

Numerical Modelling and Validation for 3-D Boulder Transport Related to Tsunami

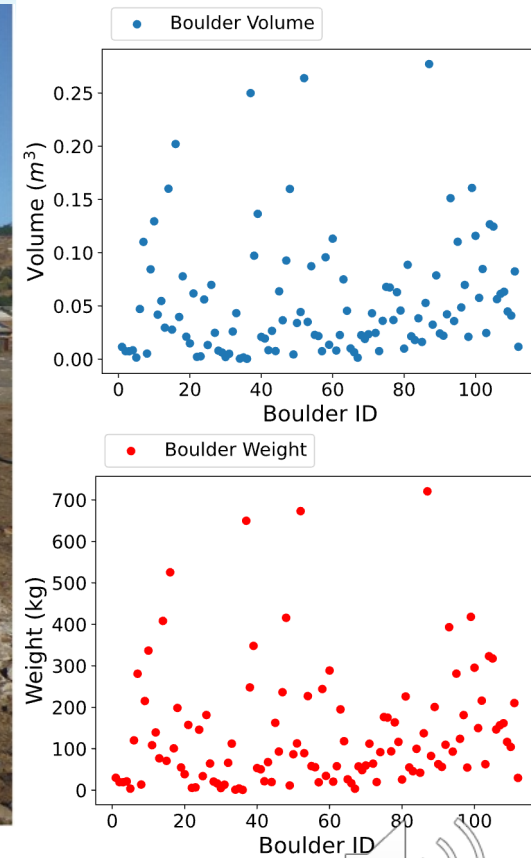
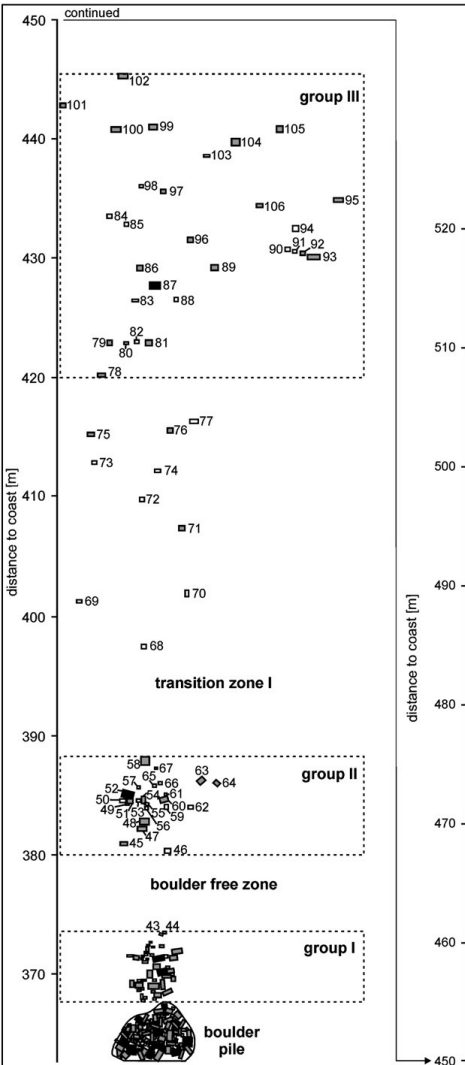
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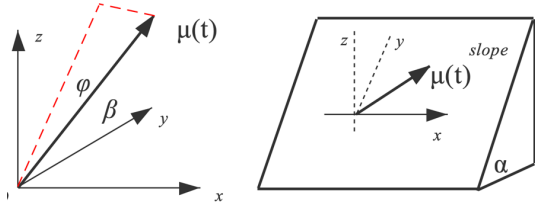
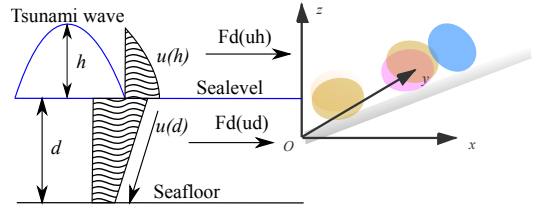
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1. Background: Boulder Transport during Tsunami



(1) Force balance on boulder



(2) Movement condition

Drag force Buoyant weight Damping force

$$F = F_d + F_a \pm F_m - F_f - F_\mu$$

Impulsive force

Frictional force

The total force balance on the boulder(sliding):

$$F_s = \frac{1}{2} C_d A_y \rho_f \left(u(t) \cos(\beta - \alpha) - \frac{ds}{dt} \right)^2 + k \rho_f V \left(a(t) \cos(\beta - \alpha) - \frac{d^2 s}{dt^2} \right) - \left(F_m \cos \alpha - \frac{1}{2} C_d A_y \rho_f (u(t) \sin(\beta - \alpha))^2 - k \rho_f V a(t) \sin(\beta - \alpha) \right) \lambda - F_m \sin \alpha$$

From the force balance we can get:

$$0 \leq \left(\frac{1}{2Lg} C_d (u(t))^2 + k \frac{a(t)}{g} \right) (\cos^2(\beta - \alpha) + \lambda \sin^2(\beta - \alpha)) - \left(\frac{\rho_s}{\rho_f} - 1 \right) (\lambda + \sin \alpha)$$

