On the parameterization of atmospheric convection with a realistic plume model

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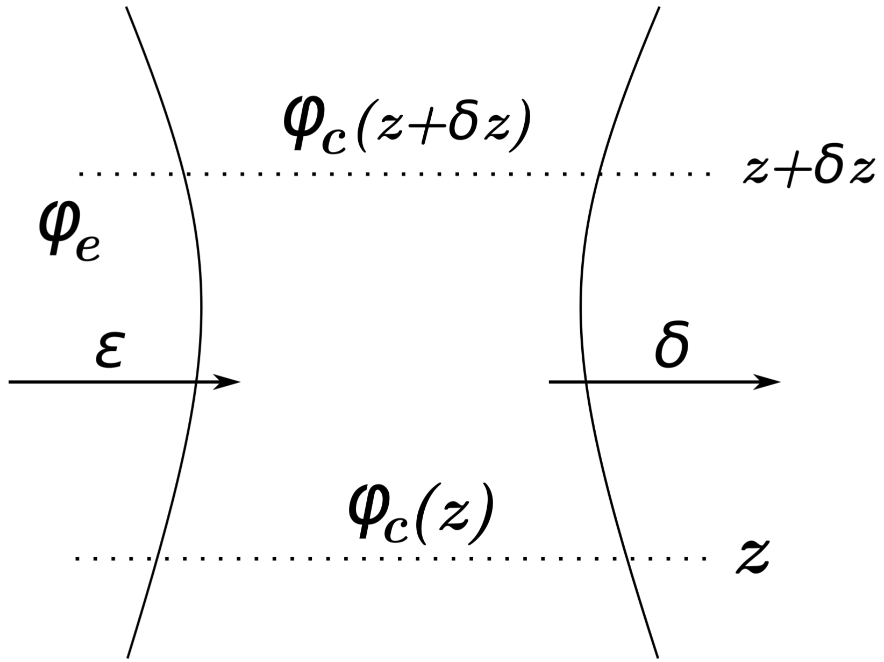
Motivation

- The parameterization of atmospheric convection is considered to be a large source of errors in the weather and climate models;
- The traditional parametrization such as AS74 [1] assumes that the vertical profile of the mass flux follows an exponential law, which requires closure for the mass flux at the cloud base;
- More recently, the one–dimensional Lagrangian model is used in order to avoid the closing problem. However, the Lagrangian model still requires parameterization for the entrainment and detrainmnet;
- In this work I show that by considering, in addition, the conservation of the kinetic energy, as in the PB55 [2] formalism, the parameterization of entrainmnet and detrainment is avoided, and a generalization of the Lagrangian model can be obtained.

[1] A. Arakawa, and W. H. Schubert. "*Interaction of a cumulus cloud ensemble with the large-scale environment, Part I*." Journal of the atmospheric sciences 31.3 (1974): 674-701. (AS74)
[2] C. H. B Priestley and F. K. Ball. "*Continuous convection from an isolated source of heat.*" Quarterly Journal of the Royal Meteorological Society 81.348 (1955): 144-157. (PB55)

The one–dimensional Lagrangian model



$$\frac{1}{2}\frac{d}{dz}w_c^2 = B - \Lambda w_c^2,$$

$$\frac{d\varphi_c}{dz} = \frac{F_{\varphi c}}{w_c} - \Lambda(\varphi_c - \varphi_e),$$

$$\Lambda = \frac{1}{\mu}\frac{d\mu}{dz},$$

It is assumed the entrainment hypothesis: $\frac{1}{\mu}\frac{d}{dz}\mu = \frac{\alpha_e}{b},$

