

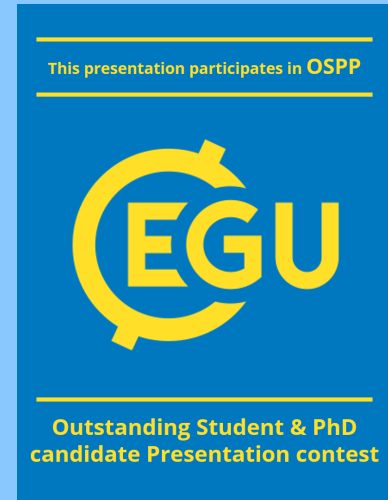
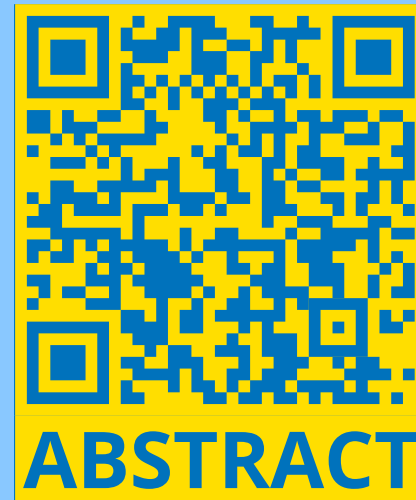
On the Evolution of Beach Shoreline According to Equilibrium Profile

Pau Luque, Marta Marcos, Lluís Gómez-Pujol and Alejandro Orfila

(EGU22, 25/05/2022, Wien)



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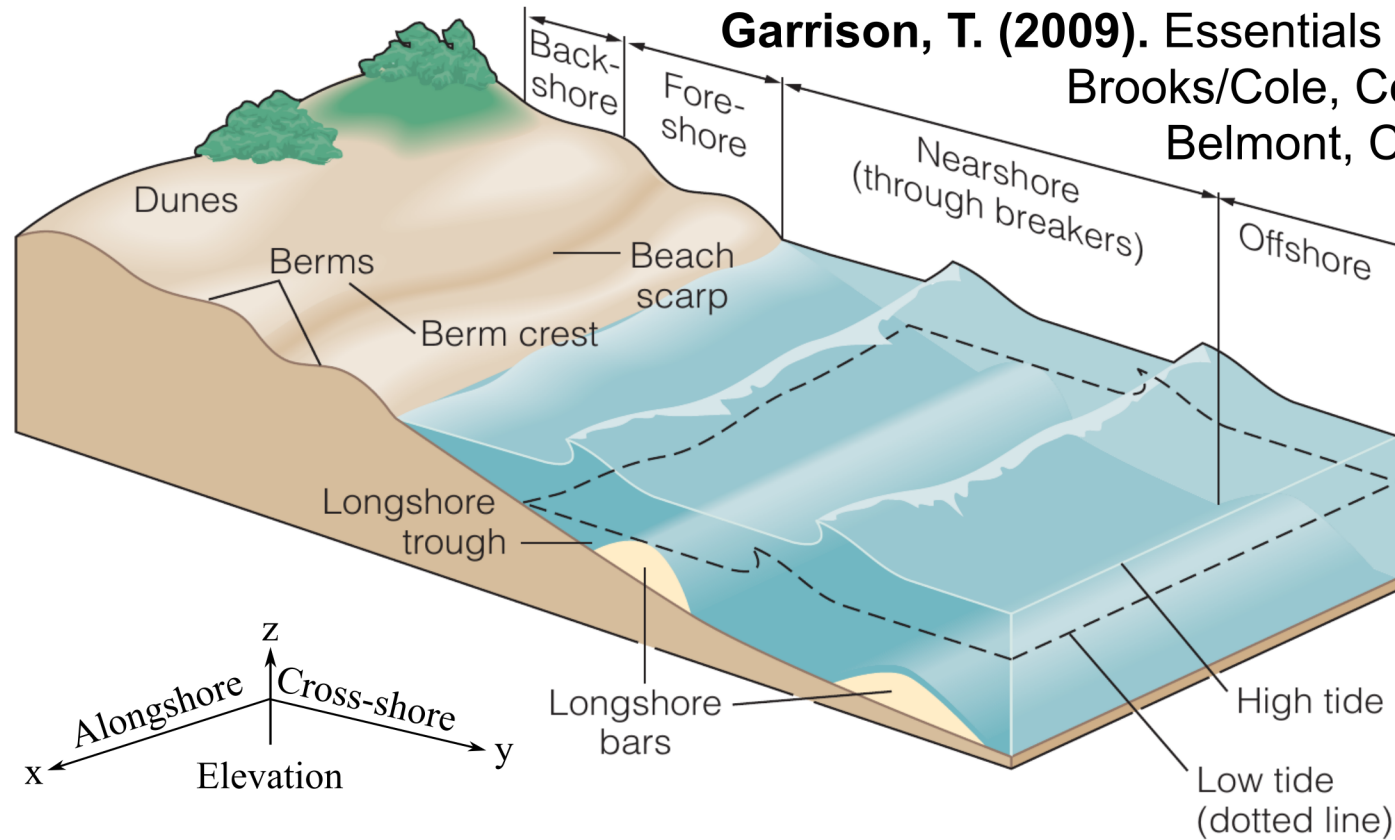


