



# Theoretical Research on the Pendulum-type Wave in Nonlinear Block-rock Mass Based on Hyperbolic Elastic Model

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01

**Background**

02

**Theoretical model**

03

**Results**

04

**Conclusions**

**01**

# Background

## ➤ Block-rock mass

Rock mass is characterized by the **block-hierarchical structures**. The deformation of rock mass is mainly concentrated in interlayers between rock blocks, and the deformation of the rock block can be ignored.

## ➤ Pendulum-type Wave

Kurlenya and Oparin found the **sign-alternating reaction of rocks to dynamic effects** in the underground explosion, and they think that there is a nonlinear sign-alternating wave in rock masses, called the **pendulum-type wave**.

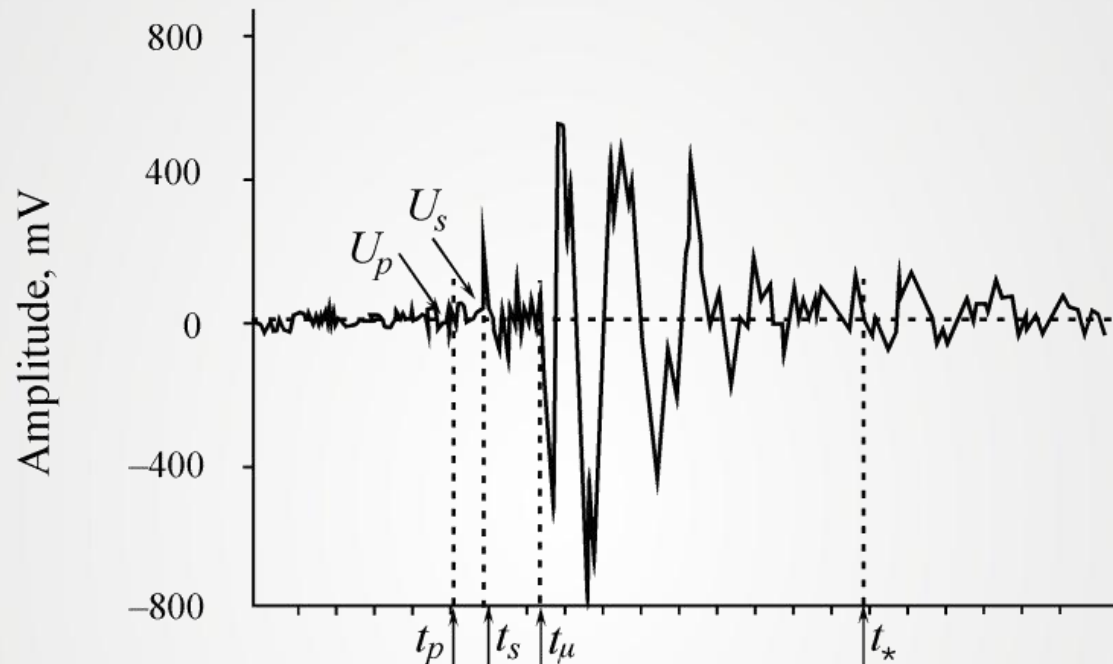


Fig.1 Pendulum-type wave isolated from rock mass

The phenomenon of pendulum-type wave can easily cause engineering disasters in deep rock mass such as **rockbursts**.

# 02

## Theoretical model

**2.1** Dynamic model of pendulum-type wave

**2.2** Hyperbolic elastic model

**2.3** Dynamic equation

The block-rock mass is simplified as an incompressible block and an interlayer with viscoelastic properties. The spring is linear, and there are two drawbacks:

- Nonlinear deformation cannot be represented.
- The interlayers can deform indefinitely.

