



Characteristics of Differential Lunar Laser Ranging

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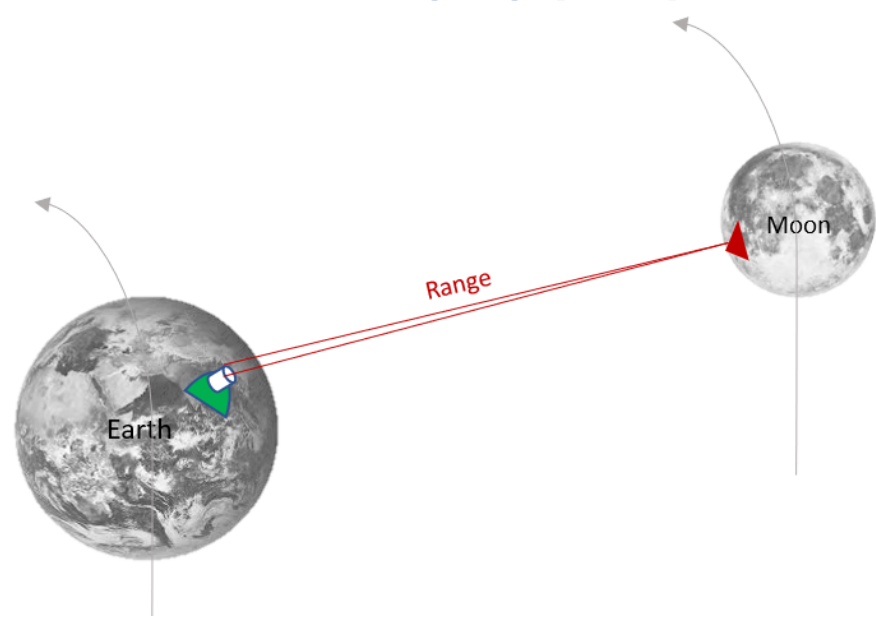
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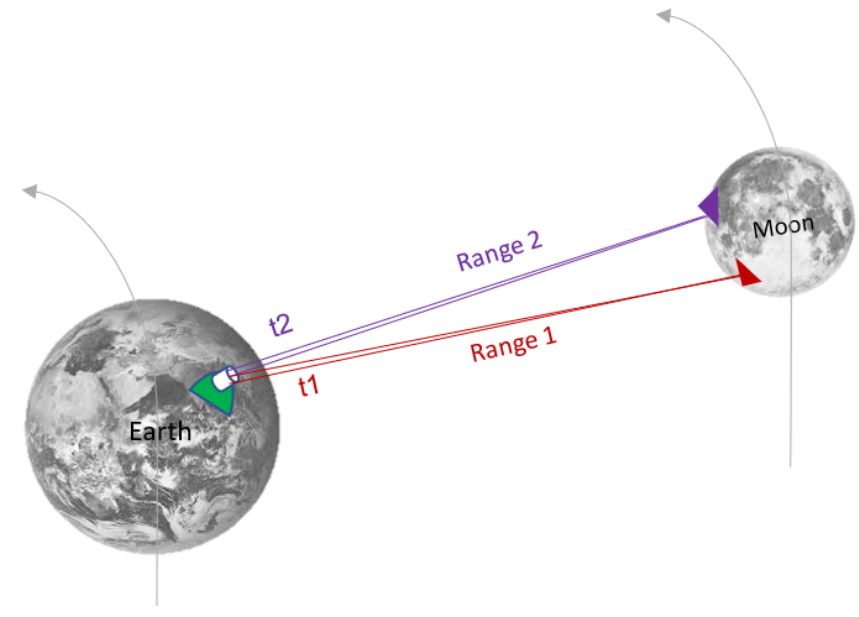


Concept of Differential Lunar Laser Ranging

Lunar Laser Ranging (LLR)



Differential Lunar Laser Ranging (DLLR)