INTRODUCTION

INTRODUCTION

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Garrison, T. (2009). Essentials of oceanography.
Brooks/Cole, Cengage Learning,
Belmont, CA, 5th ed edition.

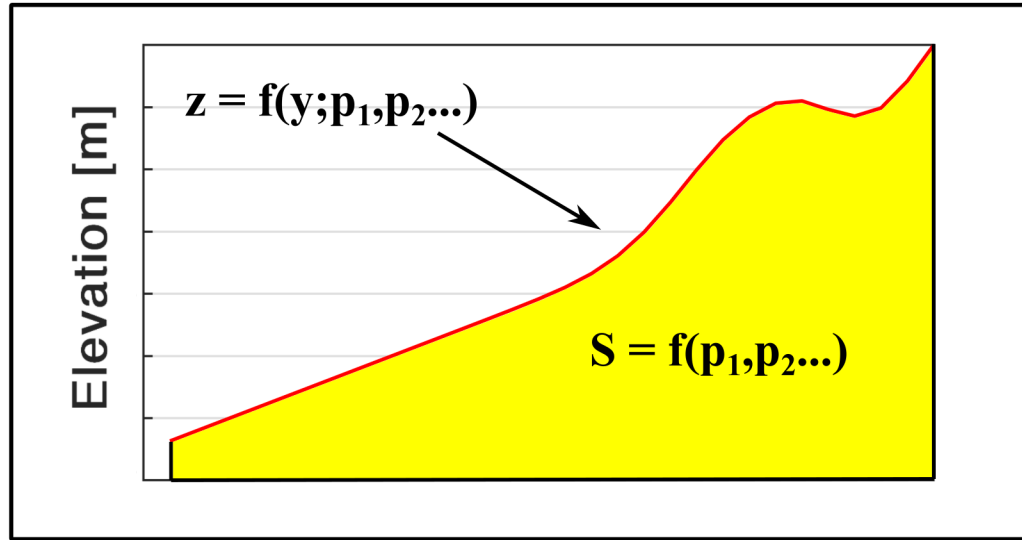


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EQUILIBRIUM BEACH PROFILE

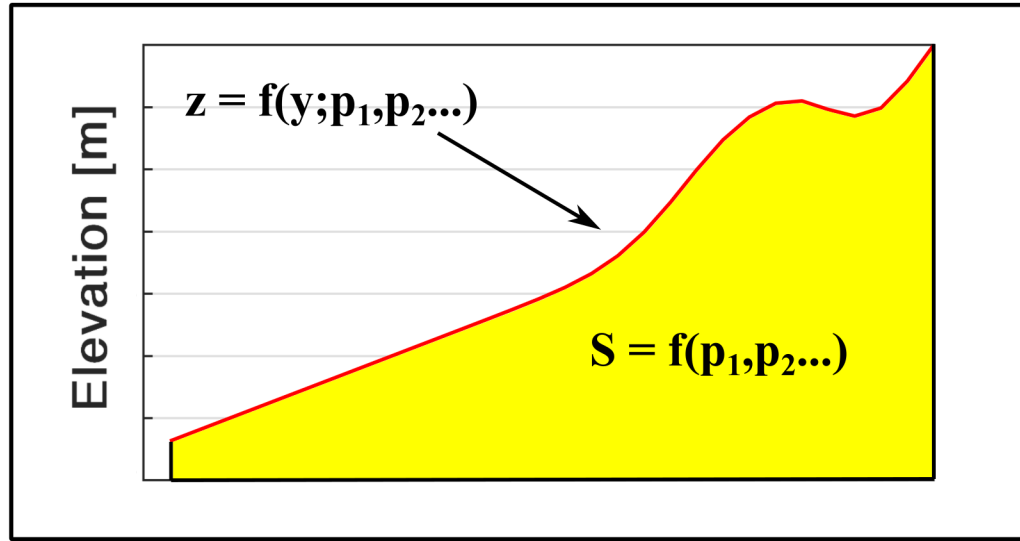


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EQUILIBRIUM BEACH PROFILE



$$dS = 0$$

$$dp_1 = \frac{\partial S / \partial p_2}{\partial S / \partial p_1} dp_2 + \dots$$

