Full-Bayesian GNSS-A seafloor positioning solution derived by the Markov-Chain Monte Carlo method

Shun-ichi Watanabe, Tadashi Ishikawa, Yuto Nakamura (Japan Coast Guard)
and Yusuke Yokota (IIS, Univ. Tokyo)

May 24, 2022 | EGU General Assembly

Purpose of this study:

To develop a platform software for GNSS-A analysis based on MCMC, enabling more flexible sound speed modeling and easier system error evaluation

JCG 海上保安庁 JAPAN COAST GUARD

INSTITUTE OF INDUSTRIAL SCIENCE THE UNIVERSITY OF TOKYO

SGO-A

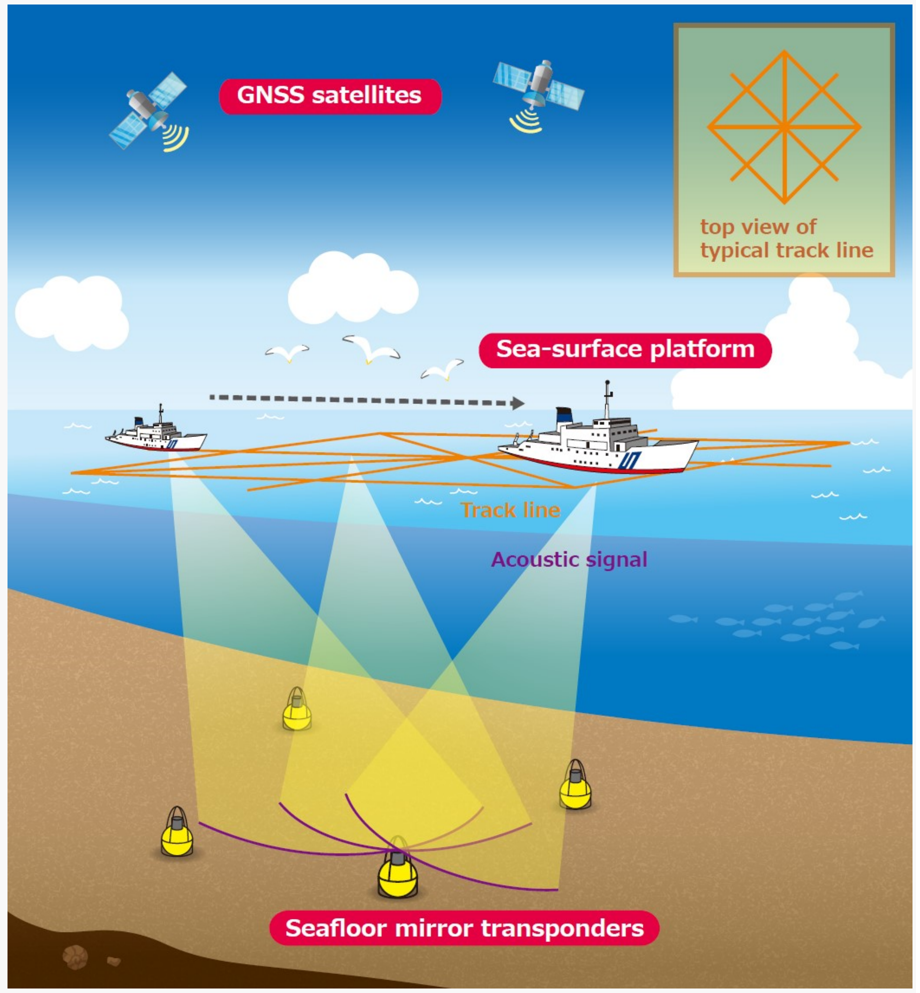
GNSS-A observation in the Japan Coast Guard