Technique	Constellation	Station switching (for one measurement)	Observation	Accuracy
LLR	one station one reflector	no	range	~8 mm (APOLLO, single shot)
DLLR	one station two or more reflectors	yes small interval: 1.5 min large interval: 15 min	range difference	~30 um

Advantages of DLLR

- significantly reduces the atmospheric error
- better knowledge of the lunar interior
- is beneficial for some relativity tests

DLLR Simulation

- data - basic information

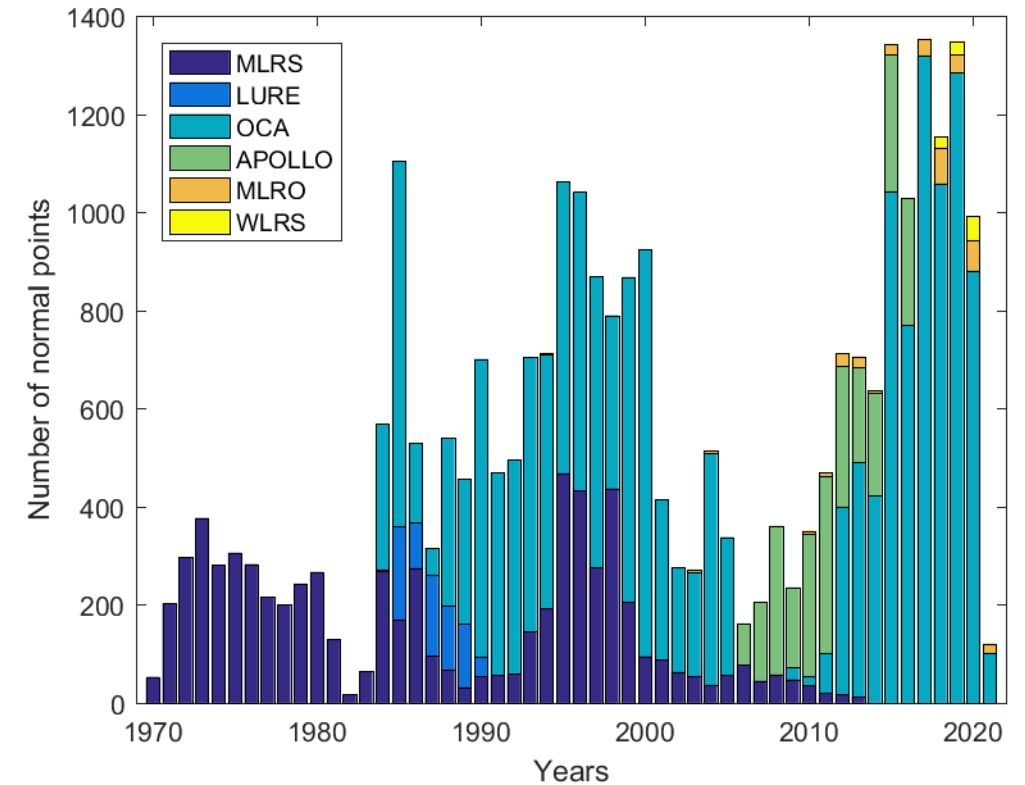
- number of simulated range differences: 28093
- time span: 04.1970-03.2021
- stations: MLRS, LURE, WLRS, OCA, APOLLO, MLRO
- reflectors: Apollo 11, 14, 15; Lunokhod 1, 2

- data distribution

- same as real LLR data
 - first time point: from real LLR data
 - second time point:
 - the first one + switching interval (e.g., 1.5 min)