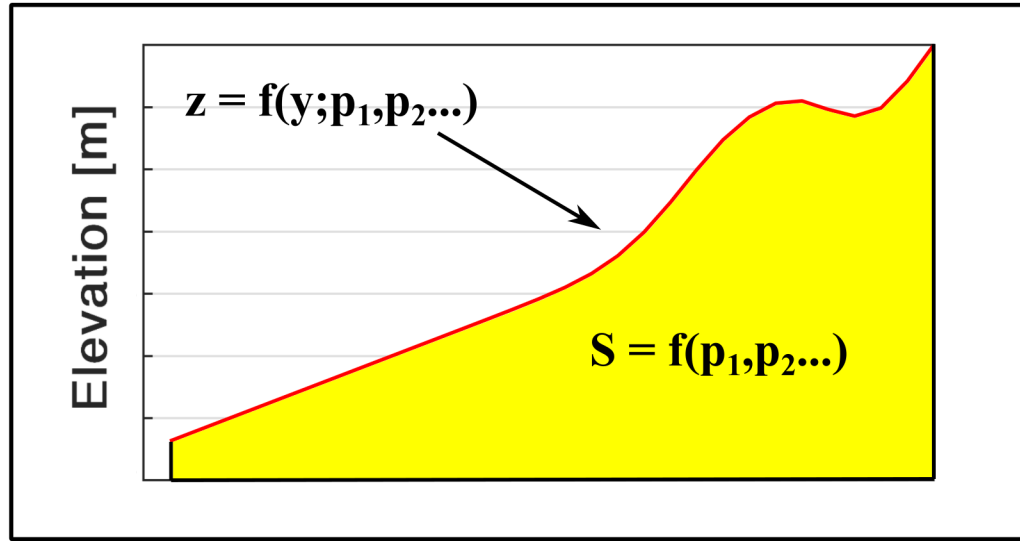


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INTRODUCTION

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EQUILIBRIUM BEACH PROFILE



OBJECTIVE:
Sea Level Rise
↓
Shoreline Evolution

$$dS = 0$$

$$dp_1 = \frac{\partial S / \partial p_2}{\partial S / \partial p_1} dp_2 + \dots$$