SGO-A the GNSS-A Seafloor Geodetic Observation Array

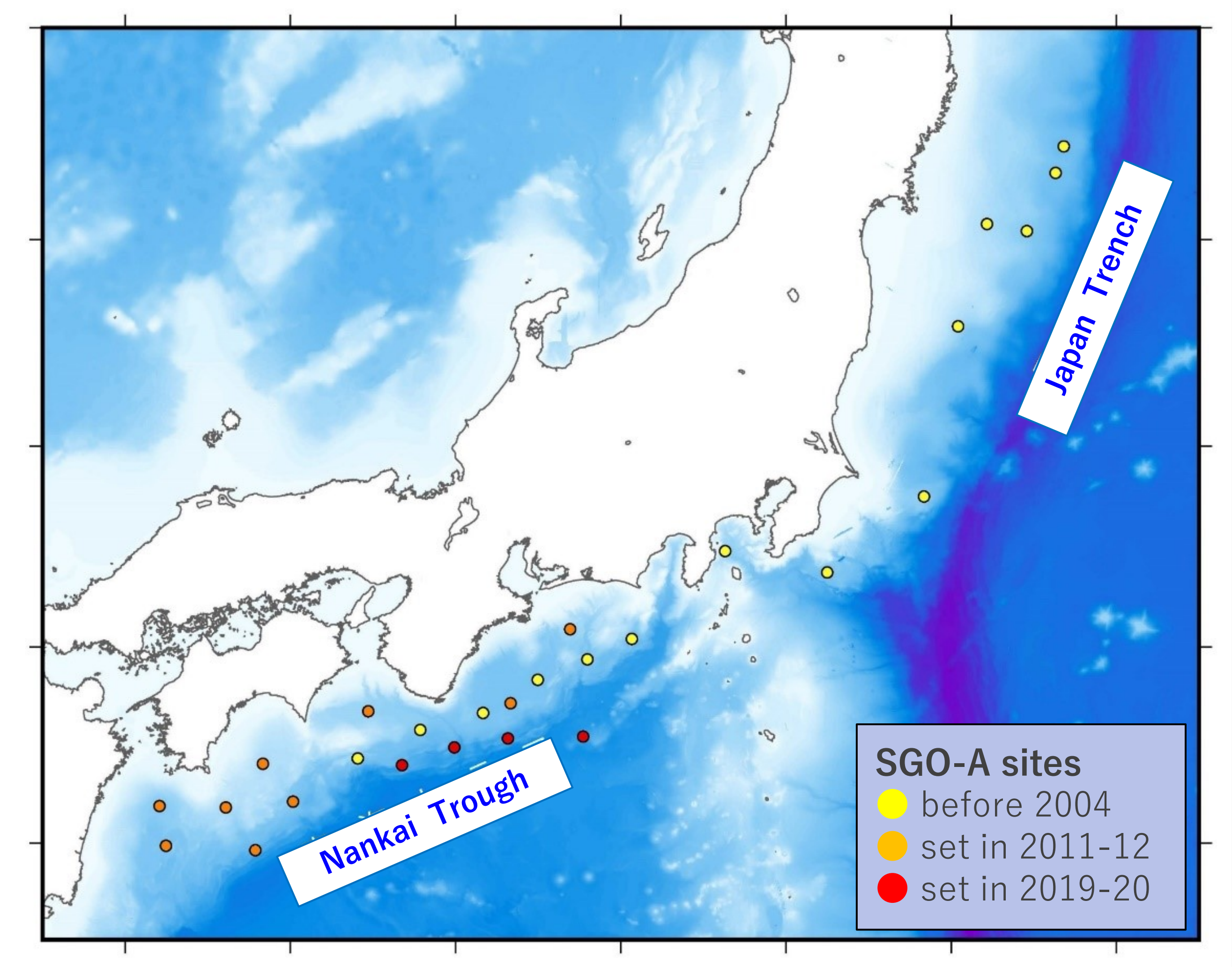
observation

GARPOS Positioning for 70% of our planet

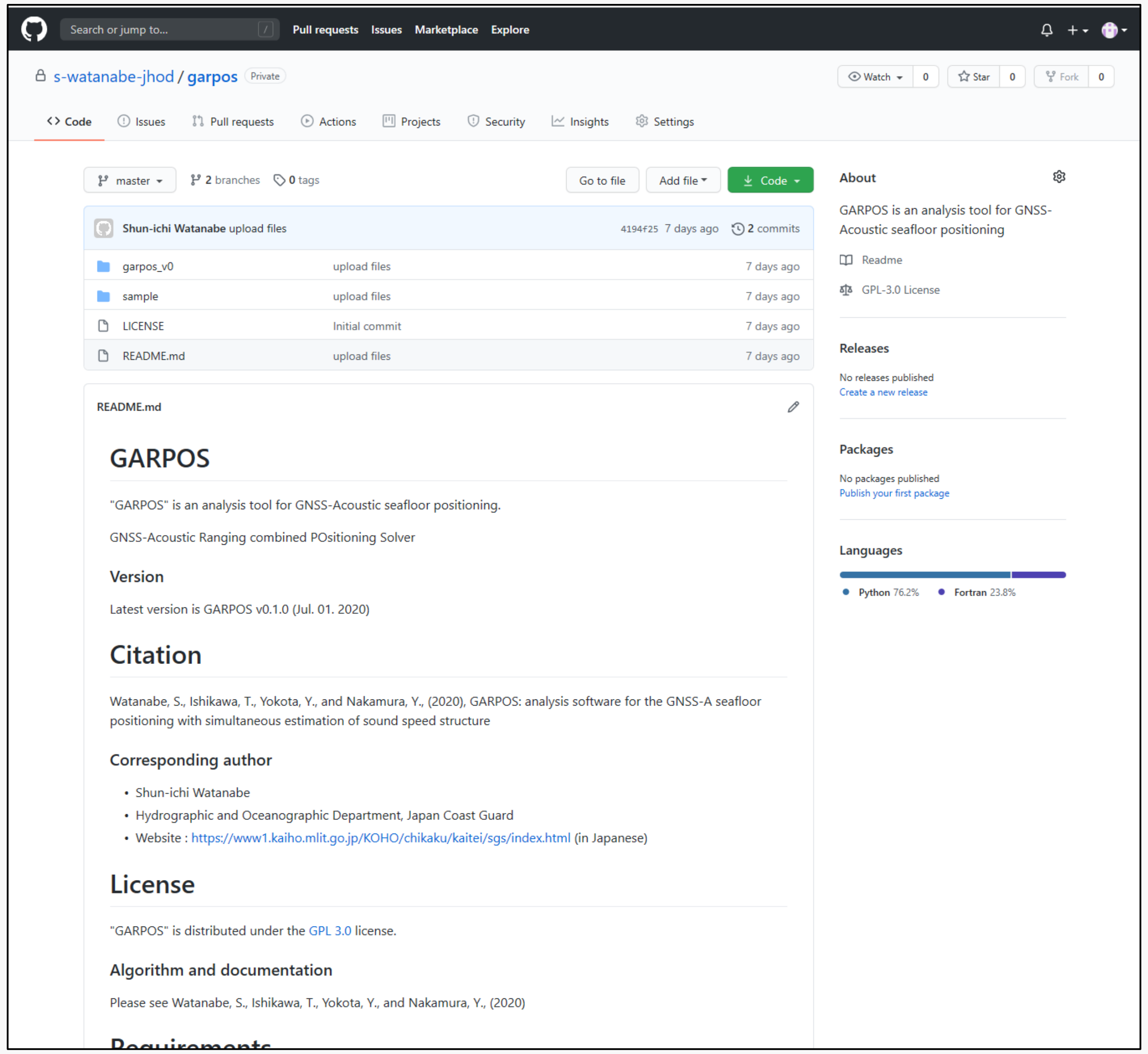
analysis



Observation system



Observation sites



Methodology: Watanabe et al. (2020, Front. Earth Sci.)

<https://doi.org/10.3389/feart.2020.597532>

<**Obs. data**> partly published on JCG web site (DOIs are minted via Zenodo)

https://www1.kaiho.mlit.go.jp/KOHO/chikaku/kaitei/sgs/datalist.html

<**Anal. software**> published on GitHub (DOI is minted via Zenodo)

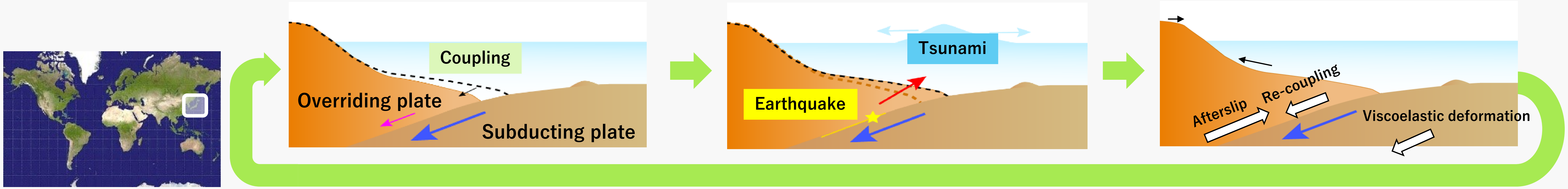
https://github.com/s-watanabe-jhod/garpos (GitHub) or https://doi.org/10.5281/zenodo.6414642 (Zenodo)

Scientific targets of GNSS-A “global” seafloor positioning in the subduction zone