- data accuracy

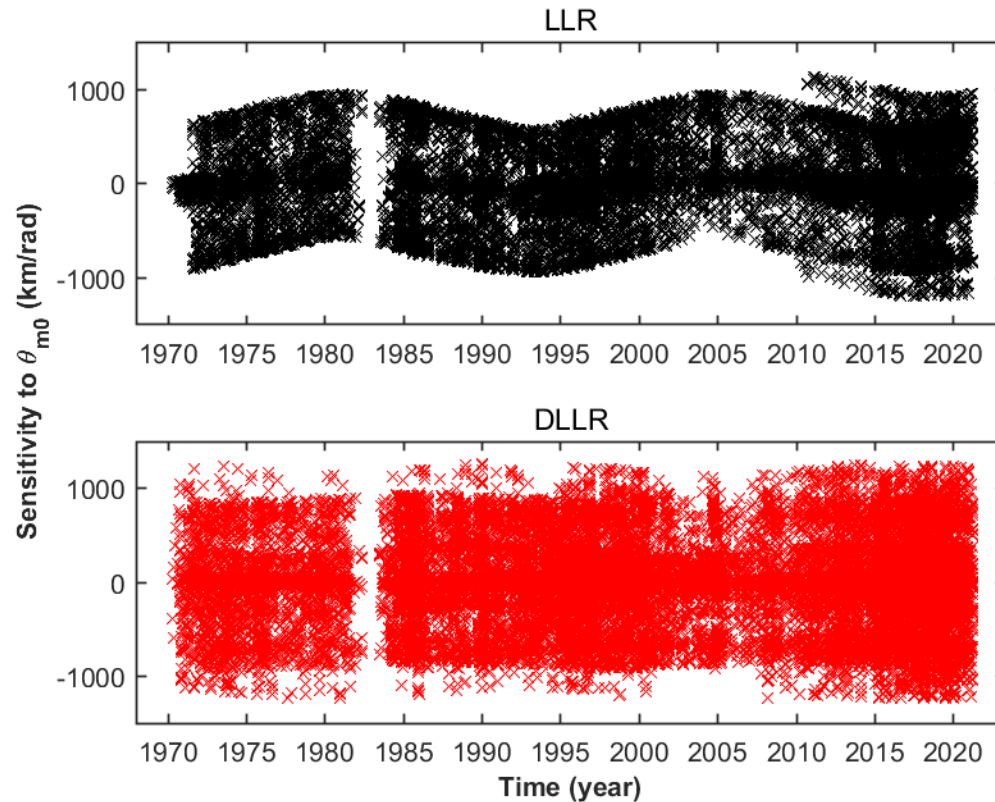
increased 200 times compared to real LLR range accuracy