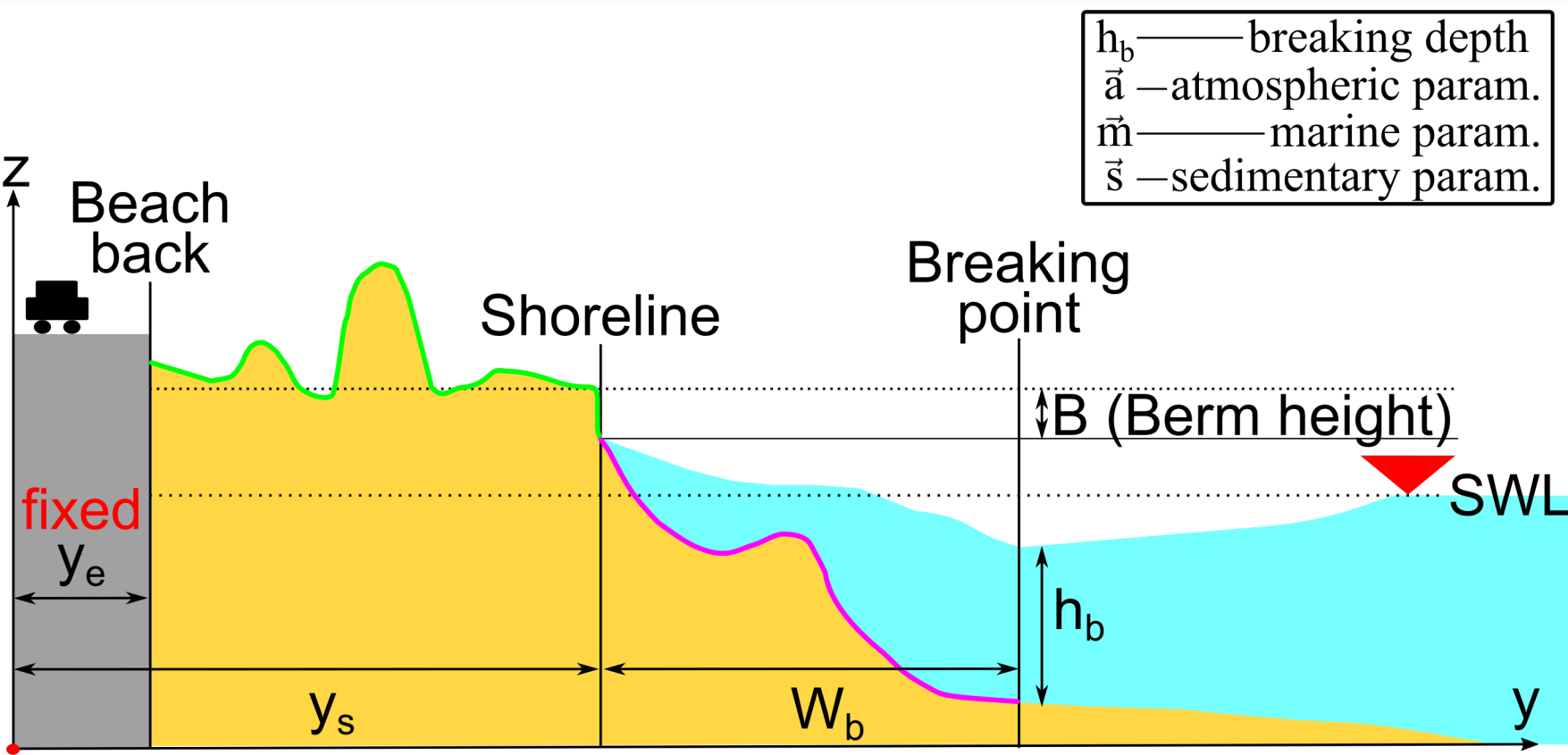


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METHODS

BEACH MODEL

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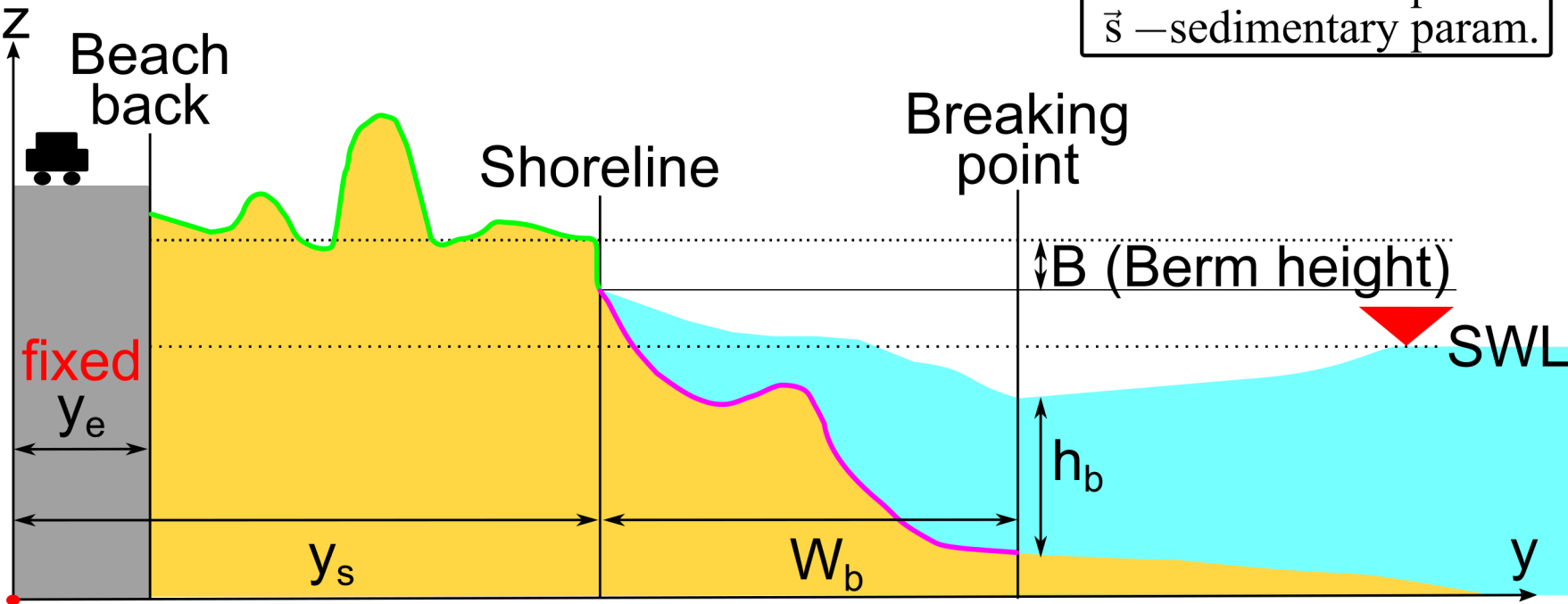
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BEACH MODEL

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$$z = f_{\text{dry}}(y; \vec{a}, \vec{m}, \vec{s}, y_s, y_e, \text{SWL})$$

h_b — breaking depth
 $\vec{a}$ — atmospheric param.
 $\vec{m}$ — marine param.
 $\vec{s}$ — sedimentary param.



