



Estimation of Ephemeris from Lunar Laser Ranging

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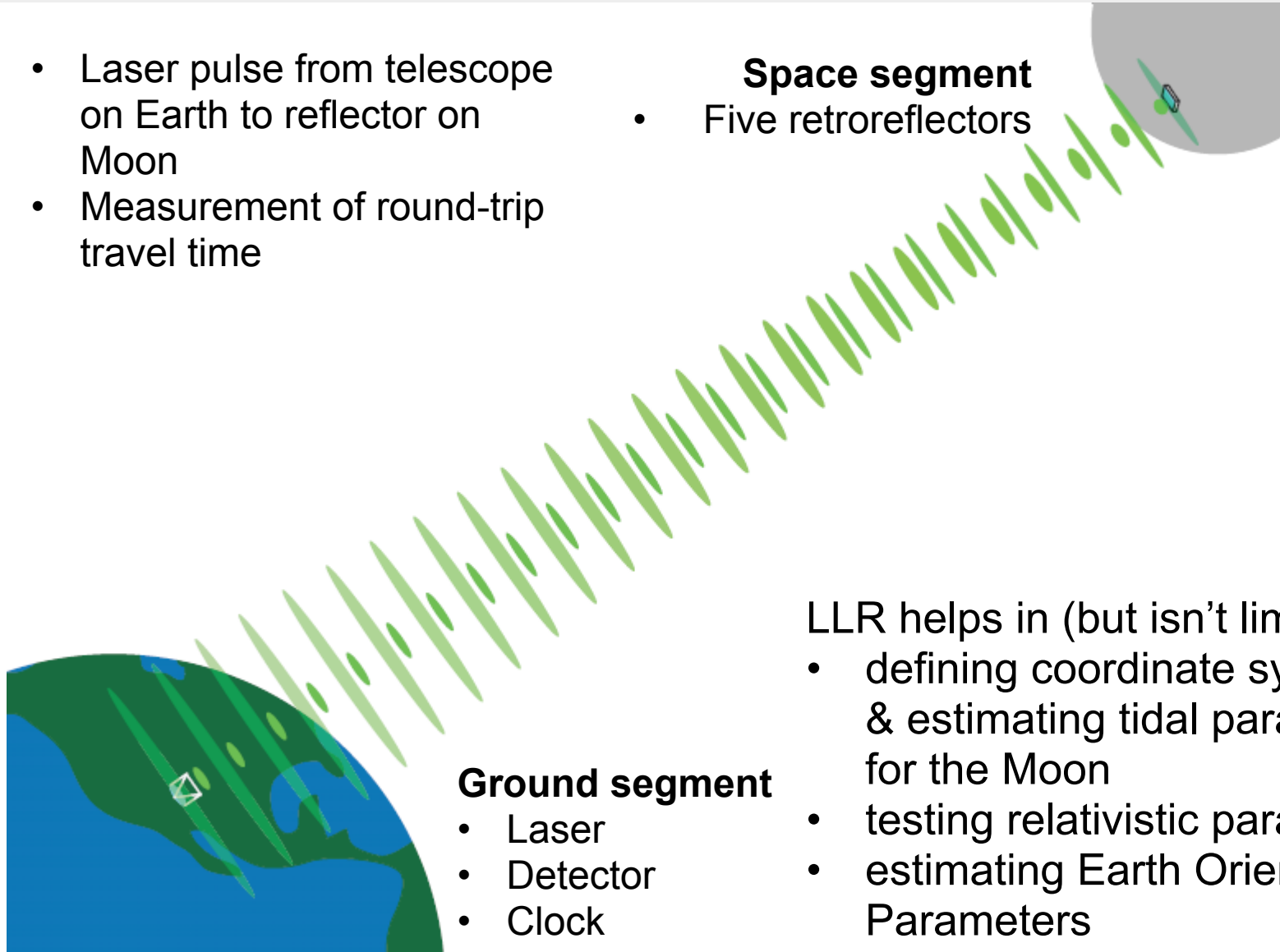
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Principle of LLR

- Laser pulse from telescope on Earth to reflector on Moon
- Measurement of round-trip travel time