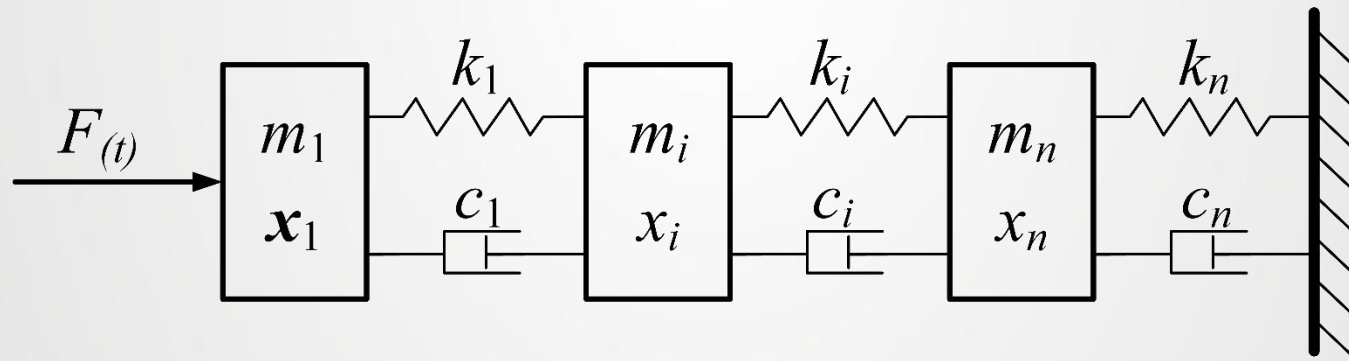


Fig.2 Dynamic model of pendulum-type wave in block-rock mass

The expression of the hyperbolic elastic model is as follows:

$$\Delta V_j = \frac{\sigma_n V_m}{K_{ni} V_m + \sigma_n}$$

$\Delta V_j$  — joint closure;

$\sigma_n$  — normal stress;

$V_m$  — maximum possible closure;

$K_{ni}$  — initial normal stiffness

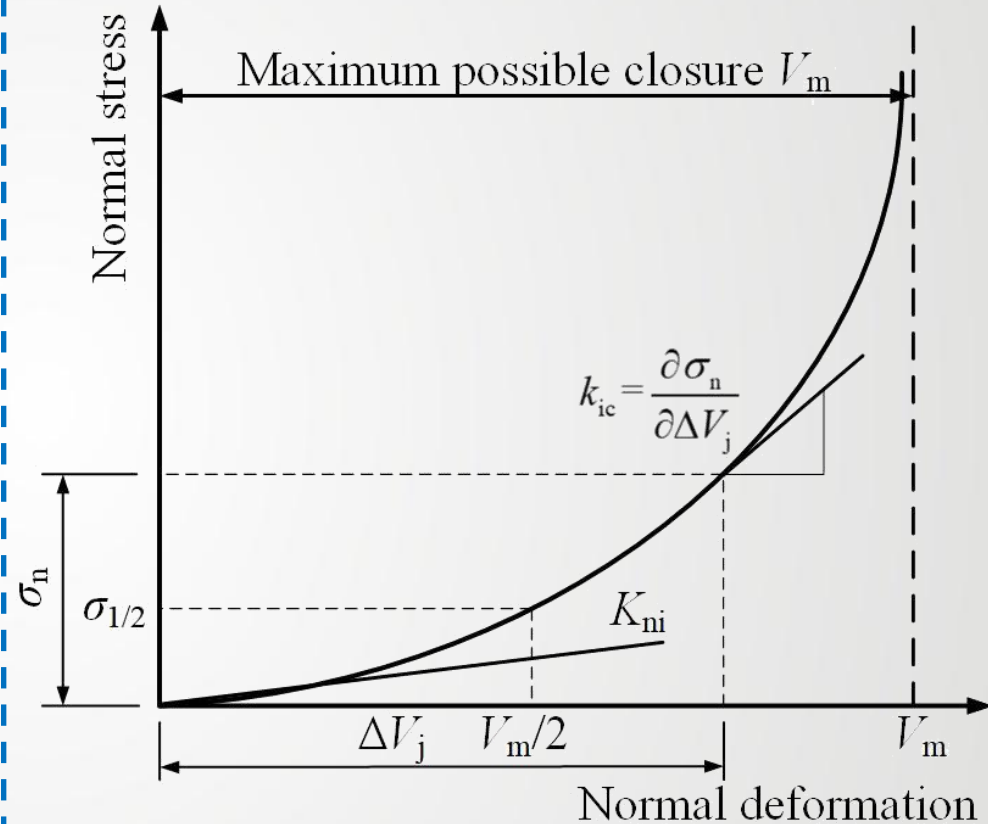


Fig. 3 Curve of hyperbolic elastic model



The dynamic equation of block-rock mass in the matrix form:

$$M\ddot{X}(t) + C\dot{X}(t) + F_s = F(t)$$

Where,  $M$  is the mass matrix;  $C$  is the damping matrix;  $F_s = [-F_{k1}, F_{k1} - F_{k2}, \dots, F_{k(i-1)} - F_{ki}, \dots, F_{k(n-1)} - F_{kn}]^T$  is the resistance vector generated by nonlinear springs.

