Experimenting with automatized numerical methods

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Experiment: Downward continuation of the gravitational potential

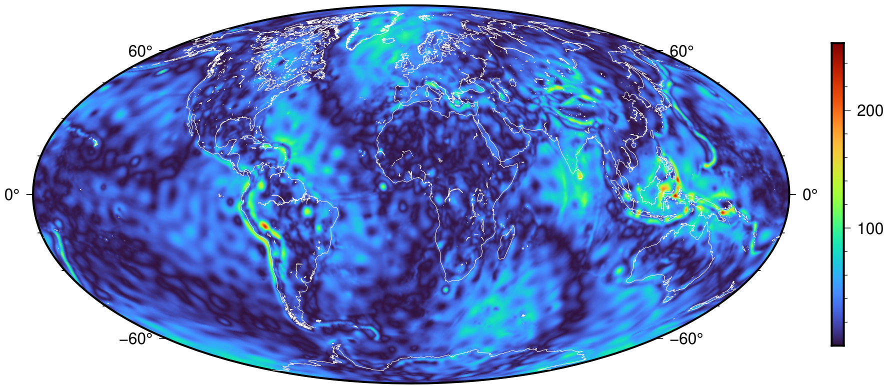

Figure: Absolute approximation error of the latest experiment (for its description **click here**). All values in m^2/s^2. The algorithm terminated after 1942 iterations with a relative RMSE of 0.000120 and a relative data error of 0.05 (noise level). Errors are mainly in regions with high local structures (like the Andean or the Pacific Ring of Fire) which can be expected for the downward continuation. Obvious is the overall pattern that resembles high-degree and high-dimensional spherical harmonics.

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Short experiment description:

- LRFMP with spherical harmonics and Abel–Poisson kernels as well as wavelets (i.e. radial basis functions)
- EGM2008 with 5% Gaussian noise evaluated at satellite tracks obtained from Mayer-Gürr, TU Graz:
  - using February and March 2014, i.e. 496170 grid points
  - height of each grid point above the sphere with radius 6371 km is visualized **here**
  - Zehentner, Mayer-Gürr, 2013: Kinematic orbits for GRACE and GOCE based on raw GPS observations. *Poster presented at the IAG Scientific Assembly 2013, 1.-6. September 2013, Potsdam, Germany.*
    Zehentner, Mayer-Gürr, 2015: Precise orbit determination based on raw GPS measurements. *Journal of Geodesy. doi 10.1007/s00190-015-0872-7*
- termination if relative data error reaches noise level or after 5000 iterations
- chosen regularization parameter: $10^{-10}\|y\|_{\mathbb{R}^\ell}$
  (best tested value after relative data error reaches noise level)

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Methods for approximating an inverse problem

- The **(Learning) Inverse Problem Matching Pursuits** determine iteratively a linear combination of dictionary elements $d \in \mathcal{D}$:

$$f_N := f_0 + \sum_{n=1}^{N} \alpha_n d_n = f_0 + \alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3 + \ldots + \alpha_n d_n$$

  via the minimization of the Tikhonov functional. In practice, e.g. in the N-th RFMP (Fischer, Michel) iteration, we determine

$$d_N := \operatorname*{arg\,max}_{d \in \mathcal{D}} \frac{\left(\langle R^{N-1}, \mathcal{T}\daleth d\rangle_{\mathbb{R}^\ell} - \lambda \langle f_{N-1}, d\rangle_{\mathcal{H}_2(\mathbb{S}^2)}\right)^2}{\|\mathcal{T}\daleth d\|^2_{\mathbb{R}^\ell} + \lambda \|d\|^2_{\mathcal{H}_2(\mathbb{S}^2)}} := \operatorname*{arg\,max}_{d \in \mathcal{D}} \frac{A_N(d)^2}{B_N(d)}$$

  and $\alpha_N := A_N(d)/B_N(d)$.
- For an increase in efficiency, a backfitting technique (Michel, Telschow) as well as a weakness parameter (Kontak, Michel) have been included.
- The learning variants (Michel, S.) include a **learning add-on** which enables $\mathcal{D}$ to be infinite.
  - The learning add-on is made up of non-linear constraint optimization in each iteration: for each type of trial function in $\mathcal{D}$, we optimize their characteristic parameters using optimization techniques. This yields a finite dictionary of candidates in each iteration.

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Their characteristics

- The methods yield their approximation as a function f_N via maximization.
- The dictionary usually consists of **diverse trial functions**: global and local trial functions are used for corresponding structures in the solution.
- The concept of the methods have been proved for **inverse gravimetry, downward continuation and the EEG and MEG problem**.
- The learning add-on automatizes the dictionary choice.
- For the **variants with the learning add-on**, we did extensive experiments with the downward continuation of satellite data:
  - **The approximation improves for decreasing noise level**. However, the satellite height is also a major influence.
  - The methods can also be used without satellite height (i.e. **pure approximation**) and with a regular and irregular grid.
  - In particular, **local errors stay local and local anomalies can be distinguished from global trends**.
  - We can also **learn a finite dictionary**: When using the learning add-on, the set of chosen dictionary elements can be used as a finite dictionary in an IPMP (without the learning add-on).
  - We **gain significant efficiency** with respect to storage, sparsity of the approximation and, possibly, also runtime. Hence, we can move forward to more realistic settings and more demanding applications.

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Next steps

- Regarding the downward continuation, we should aim to test more **realistic experiment settings**:
  - increase number of grid points:
    in particular, mathematical and computer scientific revision of the code
  - try out other noise characteristics
  - exchange spherical harmonic based data

- Moreover, we work on applying the methods to a novel application: **travel time tomography**. This is covered in another talk happening tomorrow morning:

EGU22-4037 | Presentations | SM5.6
Ray theoretical investigations using matching pursuits
Volker Michel, Naomi Schneider, Karin Sigloch, and Eoghan Totten
Wed, 25 May, 09:20–09:26 Room D3

Both aspects are work in progress and publication of results and code can be looked forward to.

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Literature and contact information

Literature on the IPMPs is listed on the website of the Geomathematics Group Siegen

https://www.uni-siegen.de/fb6/geomathe/publications/index.html?lang=de.

See, in particular, the works by Fischer, Leweke/Orzlowski, Michel, Kontak, S. and Telschow.

Thank you for your interest in our work. If you have any questions, please do not hesitate to ask them, for instance, via the contact details given at

https://www.uni-siegen.de/fb6/geomathe/staff/schneider.html?lang=de&lang=de.