Direct evidence of an oceanic dual kinetic energy cascade and its seasonality from surface drifters

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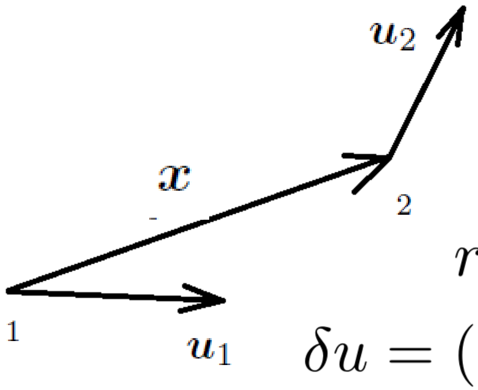
Inertial range theory

- 3D: Richardson (1922) → Kolmogorov (1941)'s *"4/5-law" for third-order structure function*. (Two-point measurements)



$$r = |\boldsymbol{x}|.$$

$$\delta u = (\boldsymbol{u}_2 - \boldsymbol{u}_1) \cdot \hat{\boldsymbol{x}}$$

$$l_f >> r >> l_\nu$$

$$\overline{\delta u^3} = -\frac{4}{5}\epsilon r$$

- Dissipation scale:

$$l_\nu = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

The "4/5-law" is a result calculated from NS (KHM) equation.

- 2D: Lindborg 1999, Bernard 1999, Yakhot 1999:

Energy inertial range:

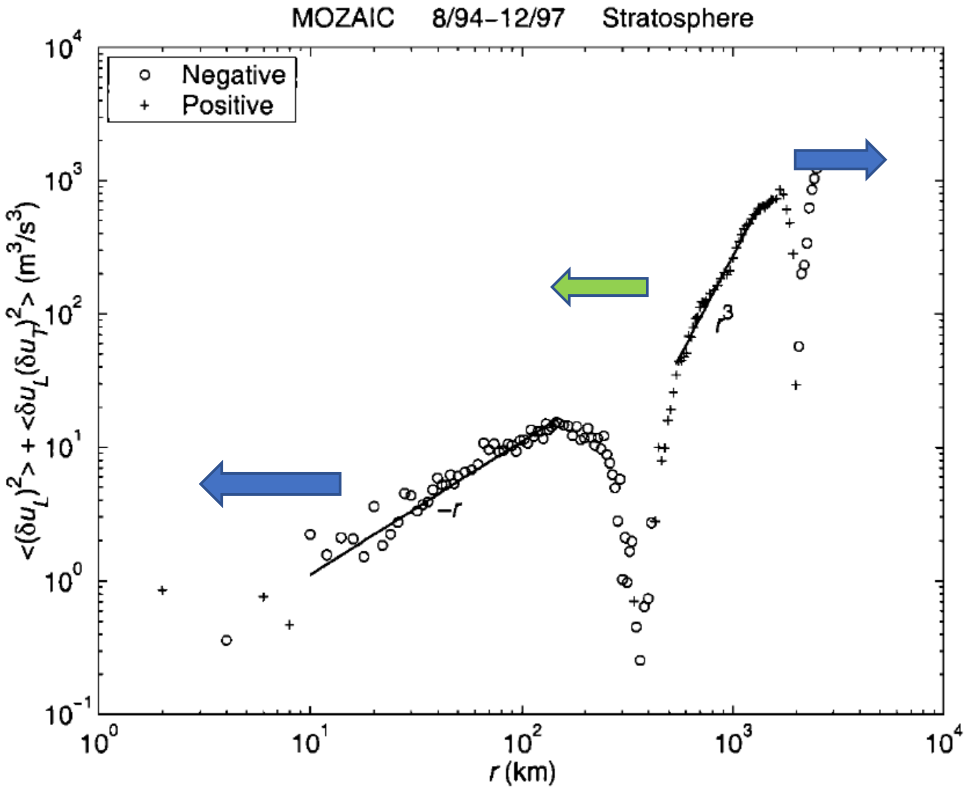
$$\overline{\delta u^3} = \frac{3}{2}\epsilon r$$

Enstrophy inertial range:

$$\overline{\delta u^3} = \frac{1}{8}\eta r^3$$

Defects of previous inertial-range theories

- Previous theories apply to *local* inertial ranges, how to obtain a *global* description?
  - How to determine inertial ranges? (power-law regimes are asymptotic)
- Previous theories apply to unidirectional energy transfer scenarios. How about the bidirectional energy transfer?
- How to obtain the forcing scale?
  - The forcing information is always unknown but important to geophysics.