R. A. Houze Jr, *Cloud dynamics*. Academic Press, 2014.

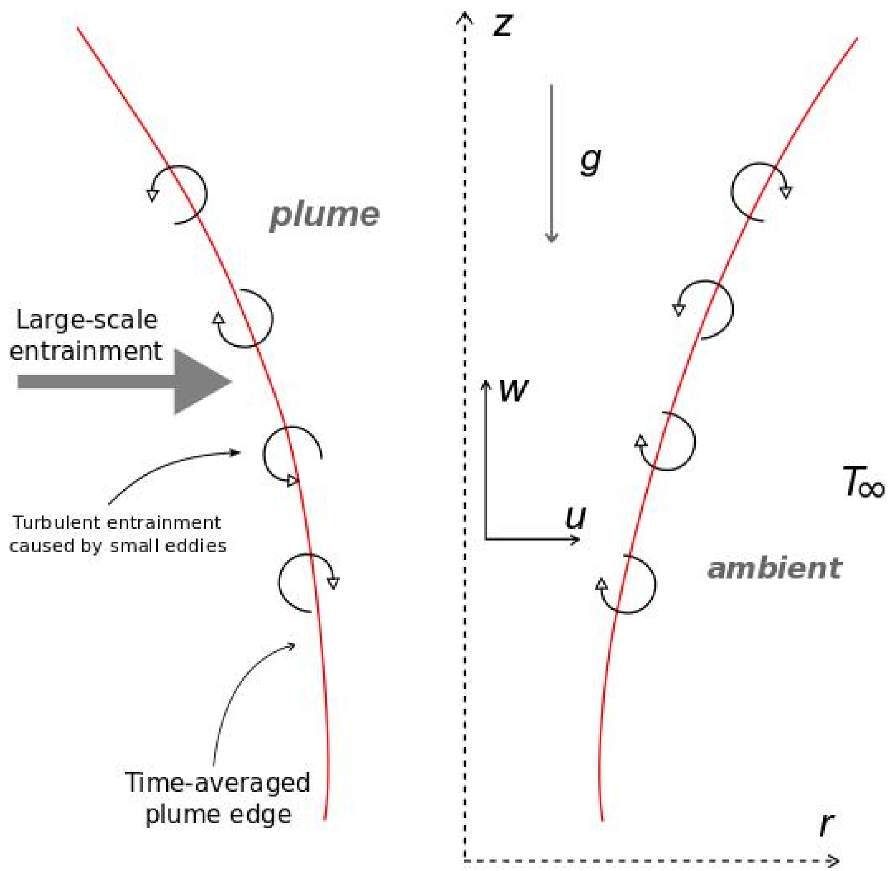
The proposed plume model

$$\frac{\partial}{\partial z}(r\rho w') + \frac{\partial}{\partial r}(r\rho u_r') = 0,$$

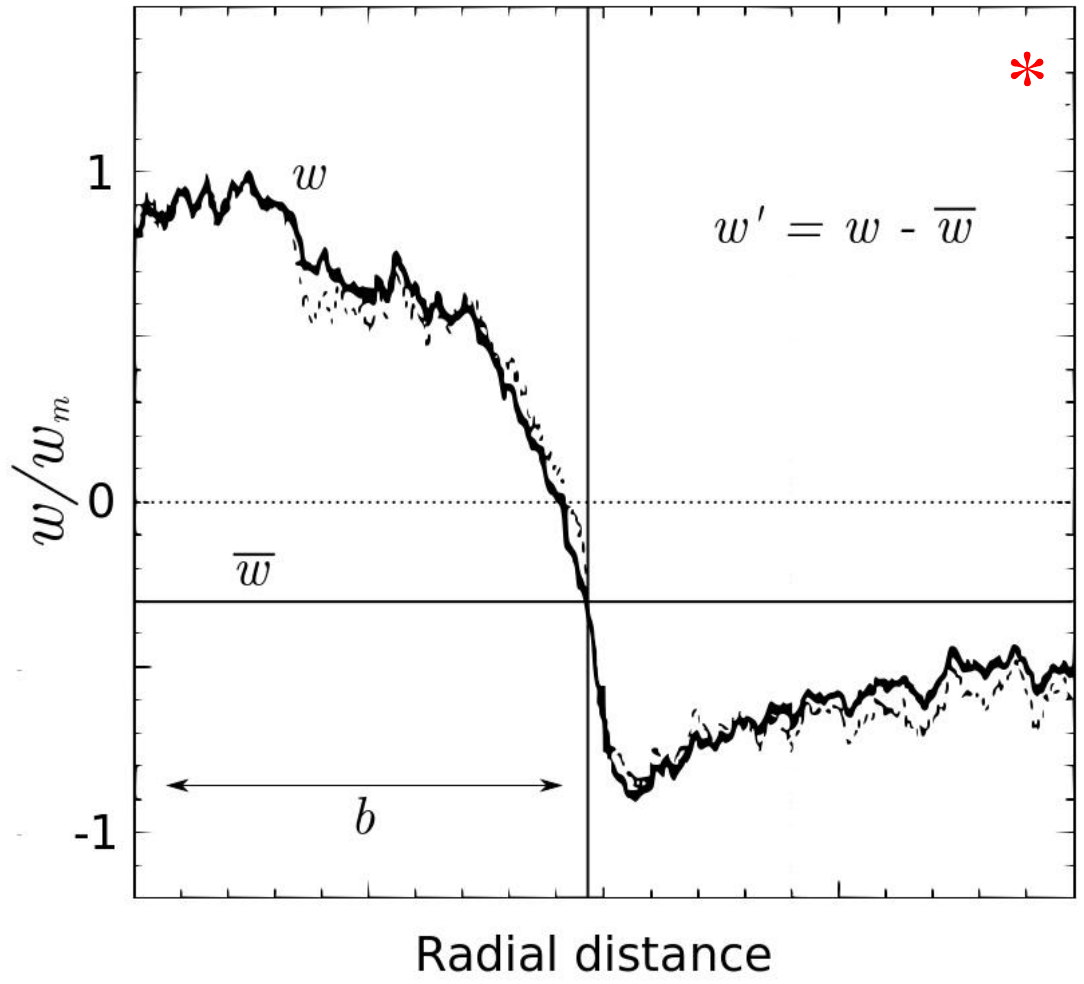
$$\frac{\partial}{\partial z}(r\rho w'^2) + \frac{\partial}{\partial r}(r\rho u_r' w') = r\rho B - \frac{\partial}{\partial r}(r\rho \tau_w),$$

