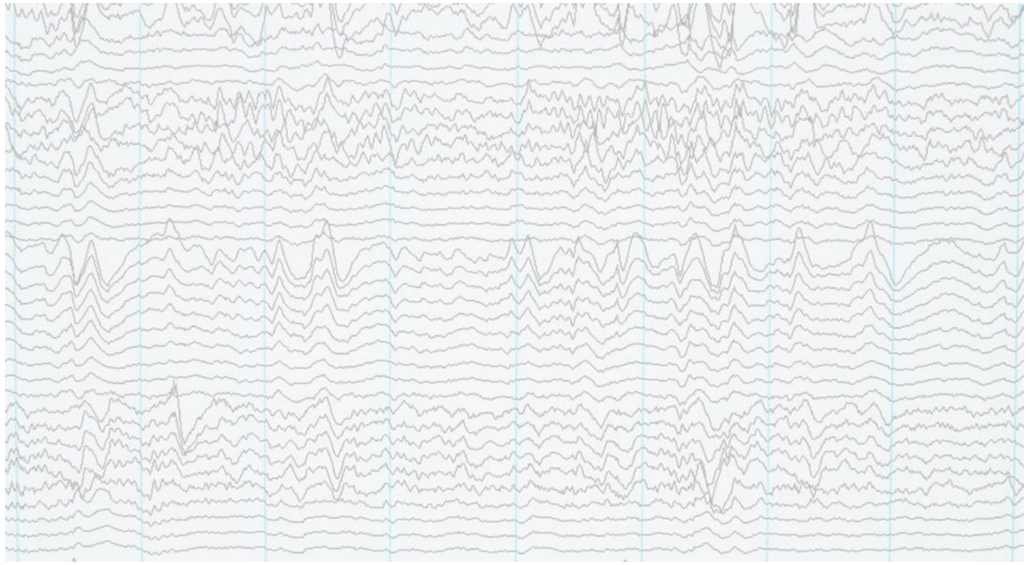

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Outline…

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Seismic AVO Inversion strategy

This is the model-based inversion which has been formulated as an optimization problem

This inversion method is based on gradient-descent optimization algorithm…

Gradient descent optimization

- we need to find the gradient/s of objective function with respect to model variables…
- we need some initial guess as a starting point…
- how to get best optimal solution…?

Theory and Method:

this inversion method relies on four components …

- seismic forward modelling
- defination of objective function
- adjoint-state solution of objective function
- l-bfgs – to find the local minima iteratively

Seismic forward modelling

Seismic amplitudes are approximated by a convolutional model:

$$d(t, \theta) = W(t) * R_{PP}(t, \theta | m)$$

R_{PP} for weak contrast can be obtained by linear equation of PP reflection amplitudes:

$$R_{PP}(t, \theta) = A(t, \theta)\,\frac{\Delta\alpha(t)}{\alpha(t)} + B(t, \theta)\,\frac{\Delta\beta(t)}{\beta(t)} + C(t, \theta)\,\frac{\Delta\rho(t)}{\rho(t)}$$

$$A = \left[\frac{1}{2}\left(1 + \tan^2\theta\right)\right], \qquad B = \left[-\frac{4\beta^2}{\alpha^2}\sin^2\theta\right], \qquad C = \frac{1}{2}\left[1 - \frac{4\beta^2}{\alpha^2}\sin^2\theta\right]$$

AVO inversion: least-square formulation of data misfit

forward modelling equation of AVO inversion…

$$f(m) = d,$$

$$m = [V_P(t), V_S(t), \rho(t)],$$

the objective function defination …

$$J = \frac{1}{2} \| d(t, \theta) - f(m) \|^2_{t, \theta}$$

Forward modelling operator …

$$f(m) = W(t) * R_{PP}(t, \theta | m)$$

The gradient of objective function with respect model variables m[V_P, V_S, ρ]

$$\nabla J = \left[\frac{\partial J(t)}{\partial V_P}, \frac{\partial J(t)}{\partial V_S}, \frac{\partial J(t)}{\partial \rho} \right]$$