JCG

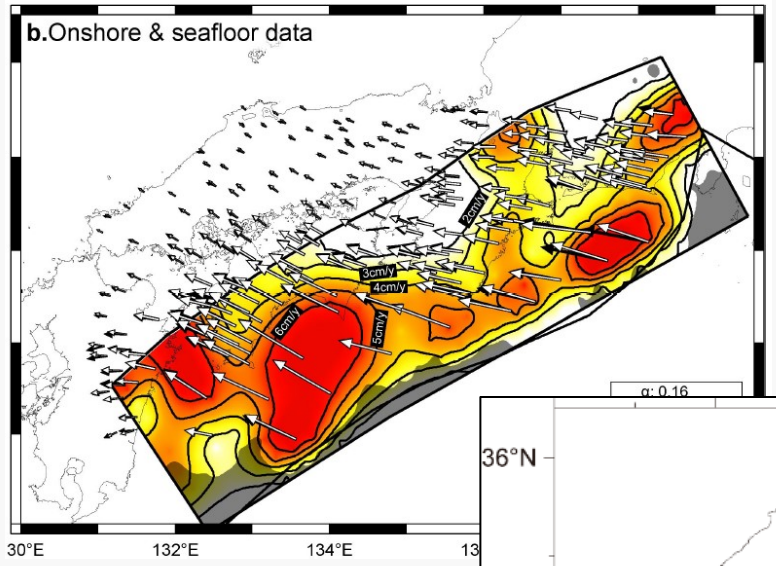
Seismic cycle

Interseismic phase — **Earthquake** — **Postseismic phase**



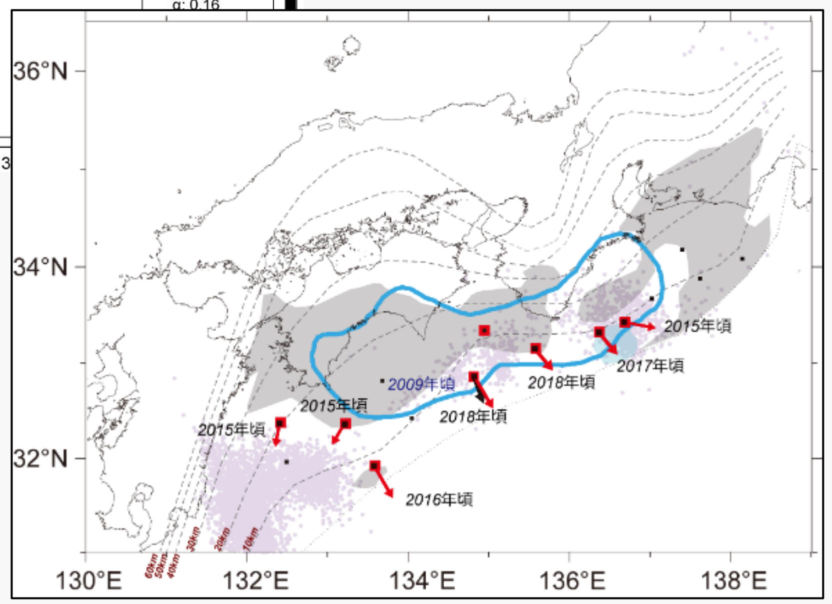
GNSS-A results in Nankai Trough and Japan Trench (2011 Tohoku-oki earthquake)

Coupling condition



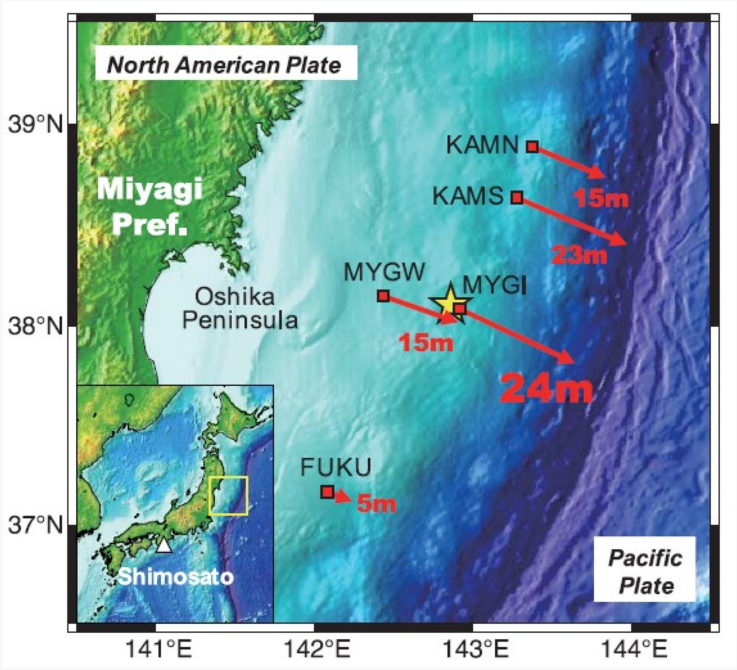
Yokota et al.
(2016, Nature)