When  $1 \leq i < n$ ,

$$F_{ki} = \begin{cases} -\frac{AK_{ni}V_m\Delta V_j}{V_m - \Delta V_j} = -\frac{AK_{ni}V_m(x_i - x_{i+1})}{V_m - (x_i - x_{i+1})}, & x_{i+1} - x_i \leq 0 \\ K_{ni}(x_{i+1} - x_i), & x_{i+1} - x_i > 0 \end{cases}$$

When  $i=n$ ,

$$F_{kn} = \begin{cases} -\frac{AK_{ni}V_m\Delta V_j}{V_m - \Delta V_j} = -\frac{AK_{ni}V_mx_n}{V_m - x_n}, & x_n \geq 0 \\ -K_{ni}x_n, & x_n < 0 \end{cases}$$

**03**

# Results

Defining the maximum compression deformation of interlayers

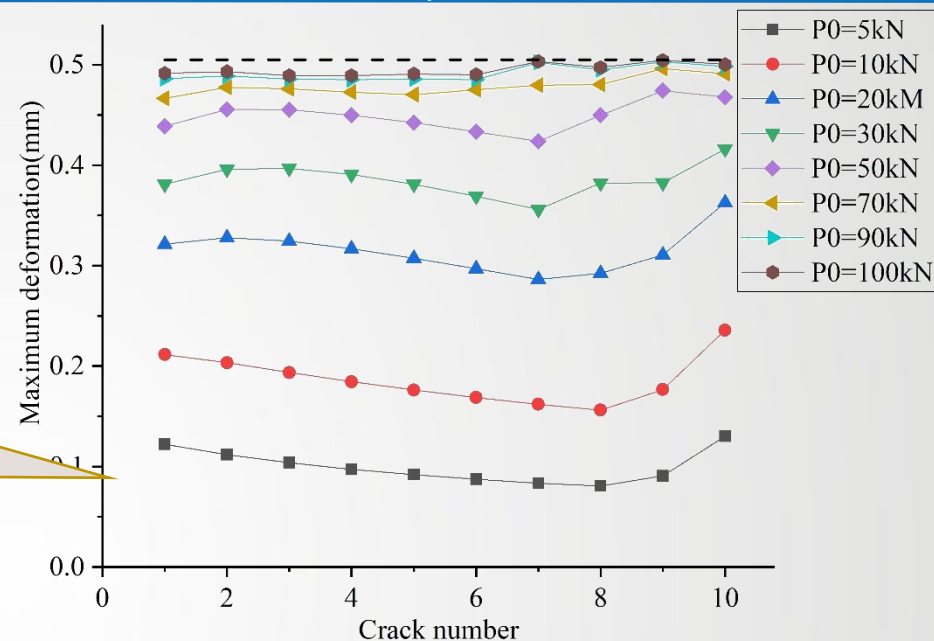
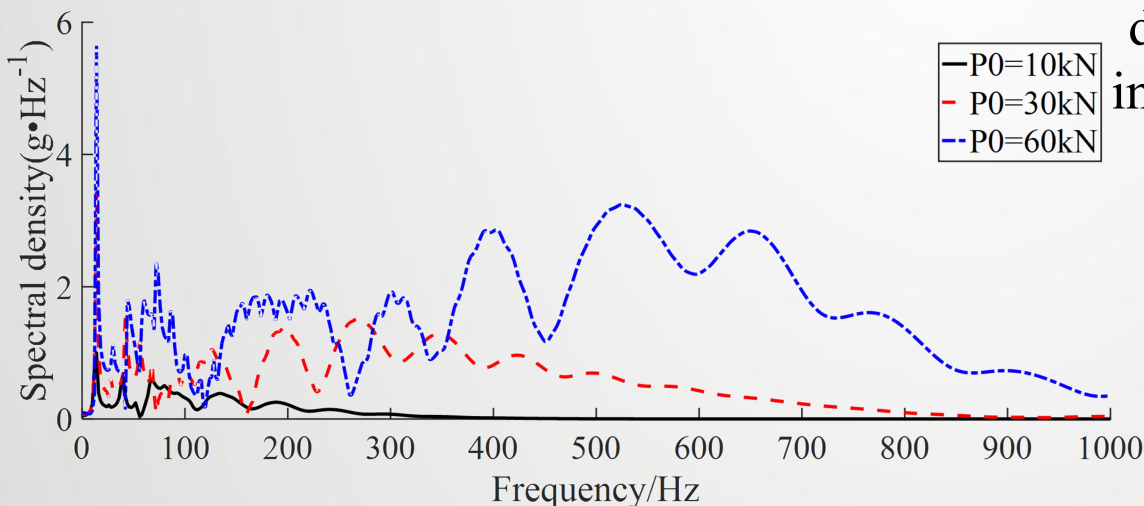