Adjoint-state solution for gradient derivation

$$\mathcal{L}(V_P, V_S, \rho, R_{PP}, \lambda) = \sum_i \int_\theta d\theta \left[d[i] - W[i] * R_{PP}[i] \right]^2 + \sum_i \int_\theta d\theta \left[R_{PP}[i] - A[i]\cdot\frac{\Delta\alpha}{\alpha} - B[i]\cdot\frac{\Delta\beta}{\beta} - C[i]\cdot\frac{\Delta\rho}{\rho} \right]\lambda_{[i]}$$

$$\mathcal{L} = J + C * \lambda$$

Lagrangian function

objective function

constraints

adjoint-state variable

$$\left[R_{PP}[i] - A[i]\cdot\frac{\Delta\alpha}{\alpha} - B[i]\cdot\frac{\Delta\beta}{\beta} - C[i]\cdot\frac{\Delta\rho}{\rho} \right] = 0$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial V_P[i]} = \int_\theta d\theta \Big\{ & \frac{A}{\alpha[i-1]}\cdot\lambda[i-1] - \frac{A}{\alpha[i]}\cdot\lambda[i] \\ & - \frac{A\Delta\alpha[i-1]}{2\alpha[i-1]^2}\cdot\lambda[i-1] - \frac{A\Delta\alpha[i]}{2\alpha[i]^2}\cdot\lambda[i] \\ & + \frac{4\beta[i-1]^2}{\alpha[i-1]^3}\cdot\sin^2\theta\cdot\frac{\Delta\beta[i-1]}{\beta[i-1]}\cdot\lambda[i-1] \\ & + \frac{4\beta[i]^2}{\alpha[i]^3}\cdot\sin^2\theta\cdot\frac{\Delta\beta[i]}{\beta[i]}\cdot\lambda[i] \\ & + \frac{2\beta[i-1]^2}{\alpha[i-1]^3}\cdot\sin^2\theta\cdot\frac{\Delta\rho[i-1]}{\rho[i-1]}\cdot\lambda[i-1] \\ & + \frac{2\beta[i]^2}{\alpha[i]^3}\cdot\sin^2\theta\cdot\frac{\Delta\rho[i]}{\rho[i]}\cdot\lambda[i] \Big\}\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial V_S[i]} = \int_\theta d\theta \Big\{ & -\frac{2\Delta\beta[i-1]}{\alpha[i-1]^2}\cdot\sin^2\theta\cdot\lambda[i-1] \\ & - \frac{2\Delta\beta[i]}{\alpha[i]^2}\cdot\sin^2\theta\cdot\lambda[i] \\ & - \frac{4\beta[i-1]}{\alpha[i-1]^2}\cdot\sin^2\theta\cdot\lambda[i-1] \\ & + \frac{4\beta[i]}{\alpha[i]^2}\cdot\sin^2\theta\cdot\lambda[i] \\ & - \frac{2\beta[i-1]}{\alpha[i-1]^2}\cdot\sin^2\theta\cdot\frac{\Delta\rho[i-1]}{\rho[i-1]}\cdot\lambda[i-1] \\ & - \frac{2\beta[i]}{\alpha[i]^2}\cdot\sin^2\theta\cdot\frac{\Delta\rho[i]}{\rho[i]}\cdot\lambda[i] \Big\}\end{aligned}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial \rho[i]} = \int_\theta d\theta \Big\{ & C[i-1]\cdot\left(\frac{1}{\rho[i-1]}\right)\cdot\lambda[i-1] - C[i]\cdot\left(\frac{1}{\rho[i]}\right)\cdot\lambda[i] \\ & - C[i-1]\cdot\left(\frac{\Delta\rho[i-1]}{\rho[i-1]^2}\right)\cdot\lambda[i-1] - C[i]\cdot\left(\frac{\Delta\rho[i]}{\rho[i]}\right)\cdot\lambda[i] \Big\}\end{aligned}$$