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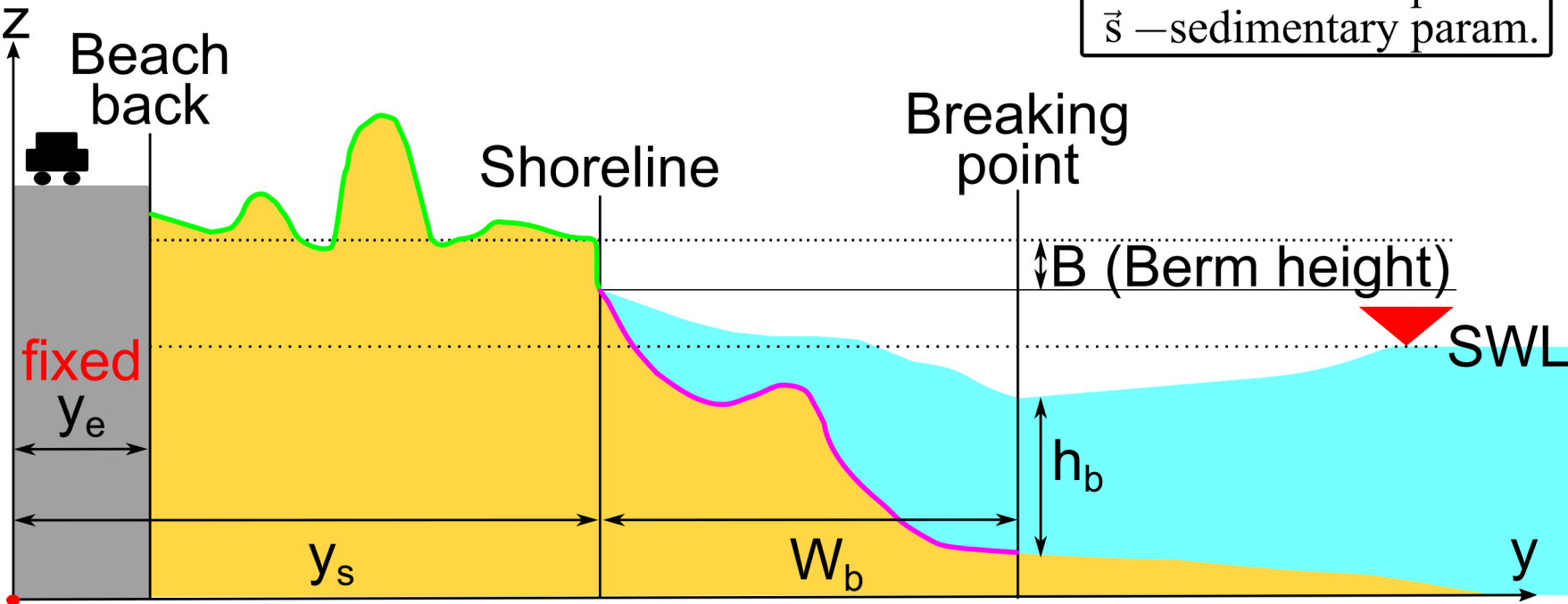
BEACH MODEL