- **Space segment**
 - Five retroreflectors



Ground segment

- Laser
- Detector
- Clock

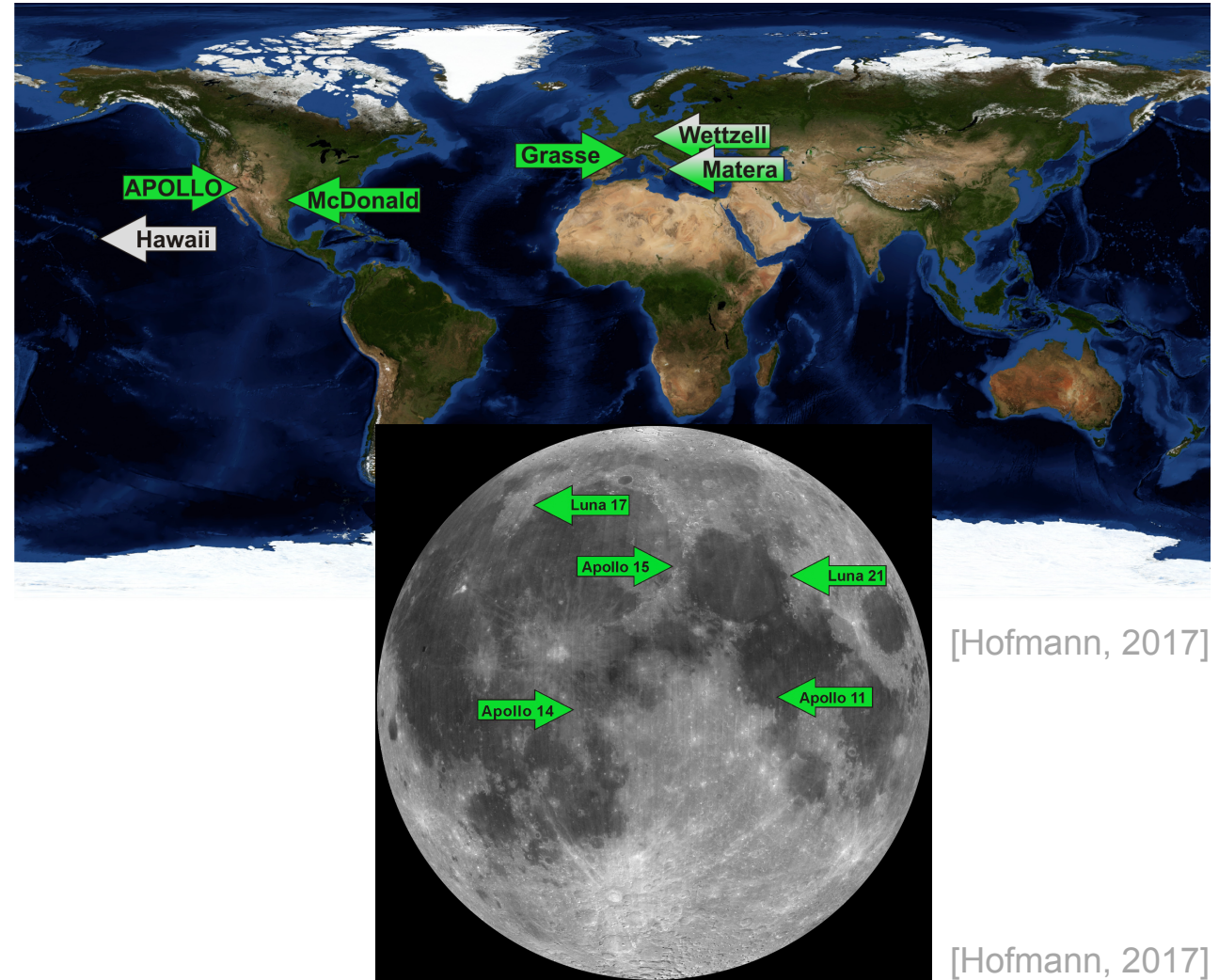
LLR helps in (but isn't limited to):

- defining coordinate systems & estimating tidal parameters for the Moon
- testing relativistic parameters
- estimating Earth Orientation Parameters

[Murphy, 2013]

LLR Analysis

- Ephemeris calculation
 - DE440 like ephemeris
 - 54 year timespan (1969 - 2023)
- Parameter estimation
 - Data modelling (station and reflector coordinates for current epoch, etc.)
 - Least squares adjustment (LSA)
 - ~ 200 parameters (can be) determined (coordinates, spherical harmonic coefficients, lunar dynamical parameters, etc.)
 - Calculation of LLR residuals