L-BFGS

$$m_{k+1} = m_k - \alpha_k H_k \nabla J, \qquad k = 0, 1, 2, 3, \ldots,$$

m_{k+1} = *updated iteration*

m_k = *current iteration*

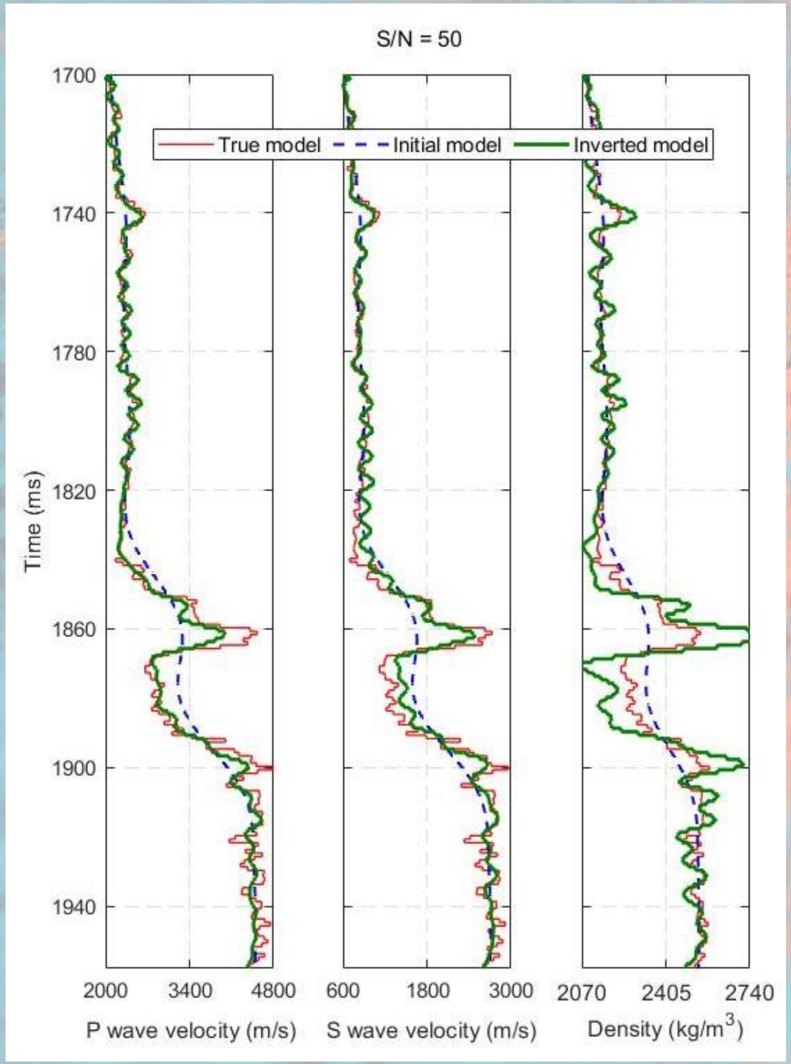