Slow slip events



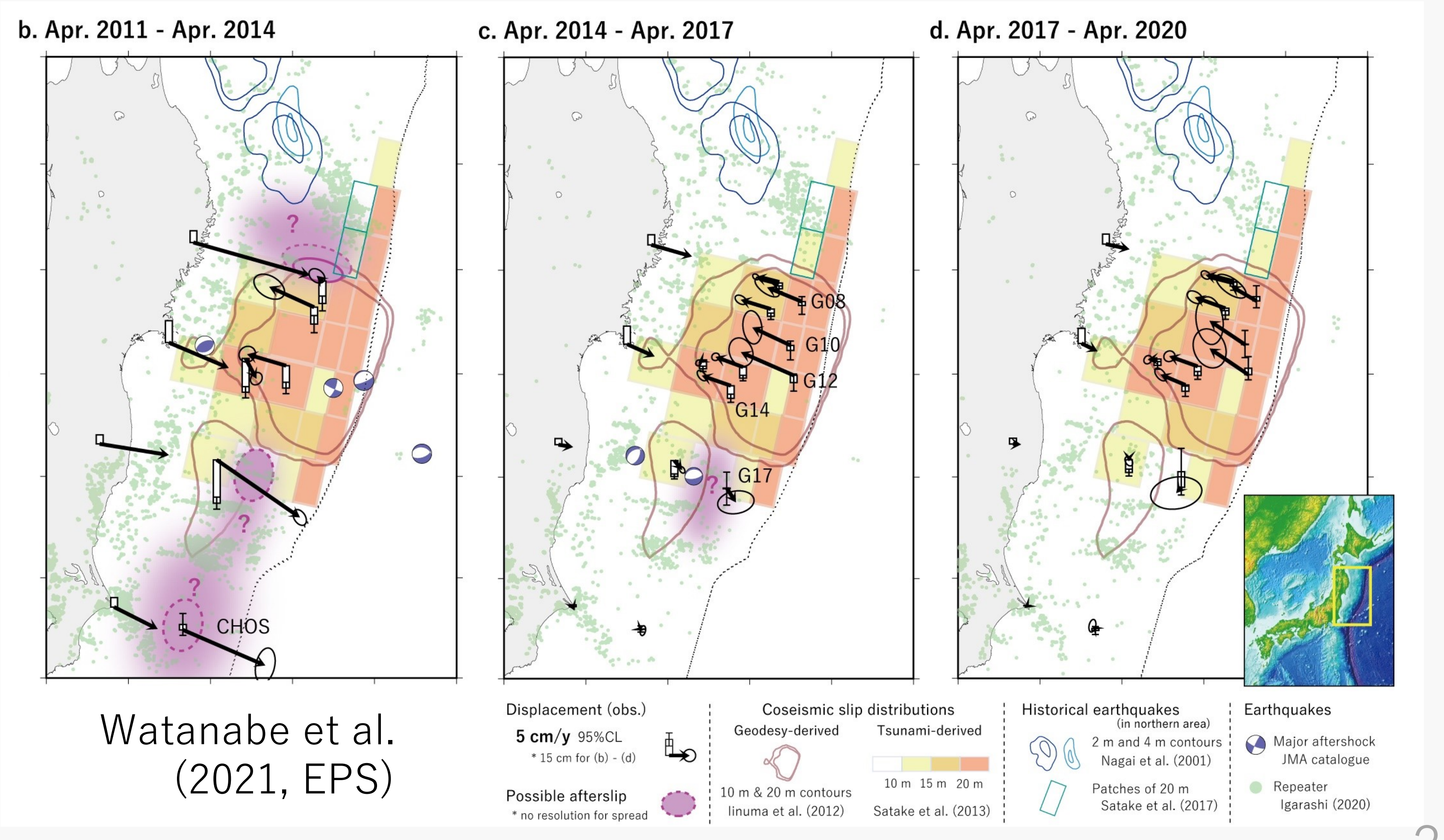
Yokota & Ishikawa (2020, Sci Adv)

Coseismic disp.



Sato et al. (2011, Science)

Transient behaviors (postseismic relaxation)



Watanabe et al.
(2021, EPS)

Observation equation used in GARPOS

$$\log(T_i^o/T^*) = \log\left(\tau_i\left(\boldsymbol{X}_j \middle| \boldsymbol{M}, \boldsymbol{Q}, \boldsymbol{\Theta}, V_0\right)/T^*\right) - \gamma_i\left(\boldsymbol{a} \middle| \boldsymbol{X}_j^0, \boldsymbol{M}^0, \boldsymbol{Q}, \boldsymbol{\Theta}\right) + e_i$$

Obs. travel time	Reference travel time	Perturbation term	Error
Obs. data	calc. with a ref. sound speed.	variation from the ref.	

→ When $\gamma_i \ll 1$, average sound speed is, $\overline{V_0} + \delta V_i \sim \overline{V_0} + \gamma_i \overline{V_0}$.

Model parameters

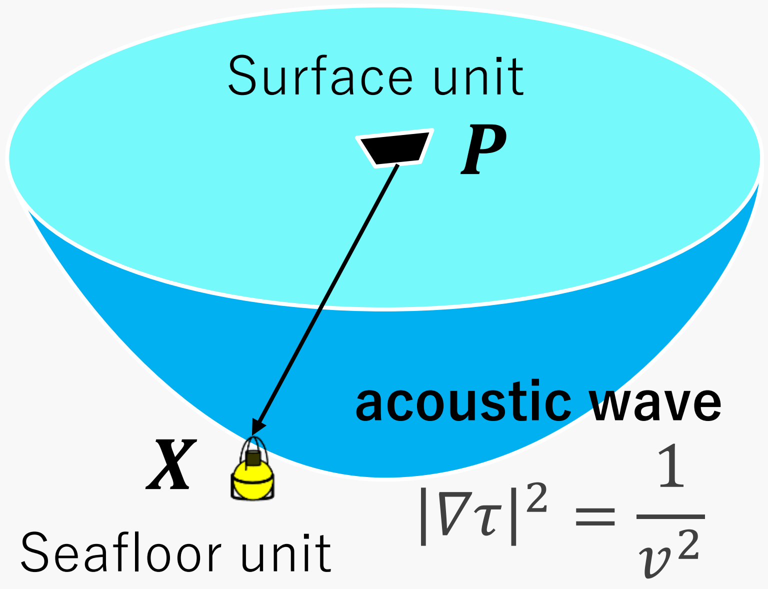
$\boldsymbol{X}_j$: j-th seafloor unit's position

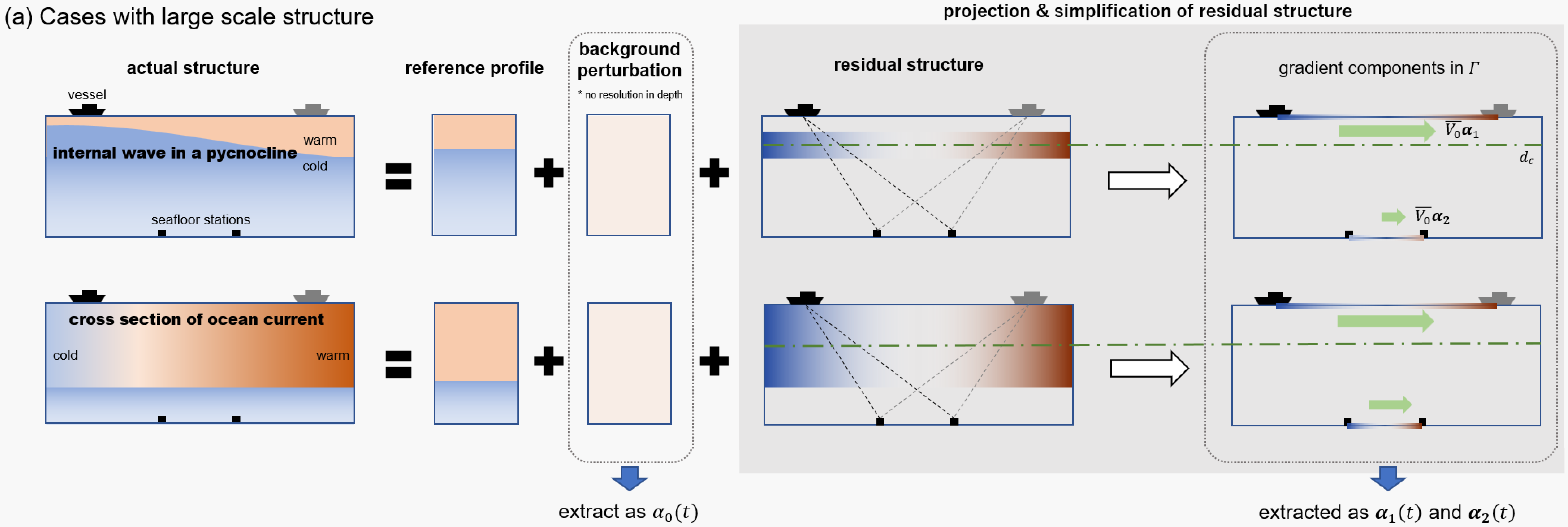
$\boldsymbol{a}$: perturbation coefficients