Third-order structure function obtained from MOZAIC data (Cho & Lindborg 2001)

❖Our solution: from Navier-Stokes equation derive

A *global forcing-scale-resolving* theory that captures the *bidirectional energy transfer.*

Derivation of structure-function expression

(with X & Bühler 2019)

- In the KHM equation, under the assumption of isotropy, we obtain a new expression for the third-order structure function:

$$F = -\epsilon_u + (\epsilon_u + \epsilon_d)\, H(K - 1/l_f),$$

$$V(r) = -4r \int_0^\infty \frac{1}{K} F(K) J_2(Kr)\, \mathrm{d}K.$$

$$V = 2\epsilon_u r - 4(\epsilon_u + \epsilon_d) l_f J_1(r/l_f).$$

$$\boldsymbol{V} = \overline{\delta \boldsymbol{u} |\delta \boldsymbol{u}|^2} = V(r)\boldsymbol{e}_r$$

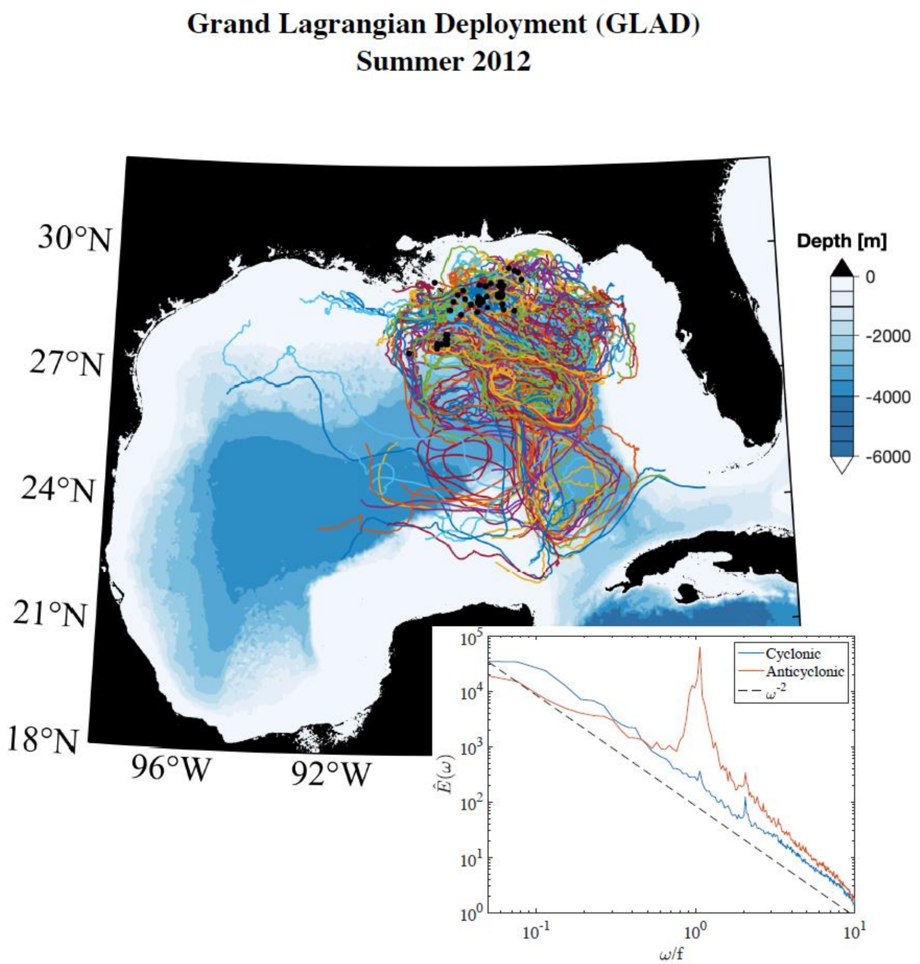
$$V = \overline{\delta u_L^3} + \overline{\delta u_L \delta u_T^2}$$

- *(1) bidirectional energy transfer (2) forcing scale l_f (3)Asymptotical*ly recovers the inertial-range theories*:*

$$V = \begin{cases} \underbrace{-2\epsilon_\mathrm{d} r}_{\text{downscale energy}} + \underbrace{\frac{1}{4}\frac{\epsilon}{l_f^2} r^3}_{\text{“enstrophy”}} + O\left((r/l_f)^5\right), & \text{when} \quad r/l_f \ll 1, \\ \underbrace{2\epsilon_u r}_{\text{upscale energy}} + O\left((r/l_f)^{-1/2}\right), & \text{when} \quad r/l_f \gg 1. \end{cases}$$

Oceanic turbulence

- The energy injection scale and energy flux direction of oceanic flow is unknown.
- Can we obtain these information from measured data?