Fig.4 Maximum compression deformation of cracks with the increase of propagation distance



Moving to high frequencies

Fig.5 Frequency response of the rock block

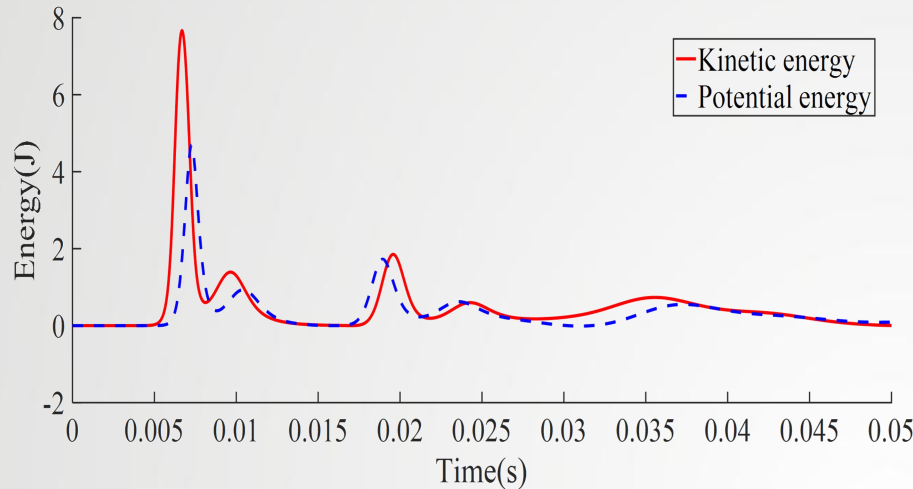


Fig.6 Kinetic energy and potential energy in a block-spring element

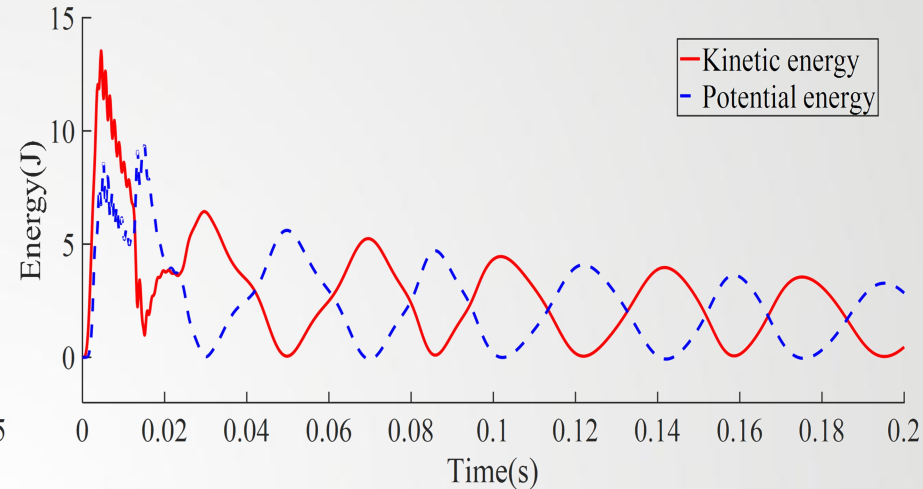


Fig.7 Kinetic energy and potential energy in block-rock mass

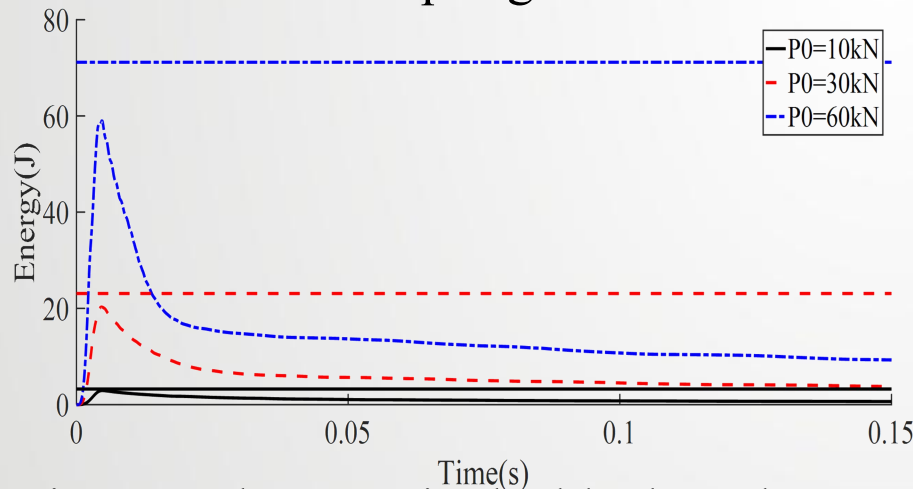


Fig.8 Total energy in the block-rock mass  
(The dashed line parallel to X-axis represents the total input energy)

The law of **transfer and transformation of kinetic energy and potential energy** in block-rock mass is obtained.

## Consideration of initial geostress

The hyperbolic elastic model can be grouped into three categories: **low stress state, high stress state and ultra-high stress state**.

$\sigma_{1/2}$  — normal stress when the deformation is half of the maximum possible closure  $V_m$

$\sigma_u$  — normal stress when the deformation reaches 95% of the maximum possible closure  $V_m$