Perturbation field model

[to consider that γ_i is an extraction from **a scalar field** $\Gamma(\boldsymbol{a})$] → $\overline{V_0}\Gamma$ is **a projection of sound speed variation**

(in linearized form) $$\Gamma(t, \boldsymbol{P}, \boldsymbol{X}) \equiv \alpha_0(t) + \boldsymbol{\alpha}_1(t) \cdot \frac{\boldsymbol{P}}{L^*} + \boldsymbol{\alpha}_2(t) \cdot \frac{\boldsymbol{X}}{L^*}$$





* **sound speed gradient is modeled with two comp., i.e., $\boldsymbol{\alpha}_1(t)$ and $\boldsymbol{\alpha}_2(t)$**

Analysis method for posterior distribution

Obs. eq.

$$\log(T_i^o/T^*) = \log\left(\tau_i\left(\boldsymbol{X}_j, \boldsymbol{M} \middle| \boldsymbol{Q}, \boldsymbol{\Theta}, V_0\right)/T^*\right) - \gamma_i\left(\boldsymbol{a} \middle| \boldsymbol{X}_j^0, \boldsymbol{M}^0, \boldsymbol{Q}, \boldsymbol{\Theta}\right) + e_i$$

rewrite ⇒

$$\boldsymbol{d} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{a} + \boldsymbol{e}$$

$\boldsymbol{x}$: seafloor units' position
$\boldsymbol{a}$: perturbation coefficients
$\boldsymbol{\theta}$: hyperparameters

Priors (controlled by hyperparameters)

$$p(\boldsymbol{d}|\boldsymbol{x},\boldsymbol{a},\boldsymbol{\theta}) = (2\pi)^{-\frac{n}{2}}|E(\boldsymbol{\theta})|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}(\boldsymbol{d}-\boldsymbol{f}(\boldsymbol{x})-G\boldsymbol{a})^T E(\boldsymbol{\theta})^{-1}(\boldsymbol{d}-\boldsymbol{f}(\boldsymbol{x})-G\boldsymbol{a})\right]$$

distribution of data error (Gaussian)

*non-diagonal terms for E are considered

$$p(\boldsymbol{a}|\boldsymbol{x},\boldsymbol{\theta}) = p(\boldsymbol{a}|\boldsymbol{\theta}) = (2\pi)^{-\frac{h}{2}}\|\Lambda_H(\boldsymbol{\theta})\|^{\frac{1}{2}}\cdot\exp\left[-\frac{1}{2}\boldsymbol{a}^T H(\boldsymbol{\theta})\boldsymbol{a}\right]$$

smoothness for perturbation term

Posterior

$p(\boldsymbol{x},\boldsymbol{a},\boldsymbol{\theta}|\boldsymbol{d})$ → determine $\boldsymbol{\theta}$ by minimizing ABIC : Conventional (empirical Bayes)

↓

estimate **joint distribution** including $\boldsymbol{\theta}$: **This study (full Bayes)**

*marginalize with $\boldsymbol{a}$ (linear comp.) to reduce the parameter dimension

$$p(\boldsymbol{x},\boldsymbol{\theta}|\boldsymbol{d}) = \frac{p(\boldsymbol{x})\cdot p(\boldsymbol{\theta})}{p(\boldsymbol{d})}\int p(\boldsymbol{d}|\boldsymbol{x},\boldsymbol{a},\boldsymbol{\theta})\cdot p(\boldsymbol{a}|\boldsymbol{x},\boldsymbol{\theta})d\boldsymbol{a}$$

⇐ analytically available

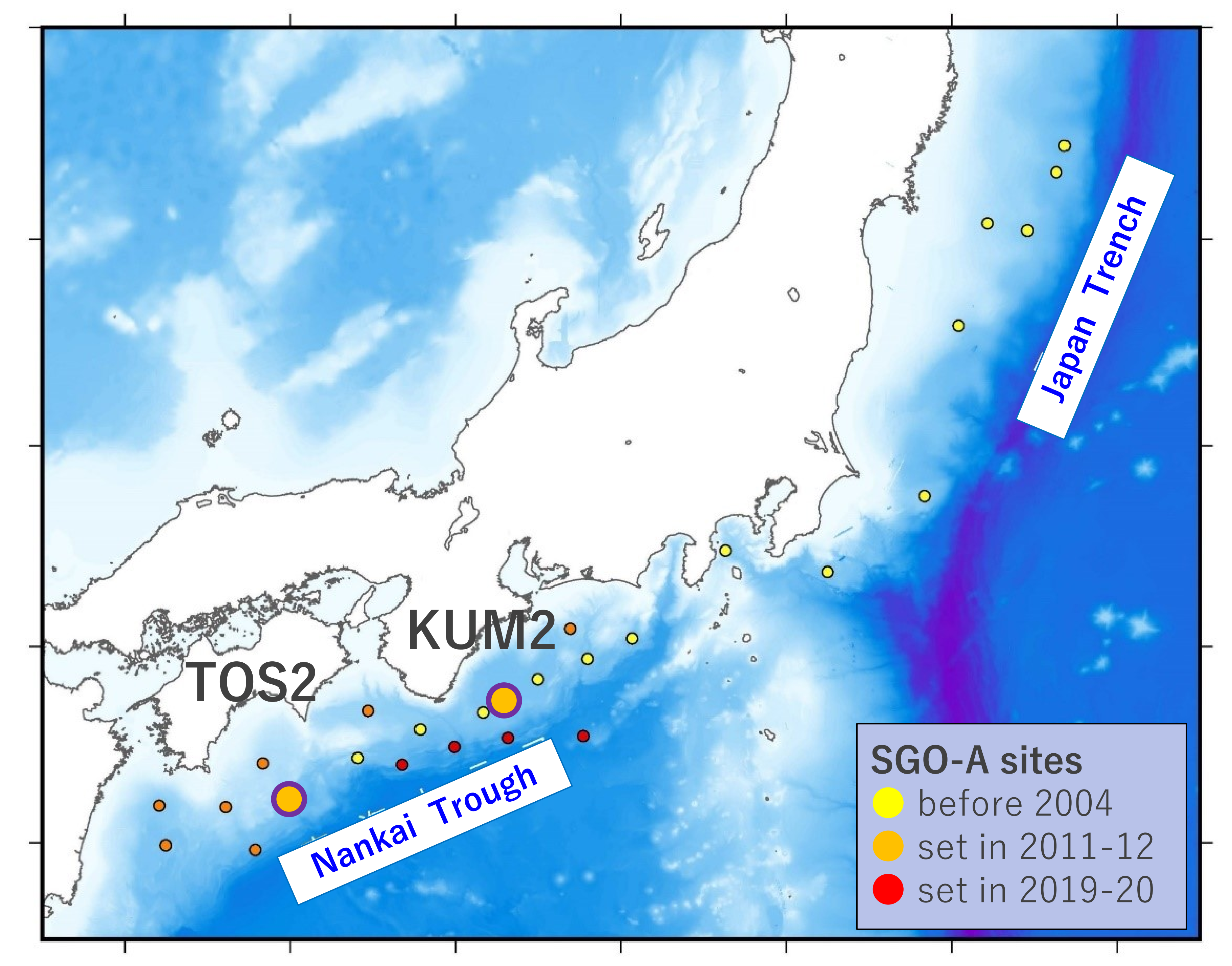
$$(2\pi)^{-\frac{n+h-m}{2}}|E|^{-\frac{1}{2}}\|\Lambda_H\|^{\frac{1}{2}}|C|^{\frac{1}{2}}\exp\left[-\frac{1}{2}s(\boldsymbol{a}^*)\right]$$