Motivation + Description

Calculation of ephemeris starting from 2000 (2-way) instead of 1969 (1-way)

- Benefit from long time span of NPs (1960 - present) and higher accuracy of NPs (over the years) for lunar initial orientation determination:
 - Position
 - Velocity
 - Euler angles (mantle)
 - Angular velocity (mantle and core)
- Reduce time consumption of ephemeris calculation

- Standard model includes: Sun, Moon, 8 planets, Pluto, Ceres, Vesta, and Pallas
- Initial position and velocity for all bodies from DE440 ASCII files
- GM for all bodies from (Park et al. 2021)
- Ephemeris model DE430 like, undistorted total MOI matrix of the Moon like INPOP17

1way = Ephemeris calculation from 1969 to 2023

2way = Ephemeris calculation from 2000 to 1969 and from 2000 to 2023

MOI: moment of inertia

Estimated Parameters

Initial Orientation Parameters		JD 2440400.5 (1-way) 3- σ	JD 2451544.5 (2-way) 3- σ	Change
Position [m]	x	0.27033	0.0663	75.49 %
	y	0.0957	0.0606	36.65 %
	z	0.06351	0.0186	71.09 %
Velocity [m/s]	x	3.3513E-07	1.5462E-07	53.86 %
	y	6.7260E-07	1.6815E-07	75.00 %
	z	4.0491E-07	1.6050E-07	60.36 %
Euler angles - mantle [rad]	1	2.8010E-08	2.6153E-08	6.63 %
	2	1.3944E-08	8.1732E-09	41.39 %
	3	4.7667E-08	3.8022E-08	20.23 %

Initial Orientation Parameters		JD 2440400.5 (1-way) 3- σ	JD 2451544.5 (2-way) 3- σ	Change
Angular velocity - mantle [rad/s]	1	2.4746E-14	2.3658E-14	4.40 %
	2	1.6417E-14	1.9639E-14	-19.63 %
	3	6.5820E-16	4.2621E-16	35.25 %
Angular velocity - core [rad/s]	1	8.8187E-12	1.2779E-11	-44.90 %
	2	6.2829E-12	6.4790E-12	-3.12 %
	3	5.492E-11	4.3871E-11	20.12 %

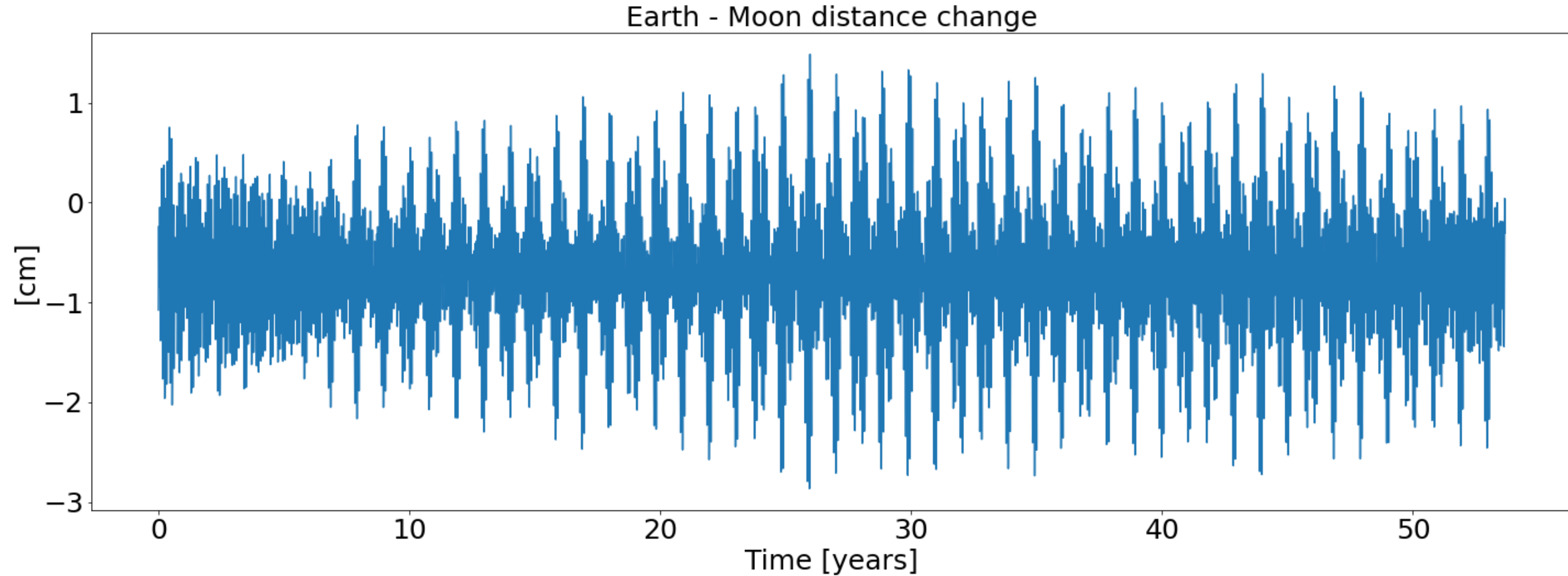
All other estimated parameters (station and reflector positions, lunar spherical harmonic coefficients, tidal time delay, etc.) show an improvement of 7% - 15% in estimated uncertainties