$$\frac{\partial}{\partial z}\left(\frac{1}{2} r\rho w'^3\right) + \frac{\partial}{\partial r}\left(\frac{1}{2} r\rho u_r' w'^2\right) = r\rho w' B - w'\frac{\partial}{\partial r}(r\rho \tau_w),$$

$$\frac{\partial}{\partial z}(r\rho \varphi' w') + \frac{\partial}{\partial r}\left(r\rho u_r' \varphi'\right) = -\frac{\partial}{\partial r}(r\rho \tau_\varphi) - r\rho w'\frac{\partial \overline{\varphi}}{\partial z} + r\rho F_\varphi',$$



The proposed plume model



$$\overline{w'} = \overline{\varphi'} = 0$$

We assume generic self–similar radial profiles:

$$w' = w'_c f_w\left(\frac{r}{b}\right);\ \varphi' = \varphi'_c f_\varphi\left(\frac{r}{b}\right);\ u'_r = u'_c f_u\left(\frac{r}{b}\right);$$
$$\tau_w = w'^2_c f_\tau\left(\frac{r}{b}\right)\ ;\tau_\varphi = w'_c\varphi'_c j_\varphi\left(\frac{r}{b}\right),$$

* Based on the airborne measurements of C. Mallaun, A. Giez, G. J. Mayr, and M. W. Rotach. *Subsiding shells and the distribution of up–and down–draughts in warm cumulus clouds over land*. Atmospheric Chemistry and Physics, 19(15):9769 – 9786, 2019.

Conservation equations

$$\frac{dw_c'^2}{dz} = \frac{a_2}{a_1} B_c' - \left[\frac{\eta_1}{b} + 2 \frac{d(\ln b)}{dz} + \frac{d(\ln \rho)}{dz} \right] w_c'^2,$$

$$\frac{d\varphi_c'}{dz} = \frac{1}{a_6 w_c' b_k^2} J_c^\varphi - \frac{a_7}{a_6} \frac{\partial \overline{\varphi}}{\partial z} - \left[\frac{\eta_2}{b} + \eta_3 \frac{d(\ln b)}{dz} + \eta_4 \frac{d(\ln \rho)}{dz} \right] \varphi_c',$$

$$\frac{dB_c'}{dz} = -N^2 - \left[\frac{\eta_2}{b} + \eta_3 \frac{d(\ln b)}{dz} + \eta_4 \frac{d(\ln \rho)}{dz} \right] B_c',$$

$$\frac{db}{dz} = \left(\frac{3a_2}{2a_1} - \frac{2a_4}{a_3} \right) \frac{B_c' b}{w_c'^2} + \left(\frac{2a_5}{a_3} - \frac{3 f_\tau(1)}{2a_1} \right) - \frac{b}{2} \frac{d(\ln \rho)}{dz},$$

If one assume top–hat profiles, then the radial coefficients are:

$$a_1 = a_2 = a_3 = a_4 = \\ = a_6 = a_7 = 1$$

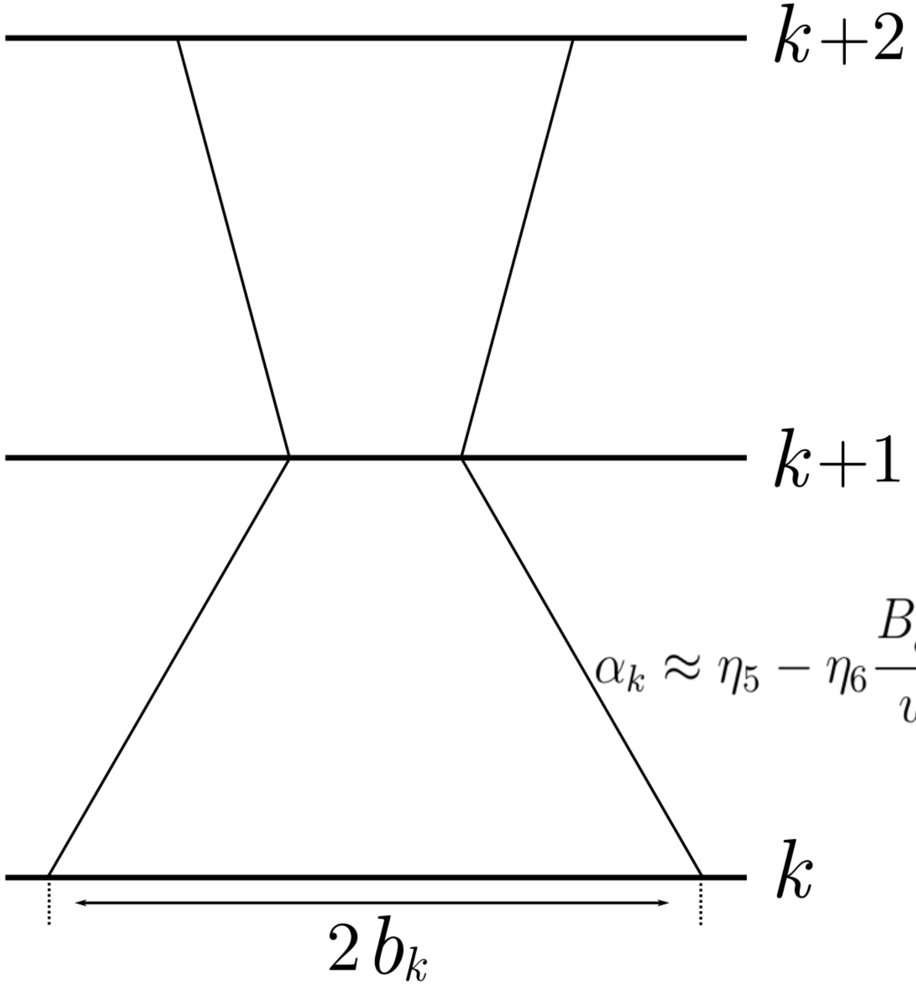
$$a_5 = \eta_1 = f_\tau(1)$$

$$\eta_2 = j_\varphi(1)$$

$$\eta_3 = \eta_4 = 0, \ \eta_5 = f_\tau(1)/2$$

$$\eta_6 = 1/2$$

Approximate analytical solutions



$$\alpha_k \approx \eta_5 - \eta_6 \frac{B'_{c,k} b_k}{w'^2_{c,k}} - \frac{b_k \gamma_k}{2\rho_k},$$

Instead of solving the system of eqs. for the whole vertical domain, a much accurate description can be obtained if one finds analytical solutions just between the vertical grids of the parent numerical model.

Between the vertical grids k and k+1, we can consider the approximations:

$$\frac{1}{b}\frac{db}{dz} \approx \frac{\alpha_k}{b_k}.$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = \frac{\gamma_k}{\rho_k},$$

Approximate analytical solutions

$$\frac{dw_c'^2}{dz} = \frac{a_2}{a_1} B_c - \Lambda_1 w_c'^2,$$

$$\Lambda_1 = \frac{\eta_1 + 2\alpha_k}{b_k} + \frac{\gamma_k}{\rho_k},$$

$$\frac{d\varphi_c'}{dz} = \frac{1}{a_6 w_c' b_k^2} J_c^\varphi - \frac{a_7}{a_6}\frac{\partial \overline{\varphi}}{\partial z} - \Lambda_2 \varphi_c',$$