SWL = Mean Sea Level +
Astronomical Tides +
Storm Surge

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BEACH MODEL

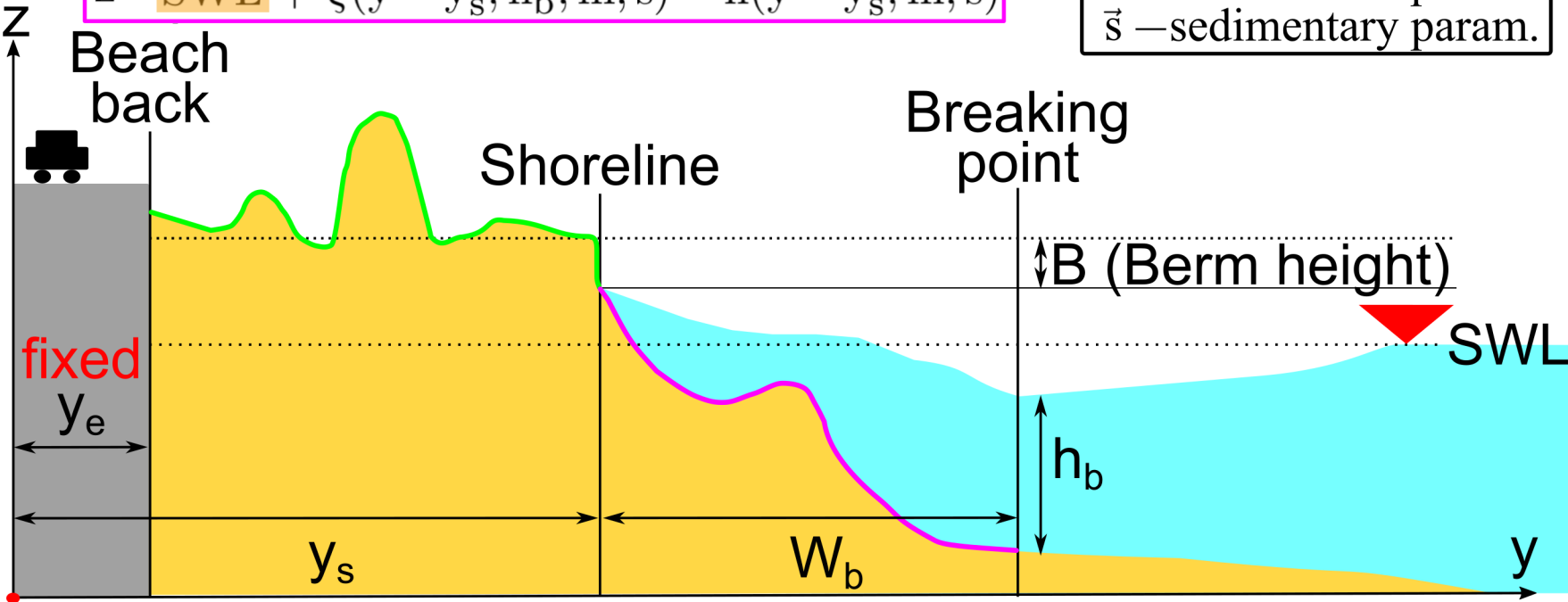
SWL = Mean Sea Level +
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$$z = f_{\text{dry}}(y; \vec{a}, \vec{m}, \vec{s}, y_s, y_e, \text{SWL})$$

$$z = \text{SWL} + \bar{\zeta}(y - y_s; h_b, \vec{m}, \vec{s}) - h(y - y_s; \vec{m}, \vec{s})$$

h_b — breaking depth
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BEACH MODEL

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z
↑
Beach

h_b — breaking depth
 $\vec{a}$ — atmospheric param.
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BEACH MODEL SIMPLIFICATIONS

- Flat dry beach $\rightarrow$ “ $z = B + \text{SWL}$ ”
- Wave setup is a function of the other variables: (Bowen, 1962) model.
- Depth shape only depends on non-varying sediment size (D50).

Bowen, A. J., Inman, D. L., and Simmons, V. P. (1968). Wave ‘set-down’ and set-up. *Journal of Geophysical Research*, 73(8):2569–2577

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EQUATIONS

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Present Approach

$$-\frac{W_b + (y_s - y_e)}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dSWL - \frac{\left(\frac{J}{1-J} - \frac{\gamma^2}{16}\right) \cdot W_b}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dh_b$$



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EQUATIONS

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Present Approach

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$$-\frac{W_b}{B + h_b} \left[\Delta SWL + \left(\frac{2/5 \cdot J}{1 - J} - \frac{\gamma^2}{16} \right) \cdot h_b \right]$$

Dean, R. G. (1991). Equilibrium beach profiles: Characteristics and applications. *Journal of Coastal Research*, 7(1):53–84.

Bruun, P. (1962). Sea-level rise as a cause of shore erosion. 88:117–130.

$$-\frac{W_*}{B + h_*} \cdot \Delta MSL$$

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EQUATIONS

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Present Approach

$$-\frac{W_b + (y_s - y_e)}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dSWL - \frac{\left(\frac{J}{1-J} - \frac{\gamma^2}{16}\right) \cdot W_b}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dh_b$$

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Present Approach

$$-\frac{W_b + (y_s - y_e)}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dSWL - \frac{\left(\frac{J}{1-J} - \frac{\gamma^2}{16}\right) \cdot W_b}{B + \left(1 + \frac{\gamma^2}{16}\right) \cdot h_b} \cdot dh_b$$

$$\int_{t_0}^t k \cdot e^{-k \cdot (t-t_0)} \cdot \left[-\frac{W_b}{B + h_b} \left[\Delta SWL + \left(\frac{2/5 \cdot J}{1-J} - \frac{\gamma^2}{16} \right) \cdot h_b \right] \right] dt$$

Dean, R. G. (1991). Equilibrium beach profiles: Characteristics and applications. *Journal of Coastal Research*, 7(1):53–84.

Miller, J. K. and Dean, R. G. (2004). A simple new shoreline change model. *Coastal Engineering*, 51(7):531–556.