1way = Ephemeris calculation from 1969 to 2023
2way = Ephemeris calculation from 2000 to 1969 and from 2000 to 2023

Euler angles and angular velocities estimated in lunar mantle frame

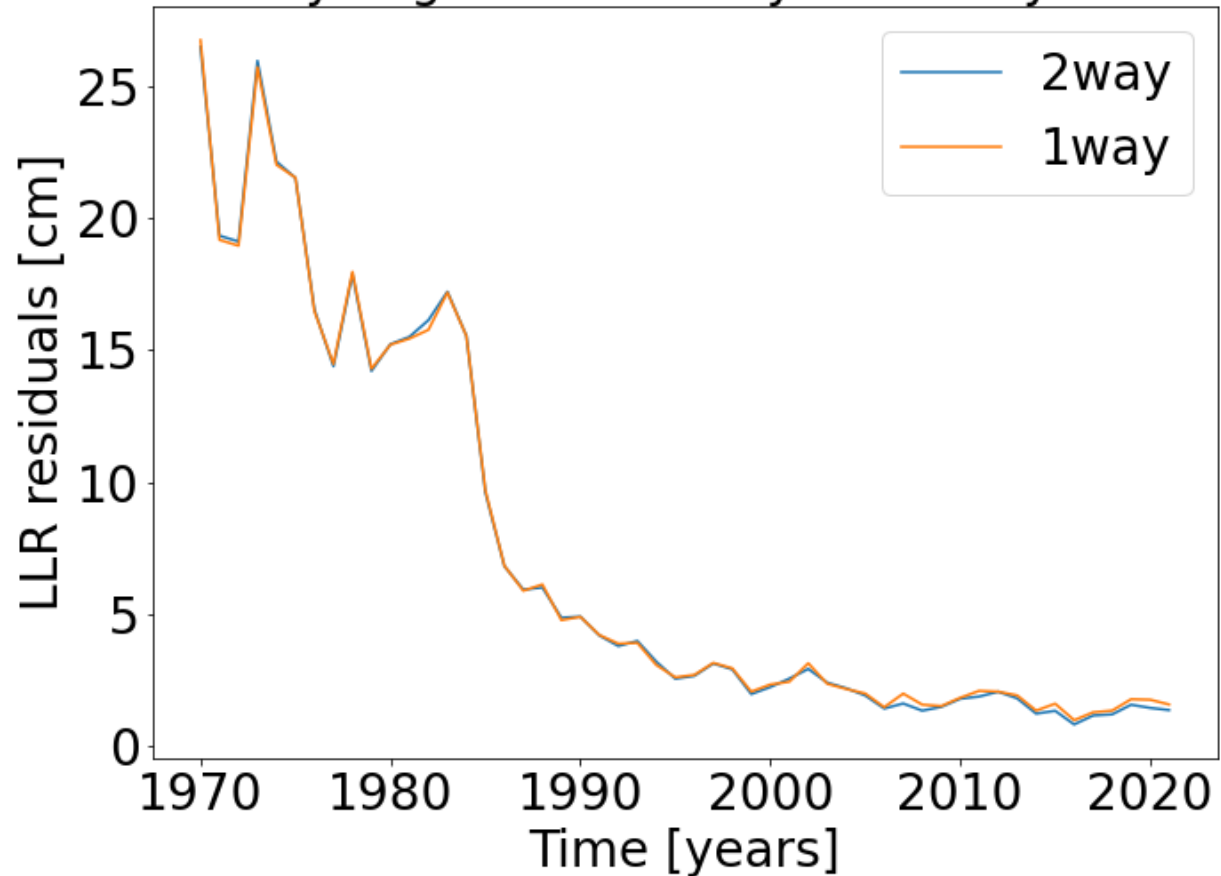
Lunar position and velocity estimated in geocentric inertial frame