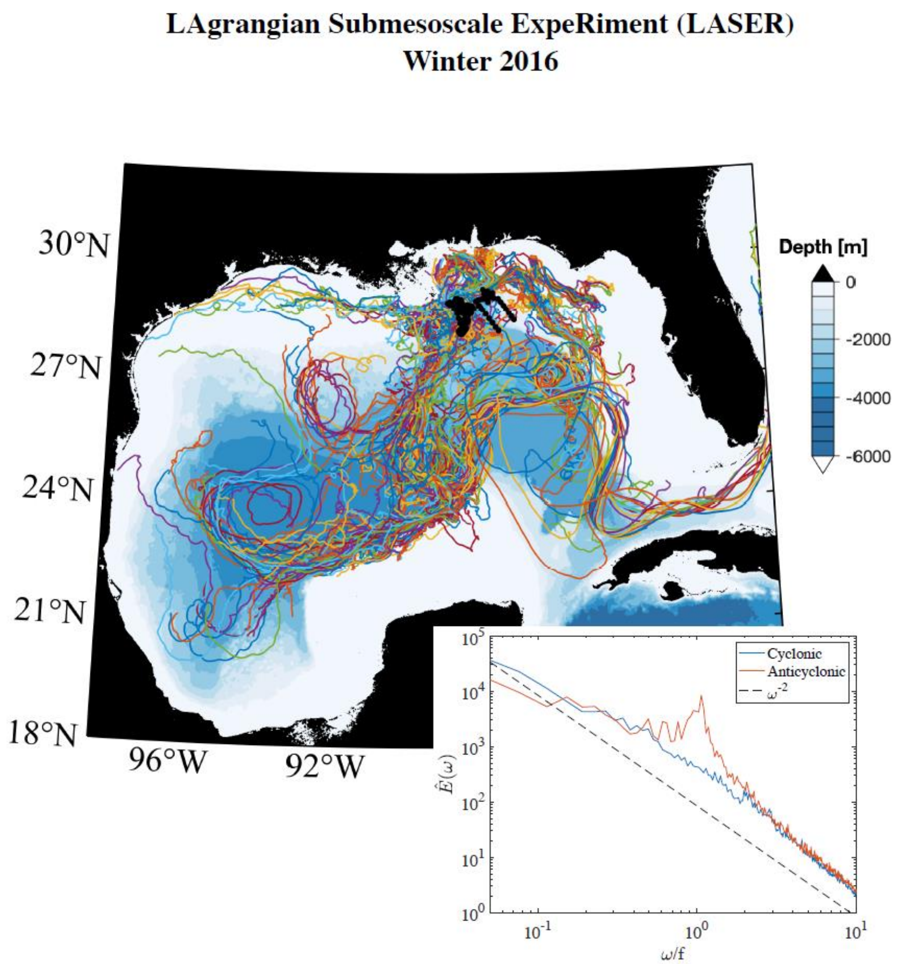


Figure: ocean drifter experiments. (with Balwada, Marino and Feraco)