α_k = *step length*

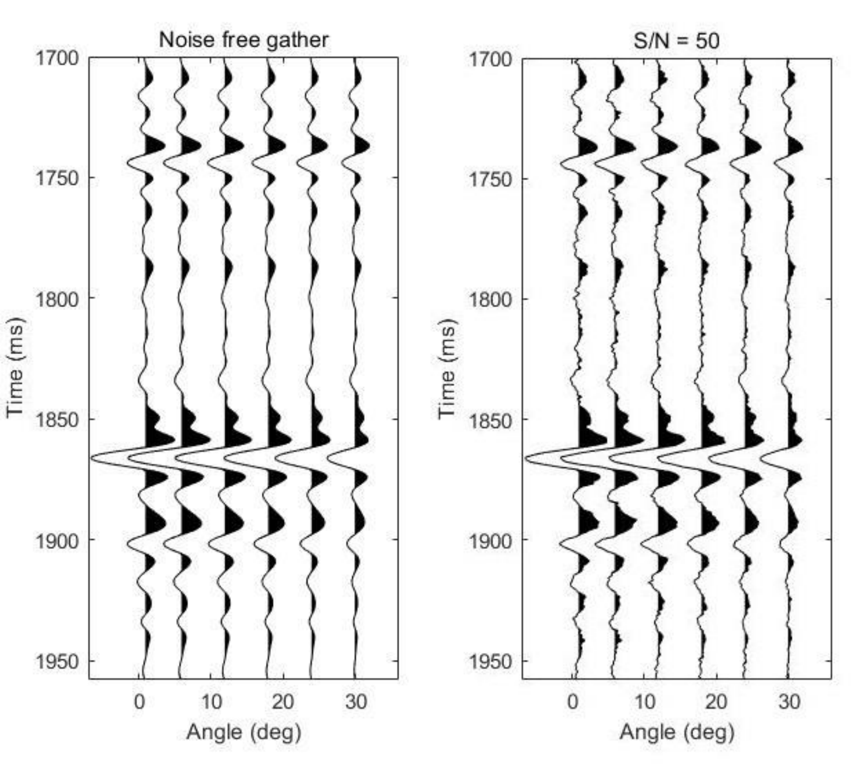
H_k = *inverse Hessian approximation*

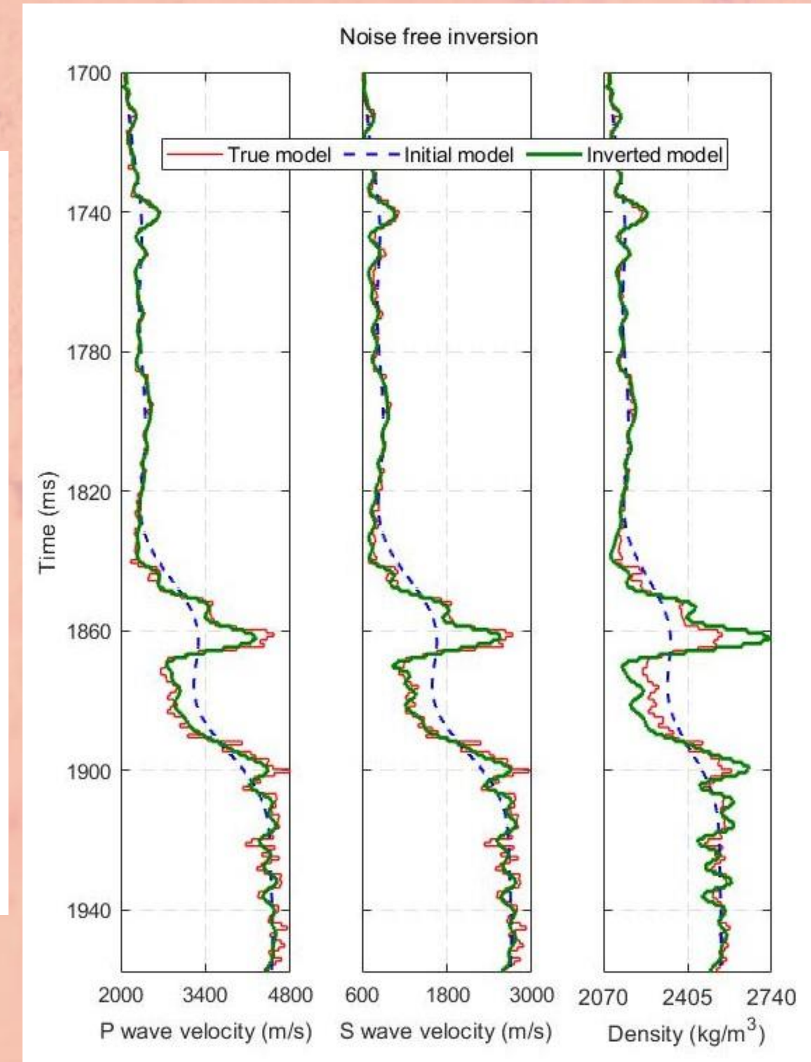
∇ = *gradient*