E	Covar. for data error
Λ_H	Constraint for $\boldsymbol{a}$ (roughness)
C	Covar. for $\boldsymbol{a}$ estimated for given $(\boldsymbol{x},\boldsymbol{\theta})$
$s(\boldsymbol{a}^*)$	Objective func. for given $(\boldsymbol{x},\boldsymbol{\theta})$

⇓

MCMC sampling with the Metropolis-Hastings algorithm

Application to the actual data



We tested for the GNSS-A data obtained in the Nankai Trough region

Site	Latitude	Longitude	Height	Data	Observation period
TOS2	32.43 °N	134.03 °E	–1740 m	27	Nov. 2011 – Sep. 2018
KUM2	33.43 °N	136.67 °E	–1970 m	40	Feb. 2012 – Oct. 2021

※ The data with the identical transponder-set (4 units) are used

[MCMC sampling]
Total sample 50000
Burn-in period 25000
Sampling rate 50

Parameter	description	Range	Sigma for proposal
$\boldsymbol{x}$	seafloor position	$\boldsymbol{x} \in \mathbb{R}^3$	5 mm
$\log_{10} \sigma^2$	scale for error covariance	$\sigma^2 > 0$	0.01
$\mathrm{logit}(\mu_t^*)$	correlation length for errors	$10\ \mathrm{s} \leq \mu_t \leq 370\ \mathrm{s}$	0.1
$\mathrm{logit}(\mu_{MT})$	correlation of errors among the different units	$0 \leq \mu_{MT} \leq 1$	0.1
$\log_{10} \nu_0{}^2$	roughness of $\alpha_0(t)$	$\nu_0{}^2 > 0$	0.2
$\log_{10} \nu_1{}^2$	roughness of $\boldsymbol{\alpha}_1(t)$	$\nu_1{}^2 > 0$	0.2
$\log_{10} \nu_2{}^2$	roughness of $\boldsymbol{\alpha}_2(t)$	$\nu_2{}^2 > 0$	0.2