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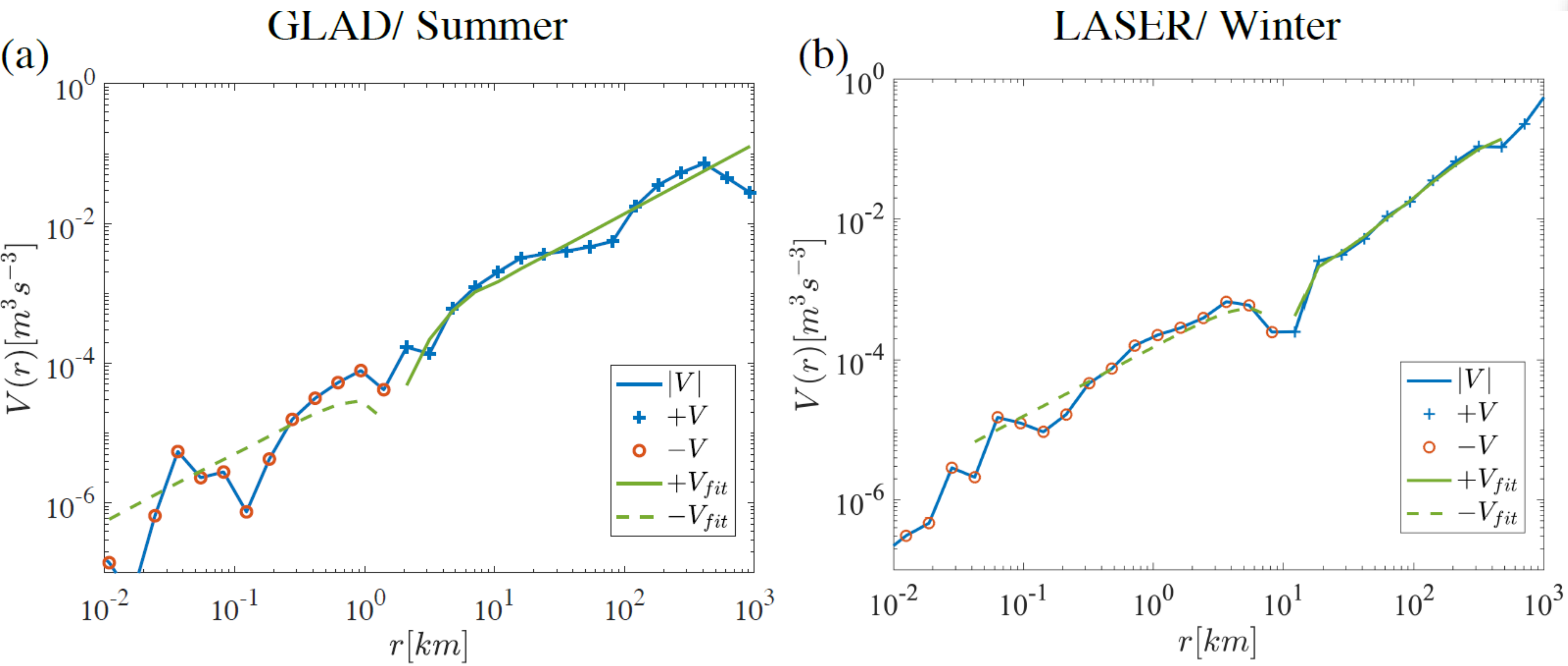

$$V = 2\epsilon_{\mathrm{u}} r - 4 \sum_{j=1}^{N} \frac{\epsilon_j}{k_{fj}} J_1(k_{fj} r)$$

Figure: third-order structure functions from measurements.