J = *data misfit / objective function*

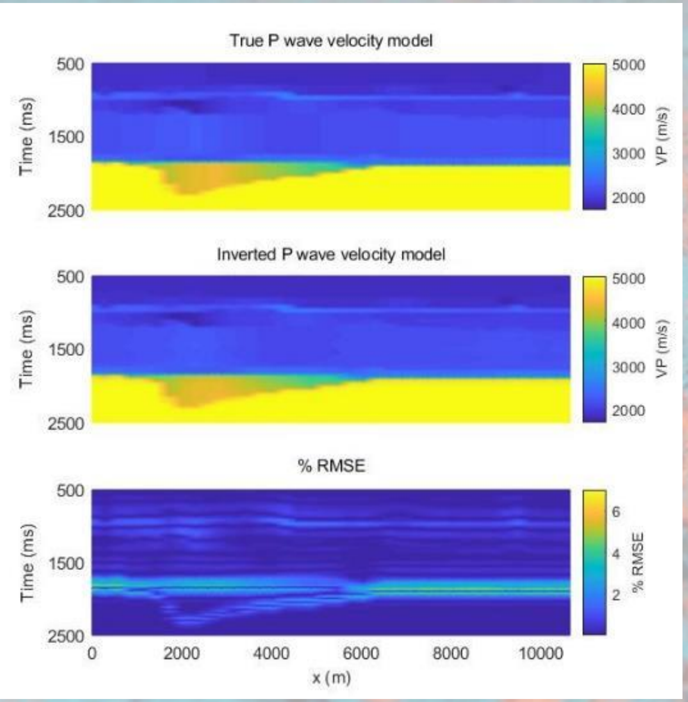
Applications… well logs example

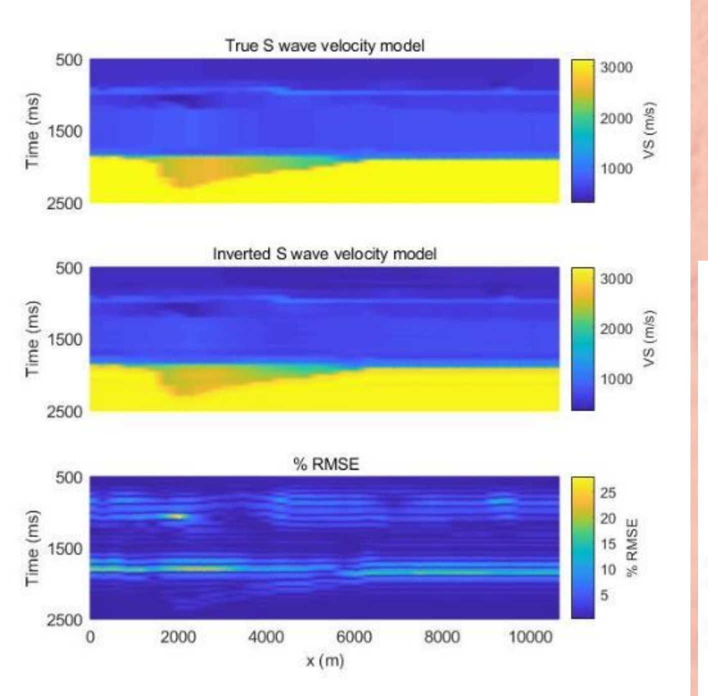