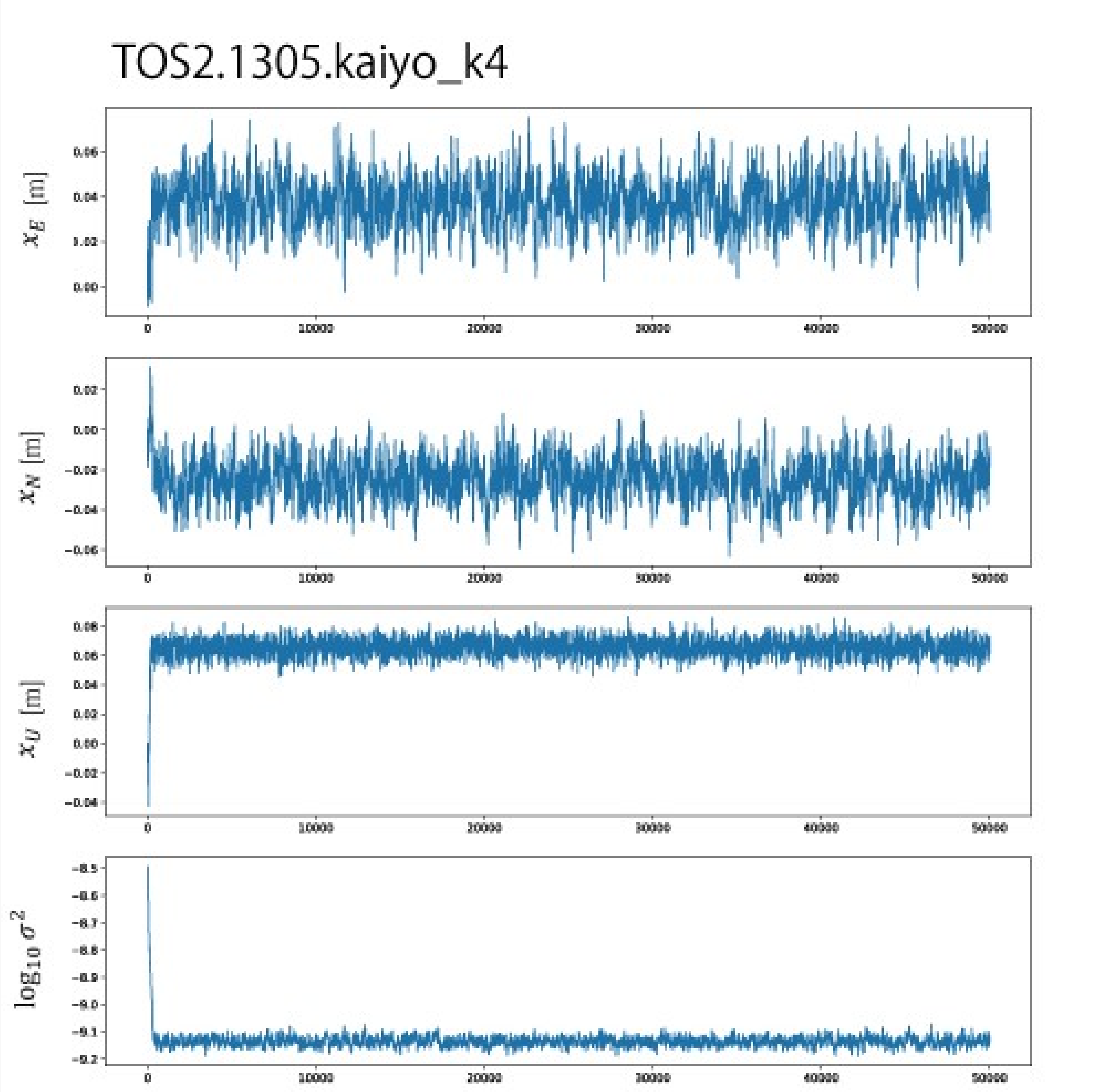
Example of MCMC sample series

Seafloor position (EW)

Seafloor position (NS)

Seafloor position (UD)

scale for error covar.



number of sample (up to 50000)

corr. length for errors

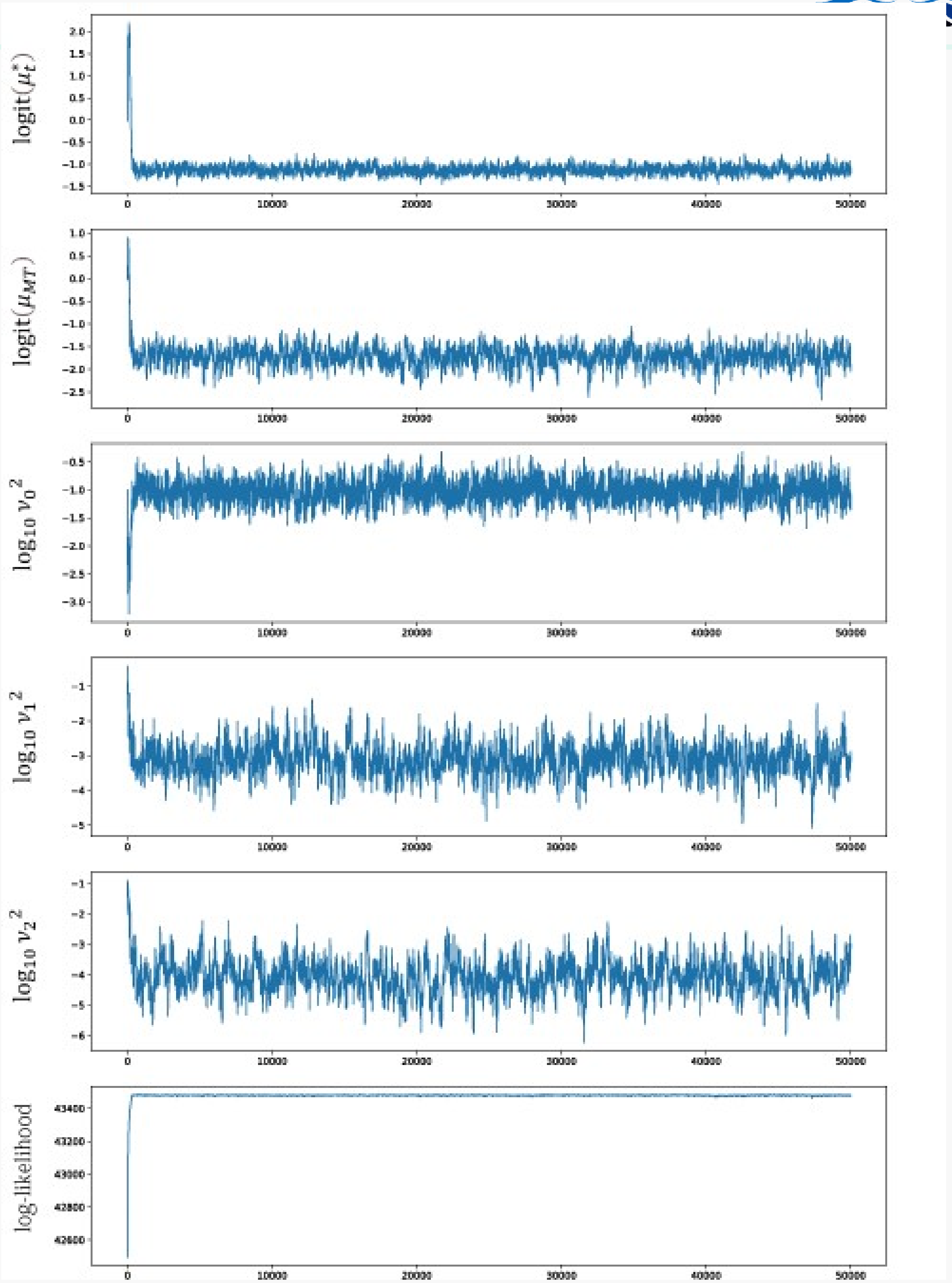
corr. of errors among diff. units

roughness of $\alpha_0(t)$

roughness of $\boldsymbol{\alpha}_1(t)$

roughness of $\boldsymbol{\alpha}_2(t)$

log-likelihood



Samples seem to converge rapidly (< 2000 samples)