Distribution of real LLR data

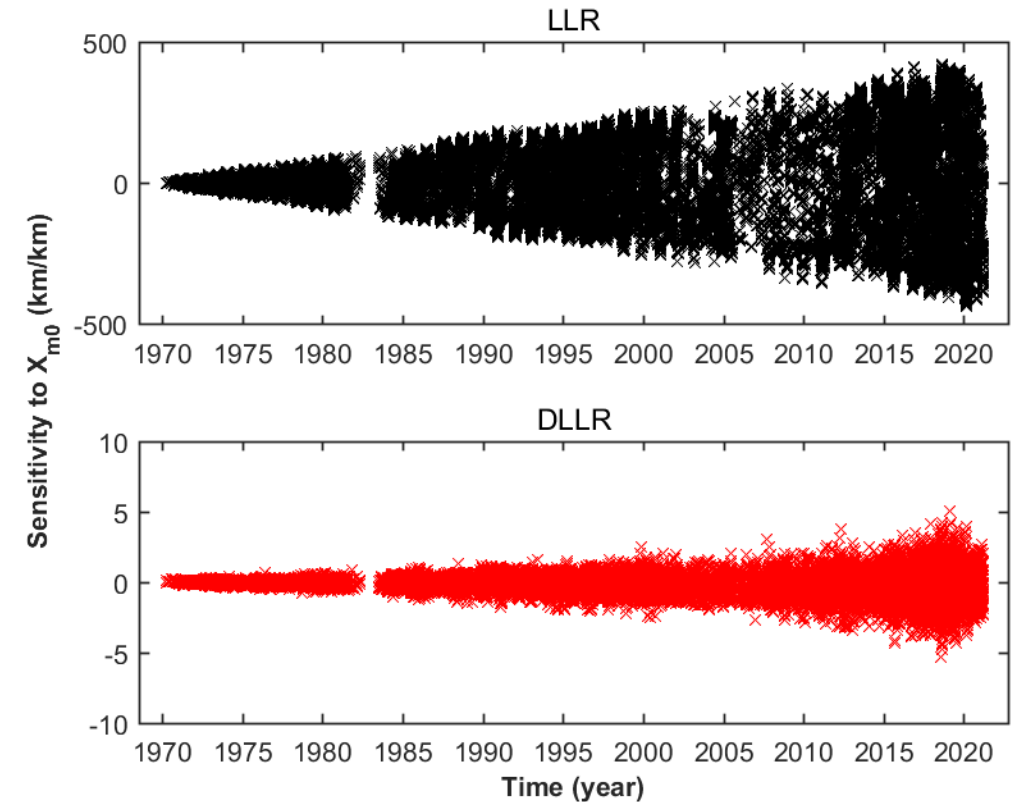
Sensitivity analysis

Sensitivity (examples)



Lunar orientation: here, the inclination of the lunar equator from the inertial XY plane (JD 2440400.5)

Group A: same sensitivity for LLR and DLLR



Lunar orbit: here, the x-component of the lunar position (JD 2440400.5)

Group B: DLLR is less sensitive than LLR

Parameter estimation

Newtonian parameters

Parameters	LLR	DLLR
	(1970-2021, data number 28093)	(1970-2021, data number 28093)
X_{m0}	8.5×10^{-5}	2.3×10^{-5}
Y_{m0}	3.0×10^{-5}	1.9×10^{-5}
Z_{m0}	2.0×10^{-5}	1.1×10^{-5}
ϕ_{m0}	8.8×10^{-9}	6.0×10^{-11}
θ_{m0}	4.4×10^{-9}	1.4×10^{-11}
ψ_{m0}	1.5×10^{-8}	1.0×10^{-10}

Relativistic parameters: Equivalence Principle (EP)

EP	LLR	DLLR
	(1970-2021, data number 28093)	(1970-2021, data number 28093)
Sun $\delta \left(\frac{m_g}{m_i} \right)_{EM}$	$(-0.6 \pm 2.1) \times 10^{-14}$	$(-0.5 \pm 0.9) \times 10^{-14}$
Galactic dark matter $\delta a_{dm} \text{ (m/s}^2\text{)}$	$(2.3 \pm 4.1) \times 10^{-17}$	$(0.05 \pm 1.4) \times 10^{-17}$

Combination of LLR and DLLR

Data group	Time span	Number of observations
real LLR	1970-2021	28093
real LLR + simulated LLR	1970-2026	36758 (28093 real LLR, 8665 simulated LLR)
real LLR + simulated LLR + simulated DLLR	1970-2026	45423 (28093 real LLR, 8665 simulated LLR, 8665 simulated DLLR)

Parameter Data group	X_{m0} [km]	Y_{m0} [km]	Z_{m0} [km]	ϕ_{m0} [rad]	θ_{m0} [rad]	ψ_{m0} [rad]	h_2	τ_m [sec]
real LLR	8.5×10^{-5}	3.0×10^{-5}	2.0×10^{-5}	8.8×10^{-9}	4.4×10^{-9}	1.5×10^{-8}	2.9×10^{-4}	65.2
real LLR + simulated LLR	4.9×10^{-5}	2.0×10^{-5}	1.3×10^{-5}	5.4×10^{-9}	3.4×10^{-9}	1.1×10^{-8}	2.7×10^{-4}	51.7
real LLR + simulated LLR + simulated DLLR	4.3×10^{-5}	1.6×10^{-5}	1.0×10^{-5}	1.0×10^{-9}	3.2×10^{-10}	1.4×10^{-9}	1.3×10^{-6}	0.4

Conclusions

DLLR can achieve a 200 times better accuracy than LLR.

DLLR can enhance our knowledge of the lunar interior.

Combination of LLR and DLLR is a good and practical way to improve the estimation of certain parameters, e.g., lunar orientation parameters.

Thank you

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