Earth-Moon Distance Change

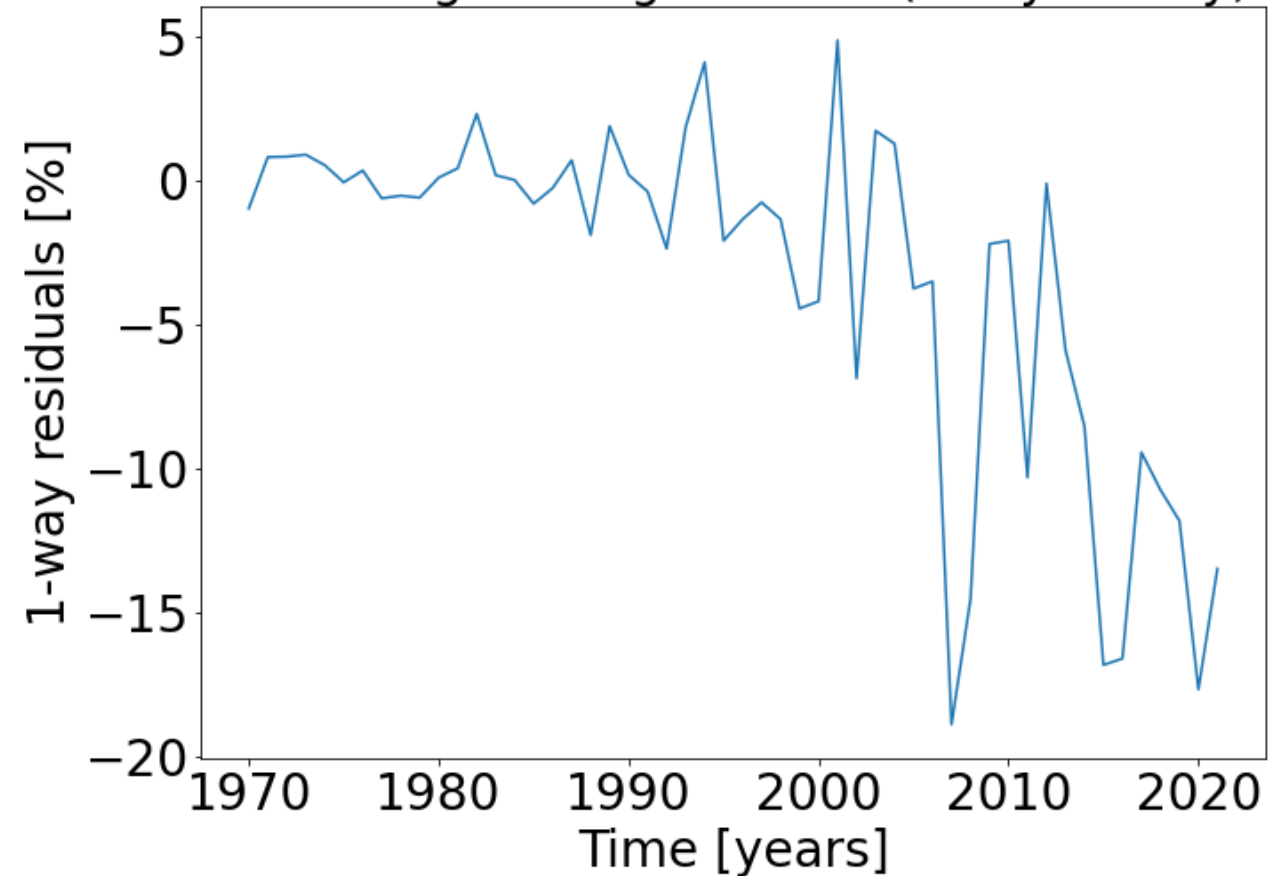


LLR Residuals

Yearly avg. WRMS: 1way and 2way calc.