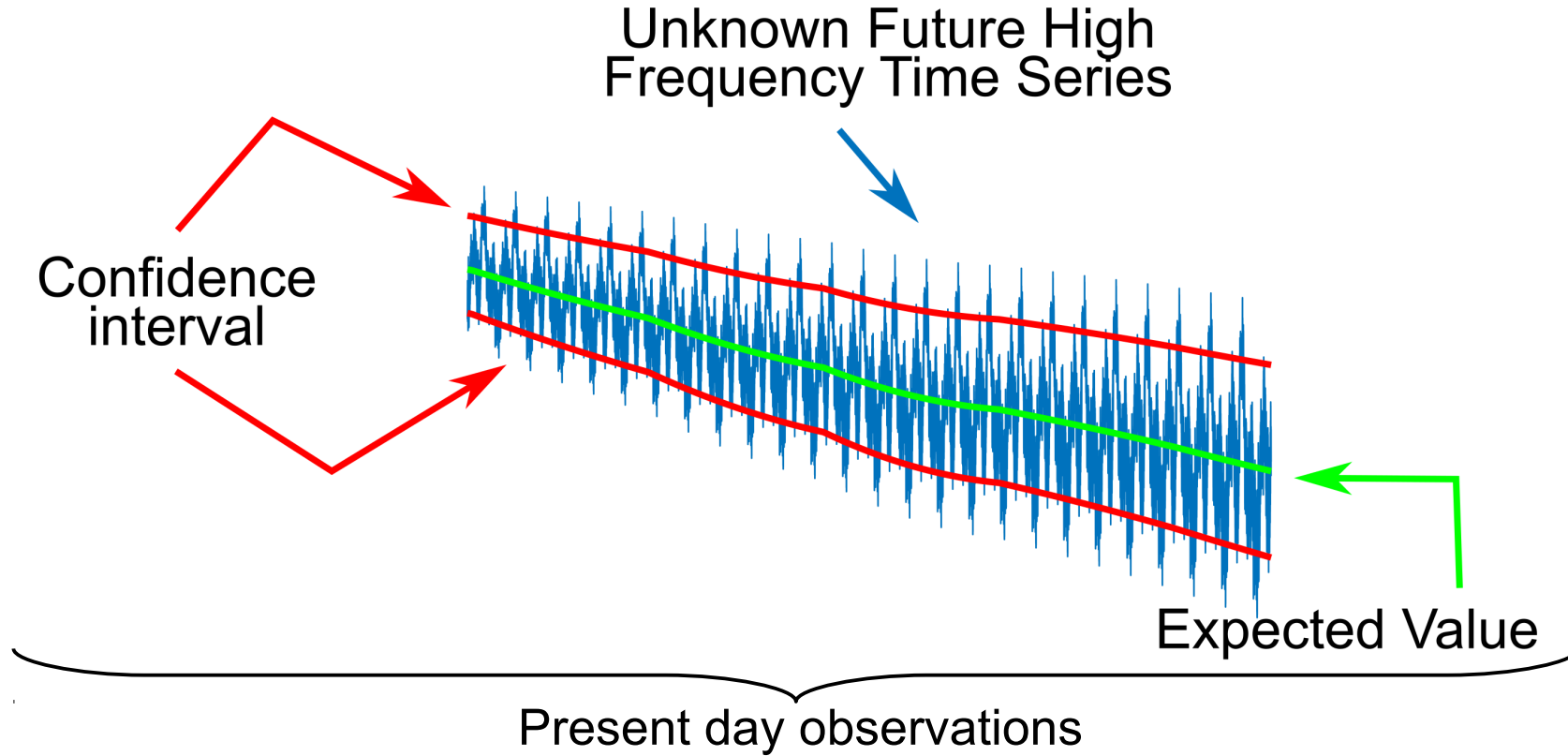
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$$-\frac{W_*}{B + h_*} \cdot \Delta MSL$$

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INTEGRATION APPROACH

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RESULTS

SOLUTIONS

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Present Approach

$$-(\text{correction} \pm \text{spread}) \cdot \rho \cdot \left[\frac{\frac{d\text{MSL}}{dt}(t)}{\theta(t)} - \frac{\frac{d\text{MSL}}{dt}(t_0)}{\theta(t_0)} - \frac{\frac{d^2\text{MSL}}{dt^2}(t)}{\theta^2(t)} + \frac{\frac{d^2\text{MSL}}{dt^2}(t_0)}{\theta^2(t_0)} \right]$$



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$$-\alpha \cdot [\text{MSL}(t) - \text{MSL}(t_0)] + \frac{\alpha}{k} \cdot \left[\frac{d\text{MSL}}{dt}(t) - \frac{d\text{MSL}}{dt}(t_0) \right] - \frac{\alpha}{k^2} \cdot \left[\frac{d^2\text{MSL}}{dt^2}(t) - \frac{d^2\text{MSL}}{dt^2}(t_0) \right]$$