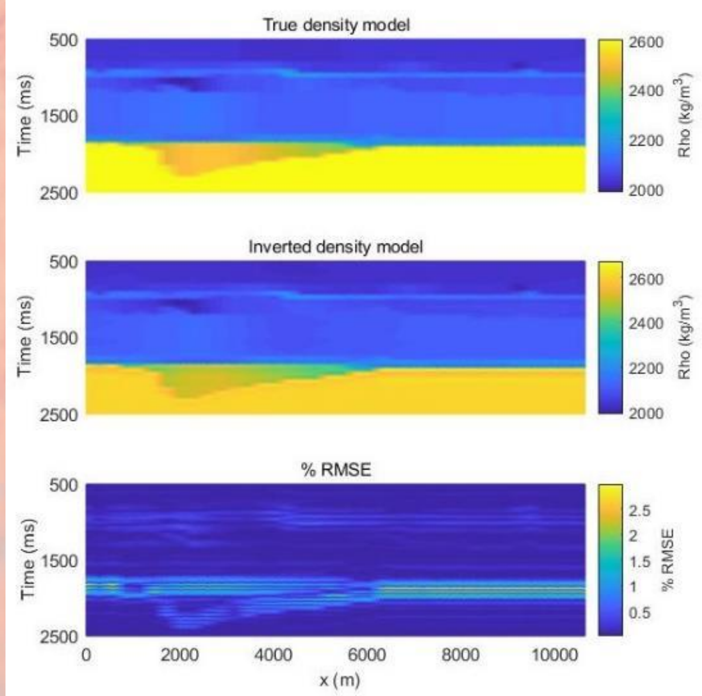


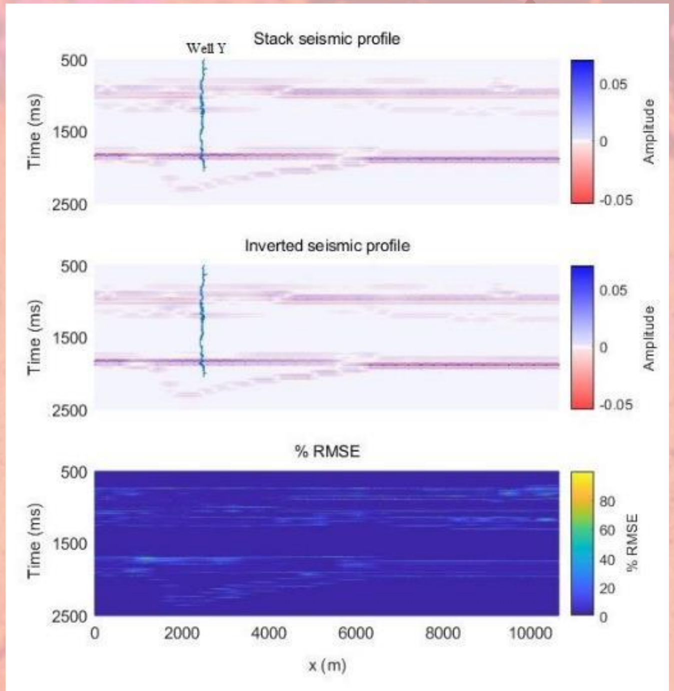


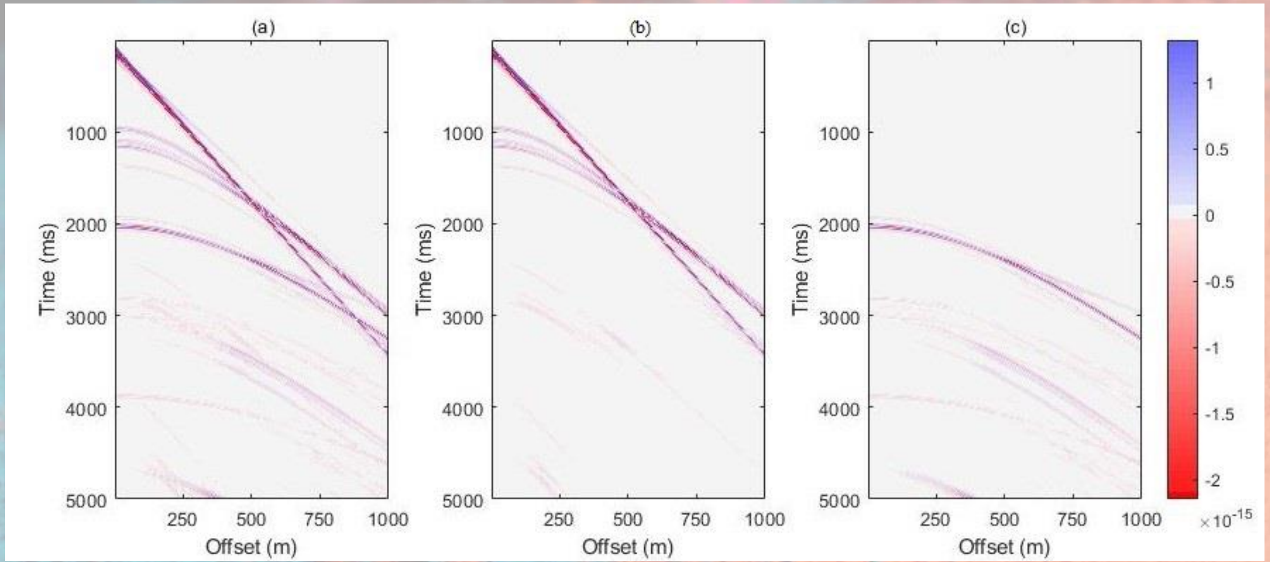
Seismic properties… 2D synthetic example