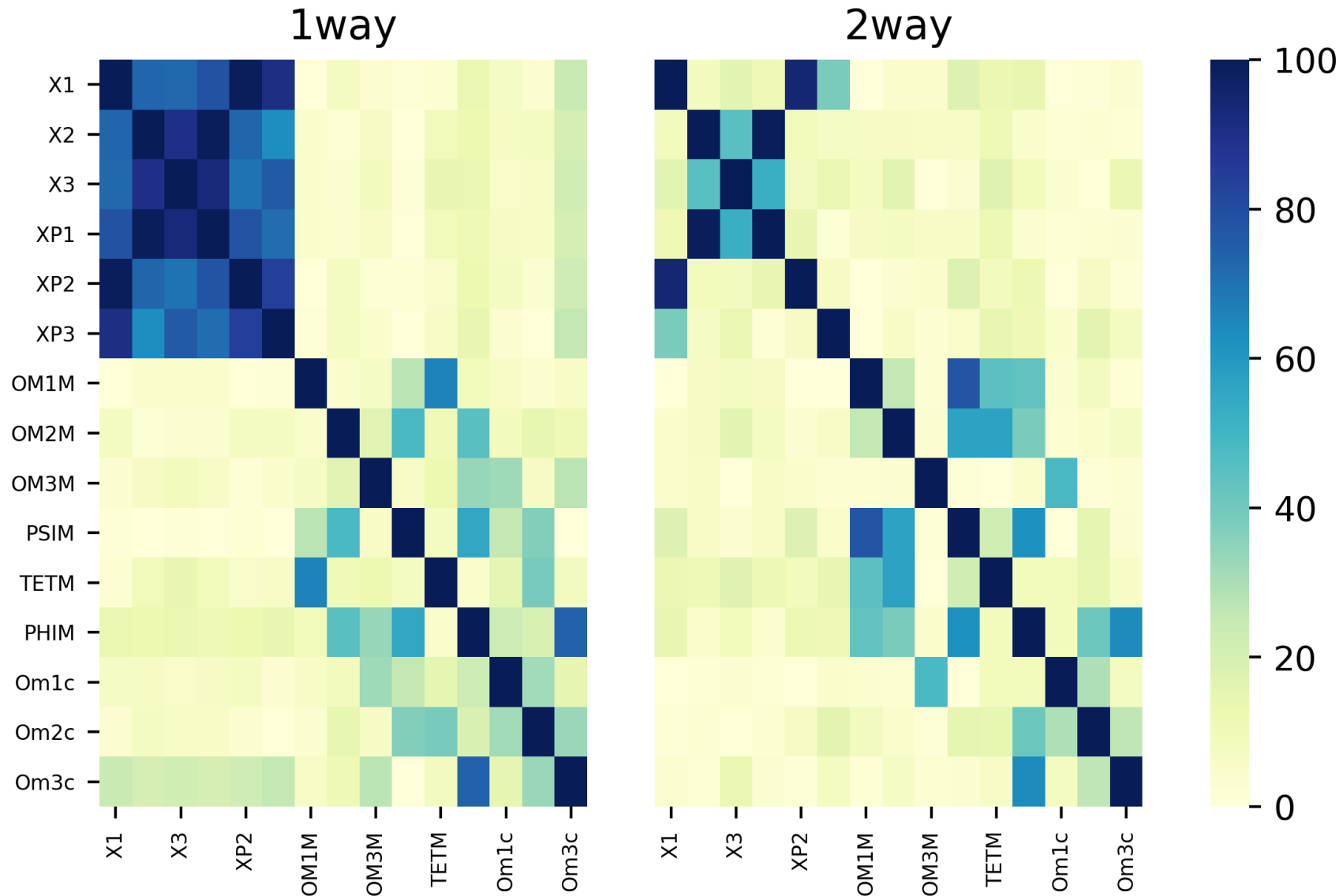
Percentage change: WRMS (2way - 1way)