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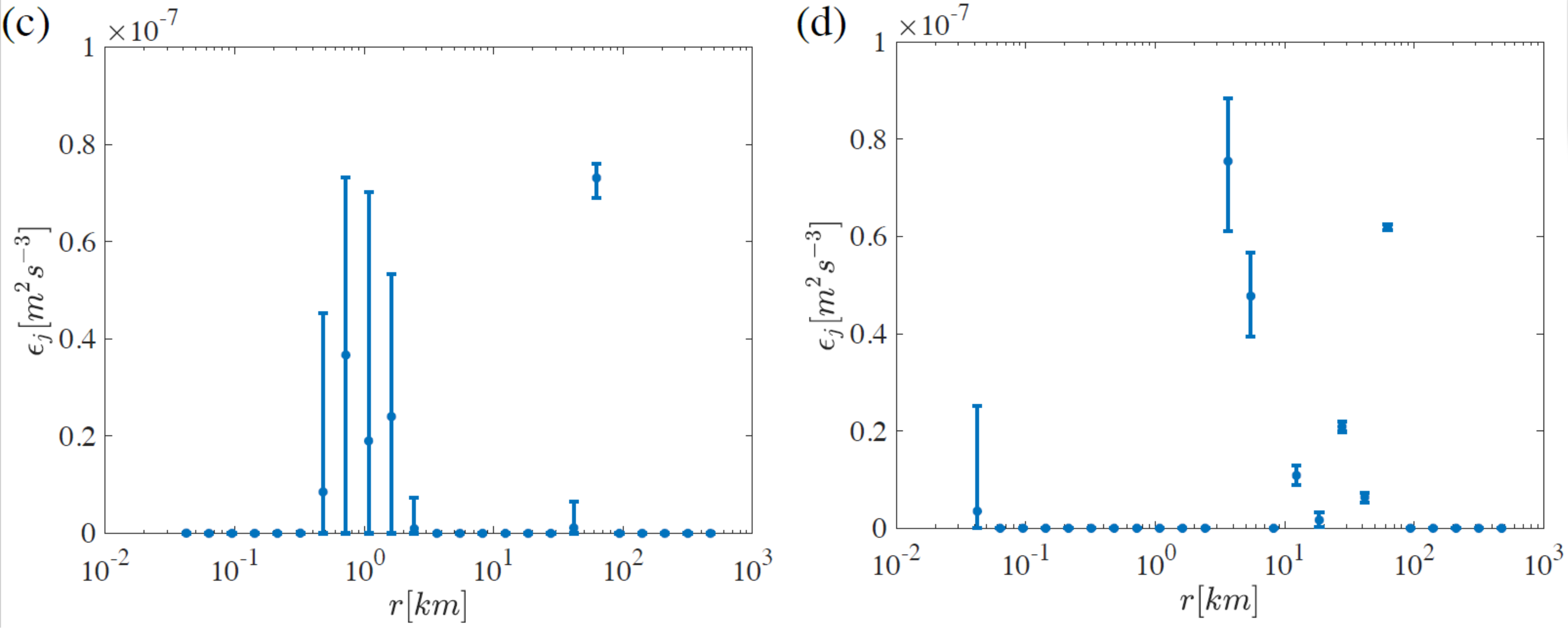

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Figure: detected energy injection at different scales.

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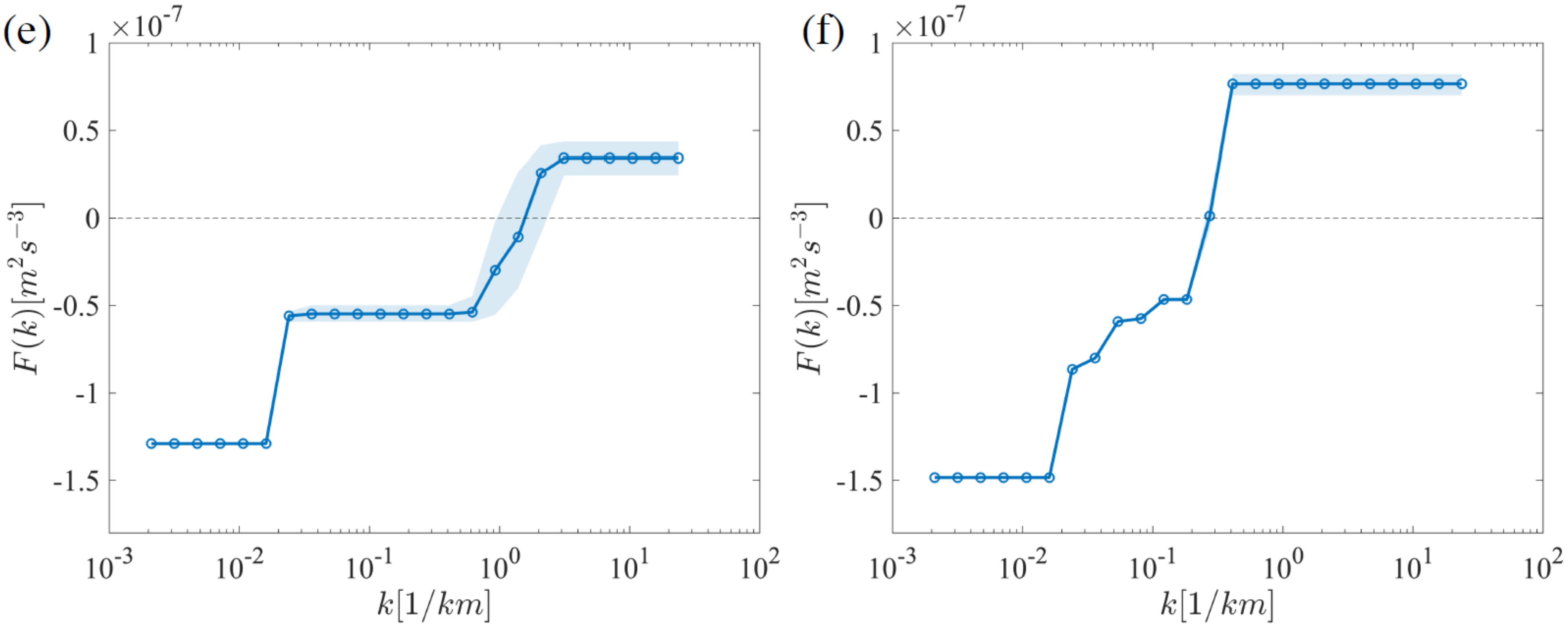