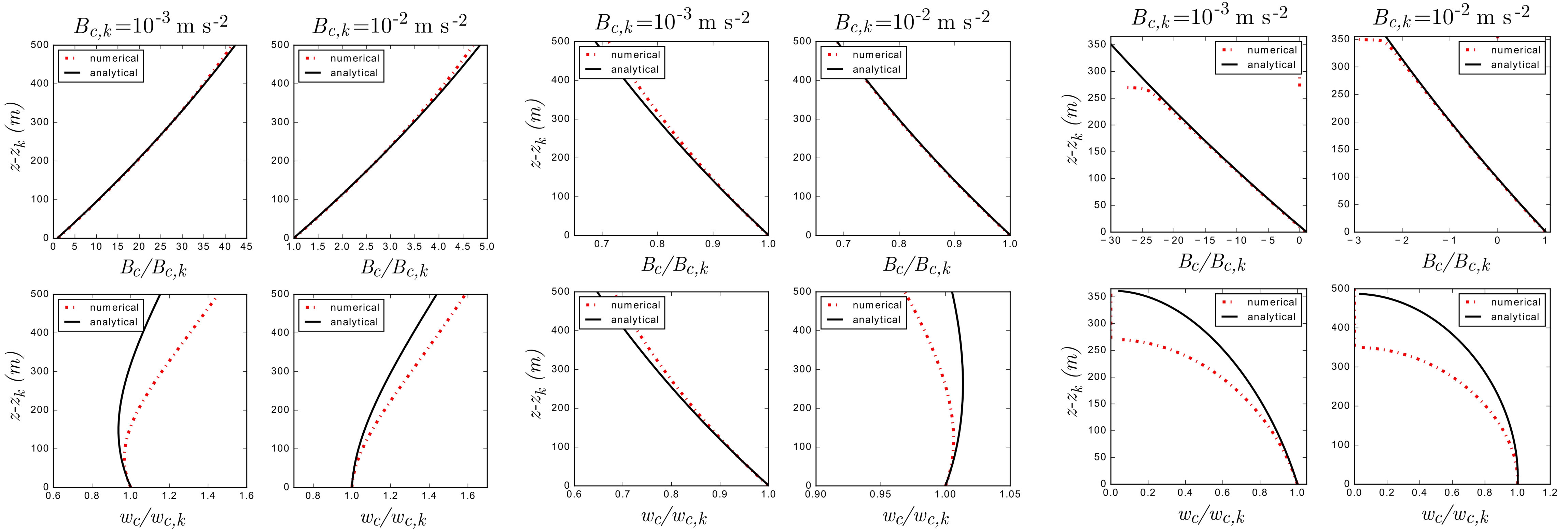
$$\Lambda_2 = \frac{\eta_2 + \eta_3 \alpha_k}{b_k} + \eta_4 \frac{\gamma_k}{\rho_k}.$$

$$w_c'^2(z) = \left[\frac{a_2}{a_1}\int_{z_k}^{z} B_c'(z') e^{\Lambda_1 (z - z_k)} dz' + w_{c,k}'^2\right] e^{-\Lambda_1 (z - z_k)},$$

$$\varphi_c'(z) = \left[\int_{z_k}^{z}\left[\frac{1}{a_6 w_c b_k^2} J_c^\varphi - \frac{a_7}{a_6}\frac{\partial \overline{\varphi}}{\partial z}\right] e^{\Lambda_2 (z - z_k)} dz' + \varphi_{c,k}'\right] e^{-\Lambda_2 (z - z_k)}.$$

Approximate analytical solutions

$N^2 = -10^{-4}$ s^{-2} $\quad$ $N^2 = 0$ $\quad$ $N^2 = 10^{-4}$ s^{-2}



The initial vertical velocity and initial radius are set to 3 m/s and 200 m, respectively. Top hat profiles have been asummed, and the vertical gradient of the density is neglected.

Convective radial velocity

In the presented approach, the continuity equation is used to obtain the radial velocity, in contrast with the existing models where the radial velocity is parameterized. Thus, from the continuity equation

$$\frac{\partial}{\partial z}(r\rho w') + \frac{\partial}{\partial r}(r\rho u_r') = 0,$$

we obtain the radial velcity:

$$u_r' = u_c' f_u\left(\frac{r}{b}\right) = \frac{1}{b\rho}\frac{d}{dz}(b^2\rho w_c') f_u\left(\frac{r}{b}\right)$$

with the radial profile:

$$f_u\left(\frac{r}{b}\right) = -\frac{b}{r}\int_0^{r/b} f_w\left(\xi\right)\xi d\xi.$$

Conclusions and future work

- The presented plume model in an improvement of the 1–D Lagrangian model in the following ways:
  - The model does not requires an entrainment closure, but it does require closure for the turbulent fluxes. This problem can be solved using an eddy viscosity model;
  - An equation for the plume/cloud radius is obtained;
  - The anelastic approximation is used instead of the Boussinesq approximation;
- Analytical solutions can be obtained between two consecutive grids of the parent numerical model;
- Future work:
  - Determine the radial coefficients and turbulent fluxes from LES and/or DNS – how they depend on the convective type (dry, shallow or deep) and wind shear?
  - Introduce a parameterization for the pressure drag;
  - Test the model against LES and observations;
  - The model can provide a framework for machine–learning (ML) since the objective of the ML will be to find the appropiate values of the radial coefficients depending on the large–scale conditions, such as CAPE or wind shear.

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Thank you for your attention!

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