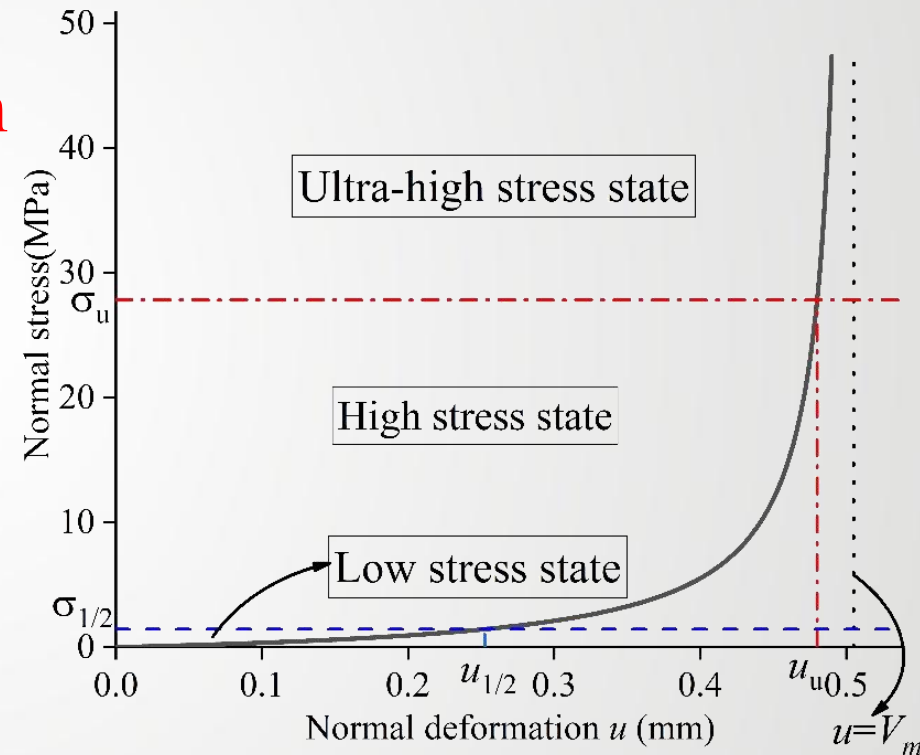


Fig.9 Division of stress states

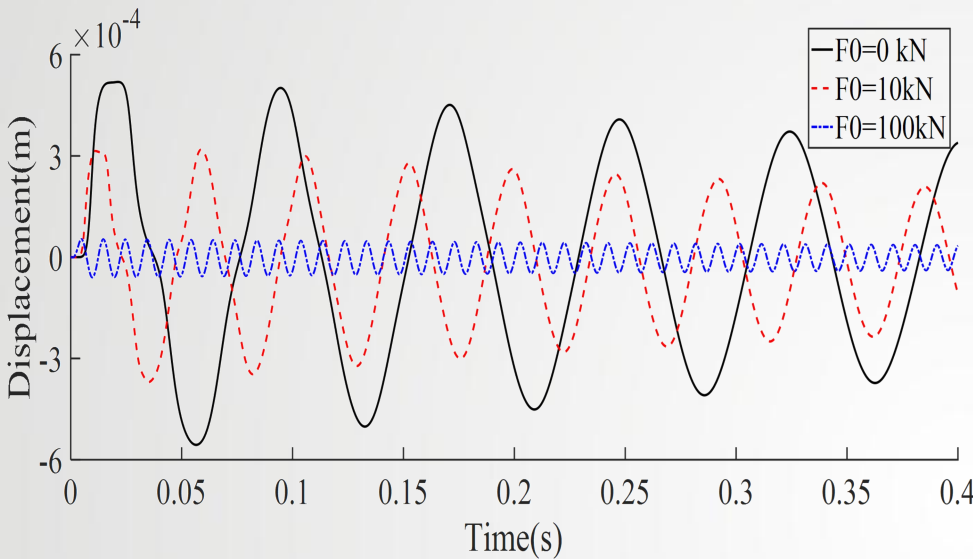


Fig.10 Displacement curves of the rock block in three initial stress states

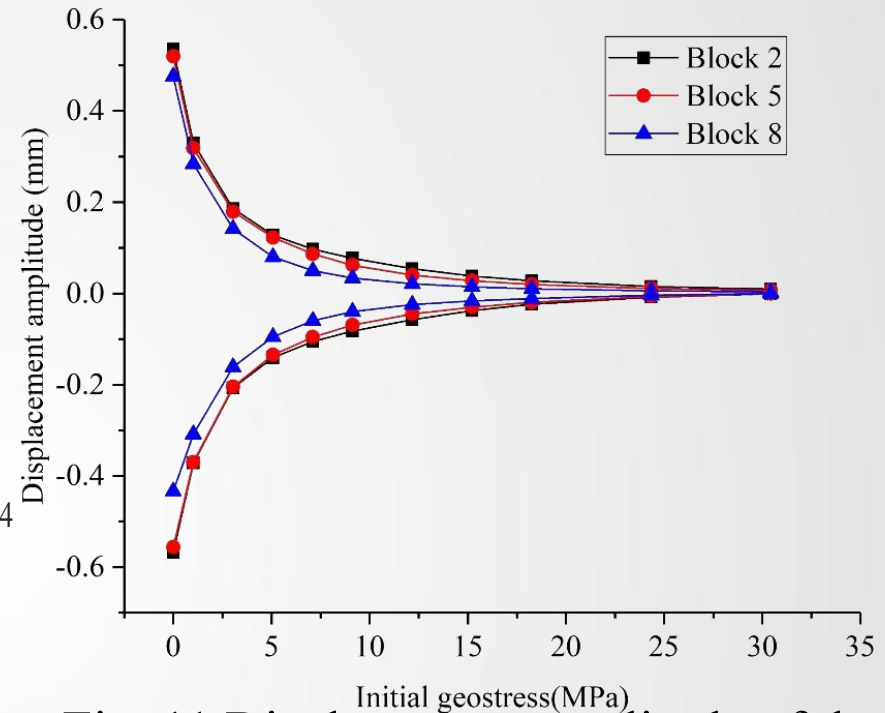


Fig. 11 Displacement amplitude of three blocks with the increase of initial geostress

With the increase of initial geostress, the displacement amplitude decrease approximately exponentially.

Based on the above analysis, we make a prediction that **in the ultra-high stress state, the phenomenon of the pendulum-type wave does not occur in block-rock mass, and the wave propagation is close to the longitudinal wave.** There are two main reasons:

- The cracks (interlayers) are basically closed under the ultra-high stress state, and the vibration displacement of rock blocks relative to the initial position is extremely weak. At this time, the deformation of rock blocks cannot be ignored or even become the main part of deformation, and the deformation of block-rock mass is no longer dominated by that of interlayers.
- When cracks of the block-rock mass are almost completely compressed, kinetic energy is no longer the main way of carrying energy.

**04**

# Conclusions



- The improved nonlinear model can not only shows the nonlinear deformation of block-rock mass, but also limits the maximum compression deformation of cracks between rock blocks.
- Kinetic energy and potential energy are constantly transformed to each other in block-rock mass, and in the free vibration stage, they are in inverse phase.
- In the ultra-high stress state, the phenomenon of the pendulum-type wave will not occur in block-rock mass, and the wave propagation is close to the longitudinal wave.



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# THANKS!

If you have any questions, please don't hesitate to contact me.

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