$$V = 2\epsilon_{\mathrm{u}} r - 4\sum_{j=1}^{N} \frac{\epsilon_j}{k_{fj}} J_1(k_{fj} r)$$

Figure: energy flux across scales.

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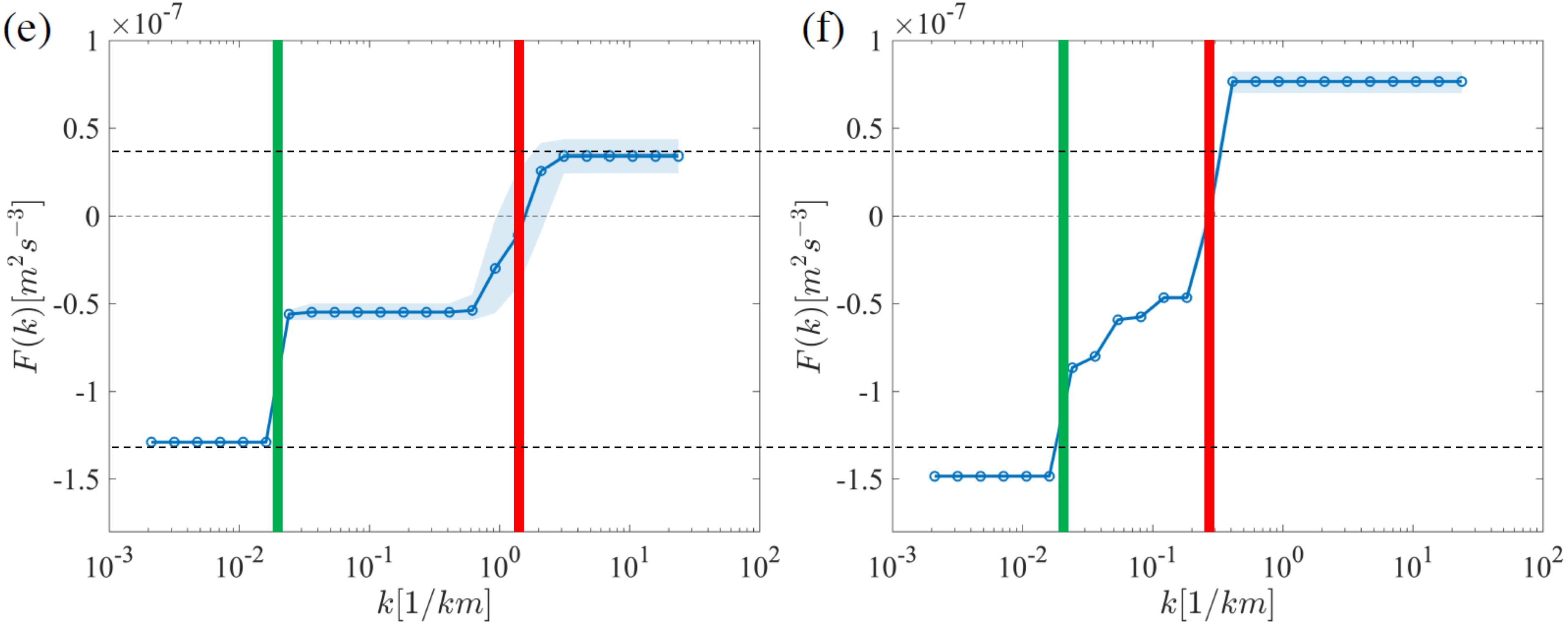

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Figure: energy flux across scales.

baroclinic instability?:

Ocean depth:

Mixed layer:

- Large and small-scale energy injection corresponds to baroclinic instability associated with ocean depth and mixed-layer depth, respectively. (winter has deeper mixed layer)

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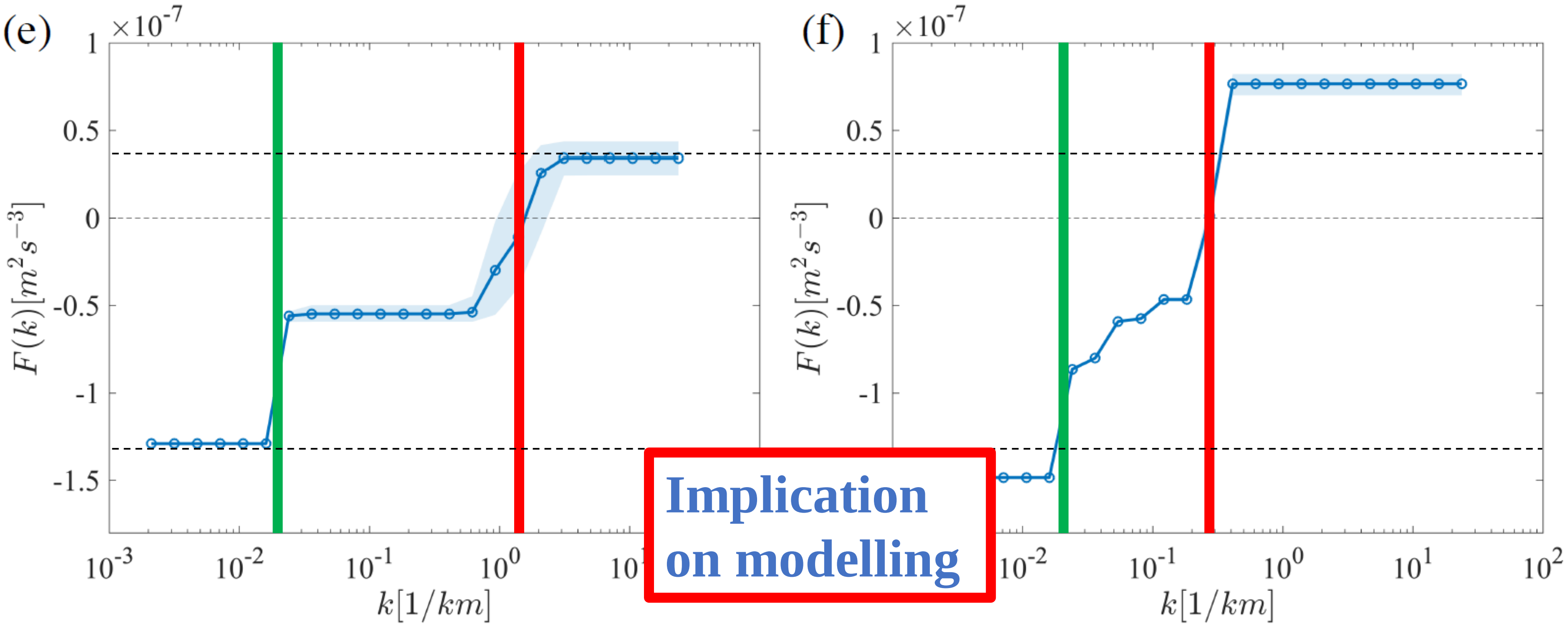

Implication on modelling

$$V = 2\epsilon_{\mathrm{u}} r - 4\sum_{j=1}^{N} \frac{\epsilon_j}{k_{fj}} J_1(k_{fj} r)$$

Figure: energy flux across scales.
baroclinic instability:
Ocean depth: (green line)
Mixed layer: (red line)

- Large and small-scale energy injection corresponds to baroclinic instability associated with ocean depth and mixed-layer depth, respectively. (winter has deeper mixed layer)

https://arxiv.org/abs/2202.08637

Summary

- For third-order structure function in *isotropic* turbulence we obtain a *global forcing-scale-resolving* theory that captures the *bidirectional energy transfer.*
- Application to drifter measurements in GoM, we find
  - Bidirectional energy flux;
  - Stronger downscale flux and larger energy-injection scale in the winter;
  - Forcing scales relate to deformation radius.
- Flaws: inhomogeneity, anisotropy, rotation, stratification, mean flow…

Thank you very much!