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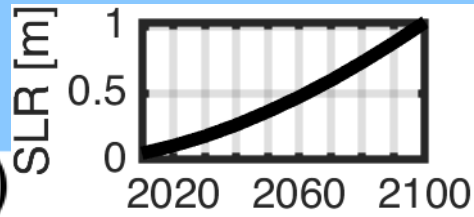
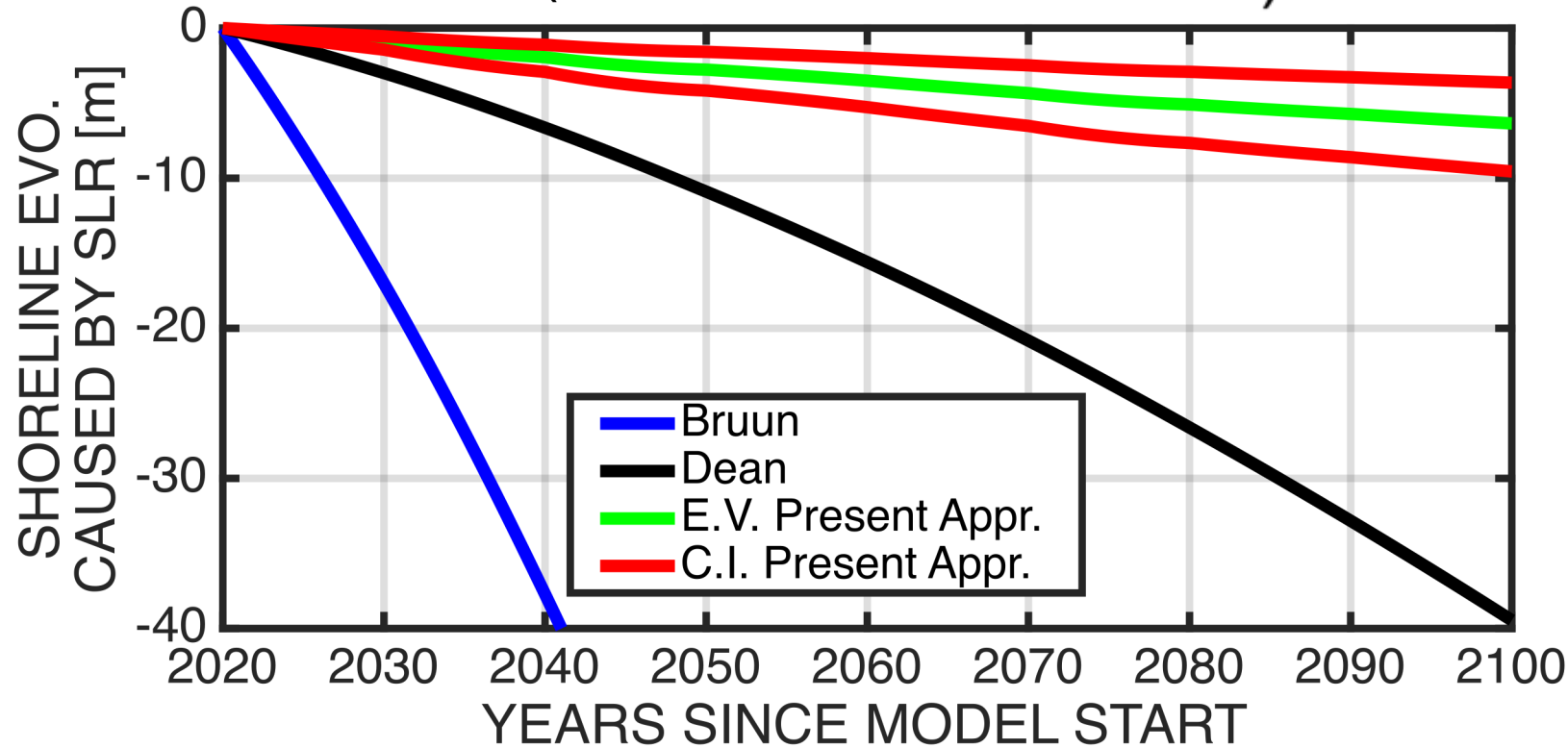
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$$-\alpha_* \cdot [\text{MSL}(t) - \text{MSL}(t_0)]$$

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COMPARISON

SON BOU BEACH (MENORCA, WEST. MED.)



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CONCLUSIONS

CONCLUSIONS

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- Generalistic equation for sea-level rise induced shoreline recession based on equilibrium beach profiles.
- Englobing relevant previous models in our generalistic equation.
- Theoretical justification of the exponential rate.
- **Shoreline recession (almost) proportional to the rate of mean sea-level rise.**



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