Example of distributions of MCMC parameter samples

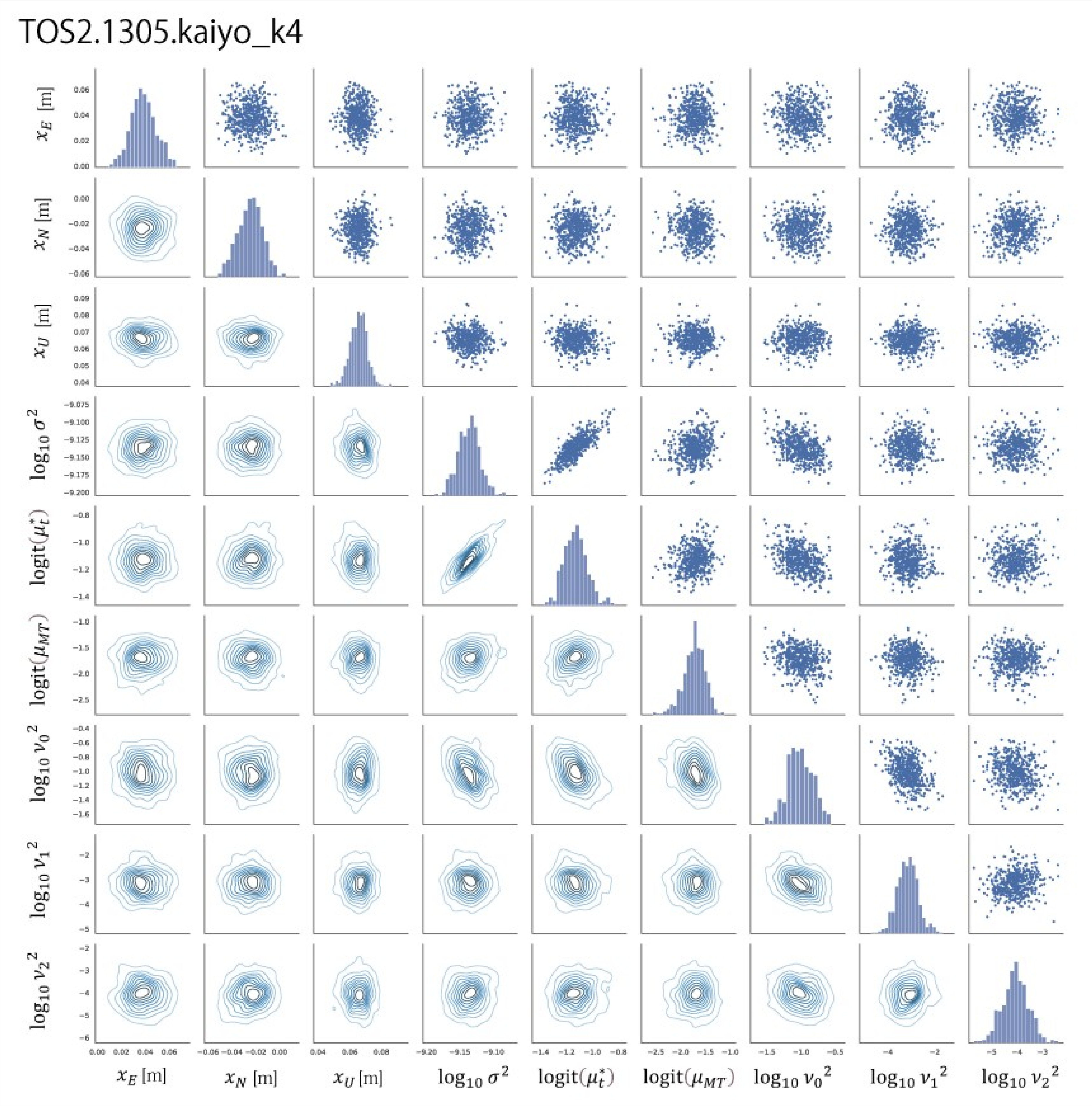
Total sample	50000
Burn-in period	25000
Sampling rate	50

Posterior for the hyperparameters

percentile	log10_nu0	log10_nu1	log10_nu2	mu_t_min	mu_m
0.025	-1.394	-4.010	-5.095	1.466	0.099
0.250	-1.144	-3.453	-4.383	1.563	0.135
0.500	-1.019	-3.141	-4.060	1.629	0.154
0.750	-0.880	-2.860	-3.731	1.694	0.171
0.975	-0.668	-2.134	-3.072	1.871	0.204

We can obtain the distributions including the correlations among the (hyper)parameters

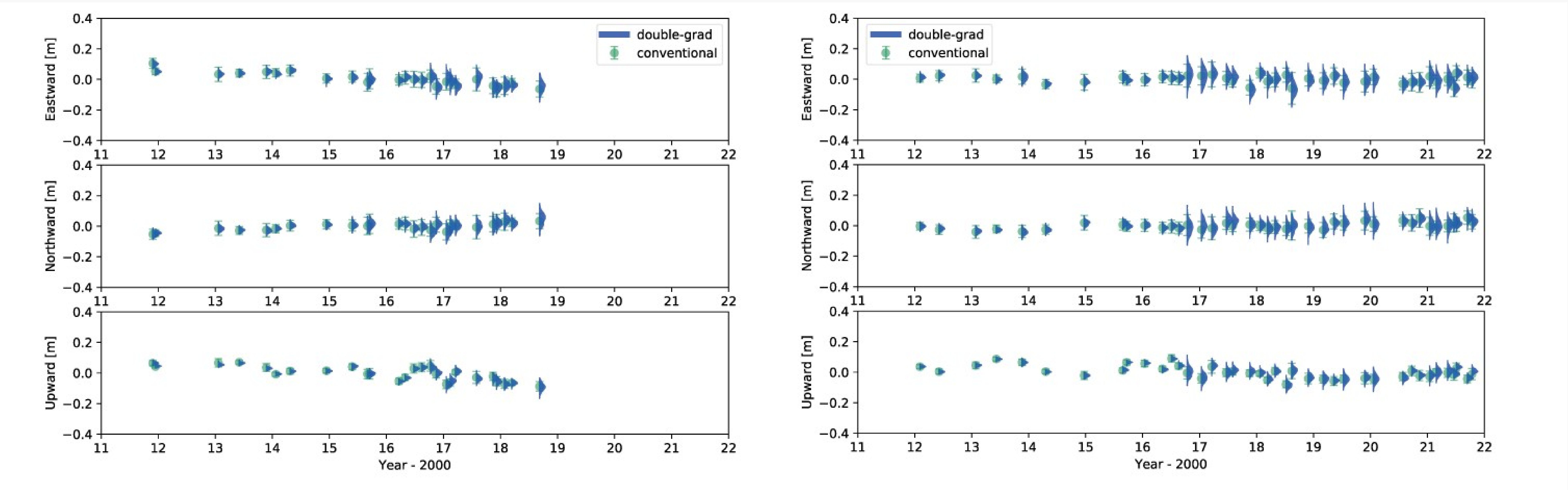
- ✓ Seafloor position is less correlated with HPs
- ✓ Histograms for position are similar to the normal distributions
- ✓ Little correlation is shown among almost all HPs, except σ^2 and μ_t.
  - ✓ That is because σ^2 and μ_t both contribute to the eigenvalues of error covariance matrix (E)



Results of seafloor displacement

Blue：MCMC solution
Green：Conventional（±3σ）

TOS2 KUM2



MCMC results are consistent with the conventional empirical Bayes (HP fixed) solutions.

-> for routine analysis, conventional method can provide “fairly-good” solutions, even for the “rough” HP search

MCMC has advantages in, for example, **(1) the extensibility of perturbation model**, and **(2) analysis for system error evaluation/correction**. -> future works

GARPOS-MCMC software will be released on GitHub/Zenodo in the future