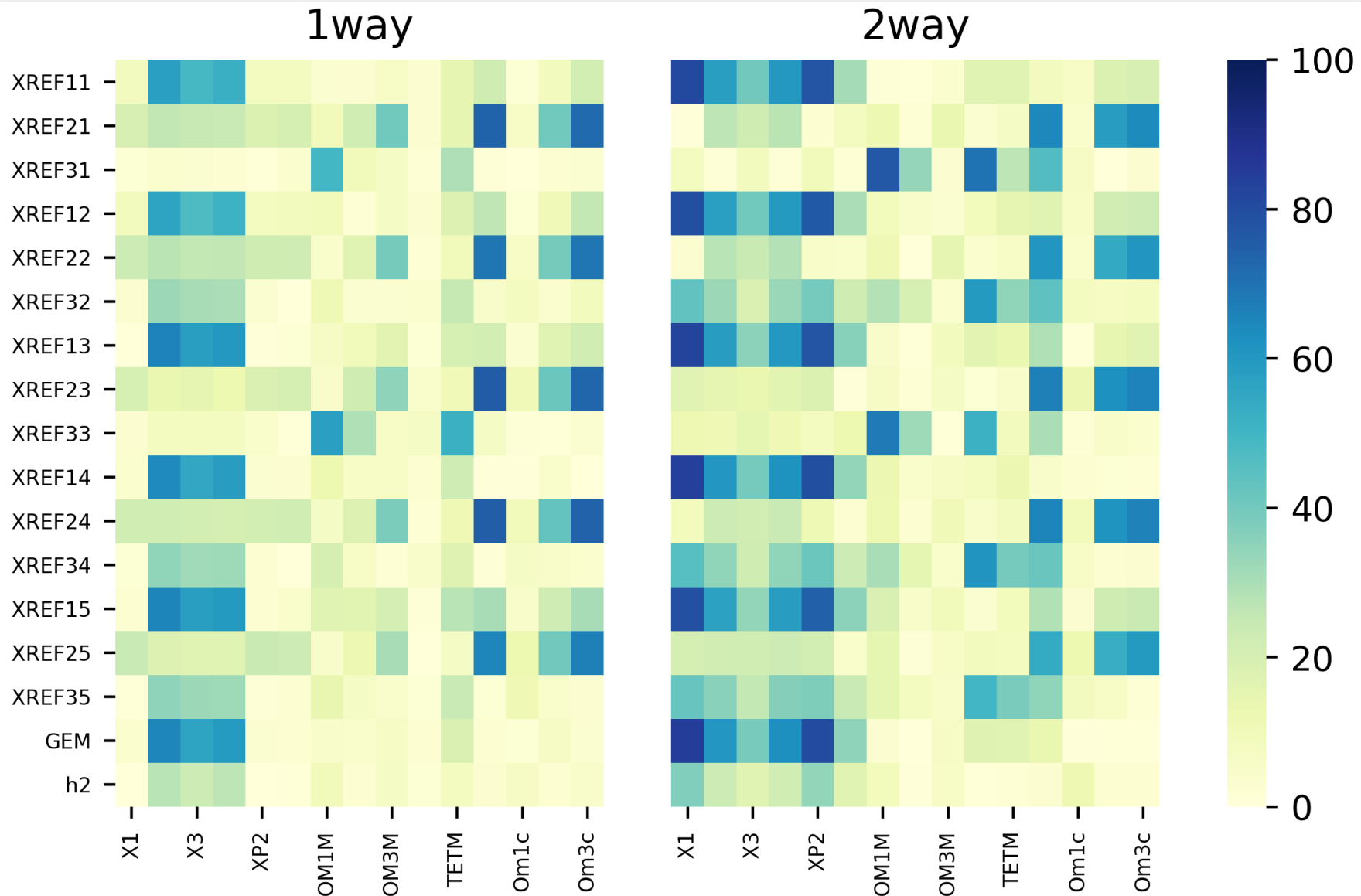
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Negative percentage = Smaller residuals = Better results

Correlations



Correlations



Further Studies and Conclusion

- Correlation of other parameters stay similar (<30% changes)
- LLR analysis using ephemeris starting in the years 1999 and 2022 show similar results (improvements in uncertainties of estimated parameters, changed correlations, etc.)
- Calculation of ephemeris starting JD 2451544.5:
 - Improves uncertainties of estimated parameters
 - Improves WRMS of LLR residuals
 - Changes correlations between parameters
 - Changes EM distance by ~7 mm
 - Reduces computation time to ~half

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Bibliography

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