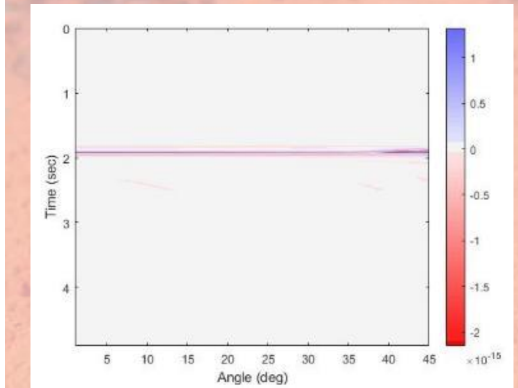


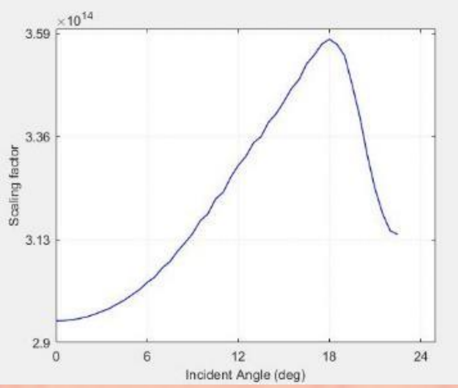


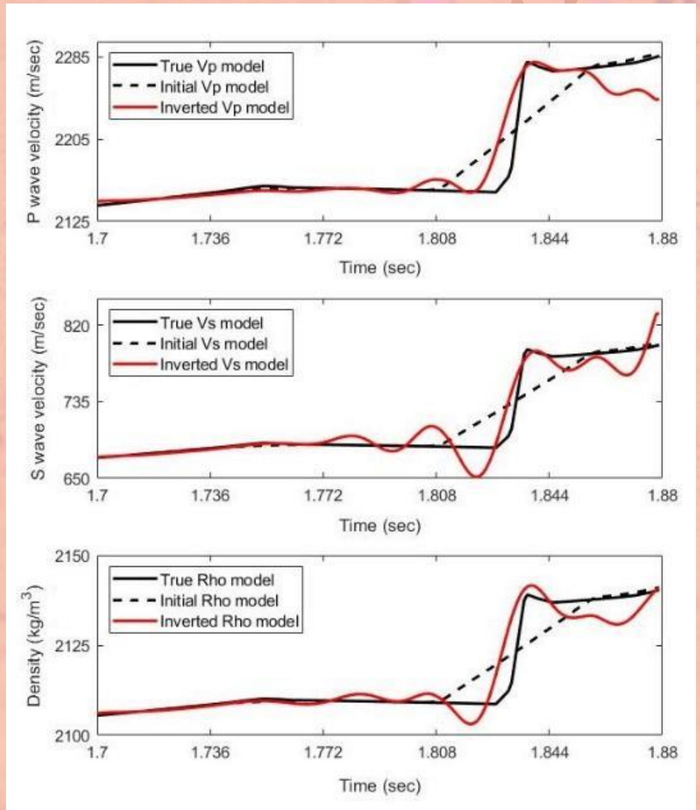


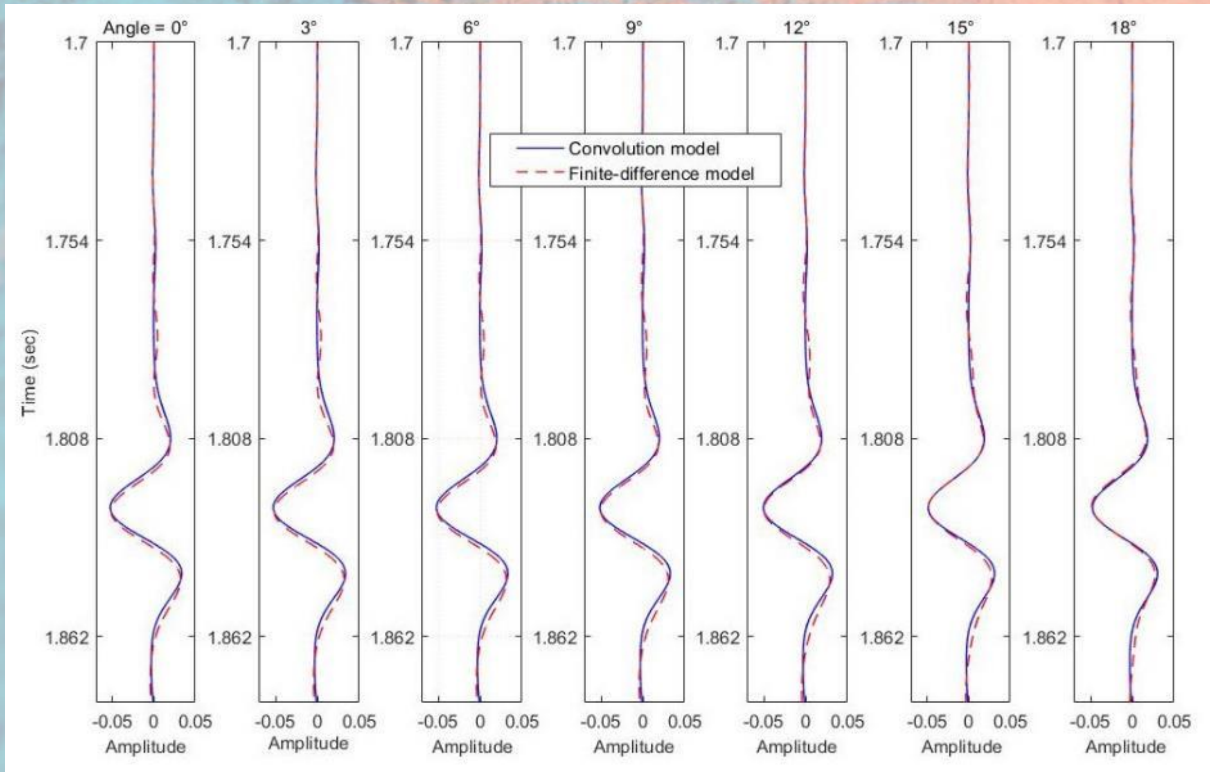












Model example 3: FD extension

Summary…

The inversion method is formulated as a least-square misfit function for the model variables estimation…

Minimization optimization (gradient descent method) that follows the negative of the gradient to the minimum of the target function...

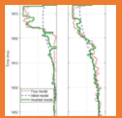
Gradient of the data misfit with respect to the un-known model varaiables are computed by using the adjoint-state method…

Best optimal solution are obtained by using non-linear L-BFGS method…

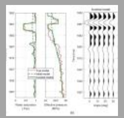
The method has been successfully implemented on synthetic datasets.

The method is capable to retrive the from the seismic gathers.

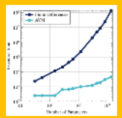
Advantages and limitations



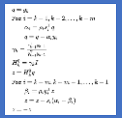
the inversion method is very general and can be extended for all the other variables. e.g.,



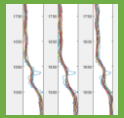
- petro-elastic and geomechanical properties, 4D seismic changes



adjoint-state provides more fast and efficient way to compute the gradient…



easy handling the big data sets due to l-bfgs implementation



However, it sensitive to the initial models and requirs to introduce the regularization weights

THANK YOU FOR YOUR ATTENTION

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I hope you have enjoyed this presentation.