Rolling:

$$0 \leq \left(\frac{1}{2Lg} C_d (u(t))^2 + k \frac{a(t)}{g} \right) (\cos^2(\beta - \alpha) + \lambda \sin^2(\beta - \alpha)) - \left(\frac{\rho_s}{\rho_f} - 1 \right) (\lambda + \sin \alpha)$$

Suspension:

$$0 \leq \left(\frac{1}{2Lg} C_d (u(t))^2 + k \frac{a(t)}{g} \right) \cos \varphi \sin \beta - \left(\frac{\rho_s}{\rho_f} - 1 \right)$$

(3) Boulder Movement

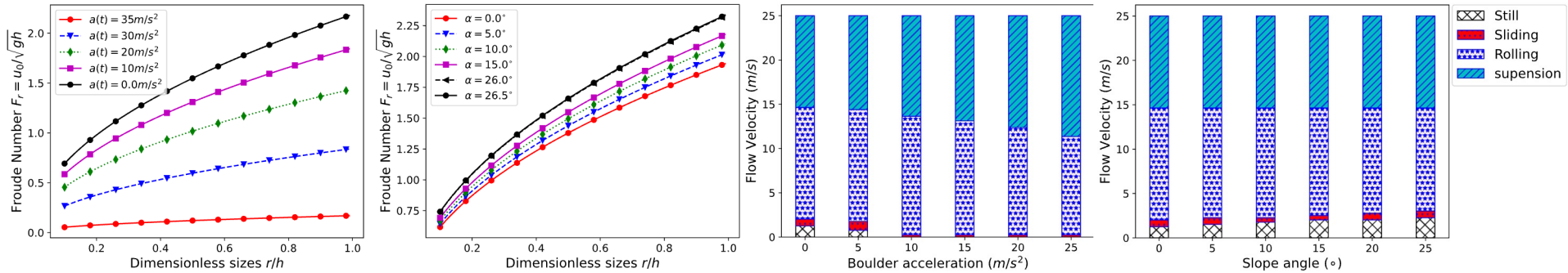
$$F = F_d + F_a - F_m - F_\mu = \left(\frac{7}{5} \rho_s + C_m \rho_f \right) V a(t)$$

$$F_s = \frac{u(t) \sin \varphi - ds/dt}{\left[\frac{u(t) \sin \varphi - ds/dt}{2} \right]^2} C_d A_y \rho_f \left(u(t) \sin \varphi - \frac{ds}{dt} \right)^2 + k \rho_f V \left(a(t) \sin \varphi - \frac{d^2 s}{dt^2} \right) - \left(\frac{u(t) \cos \varphi \sin \beta - ds/dt}{\left[\frac{u(t) \cos \varphi \sin \beta - ds/dt}{2} \right]^2} C_d A_y \rho_f \left(u(t) \cos \varphi \sin \beta - \frac{ds}{dt} \right)^2 + k \rho_f V \left(a(t) \cos \varphi \sin \beta - \frac{d^2 s}{dt^2} \right) - F_m \cos \alpha \right) \lambda - F_m \sin \alpha$$

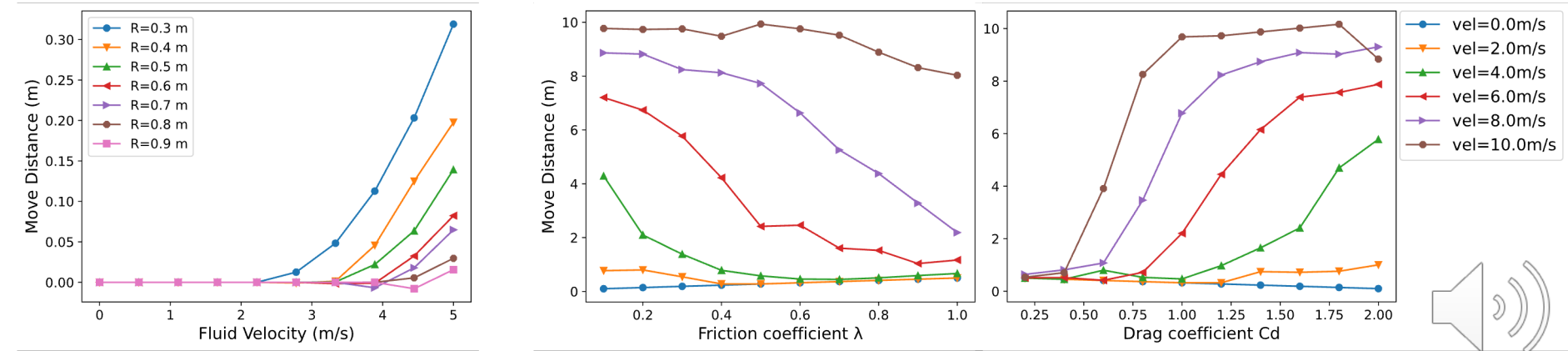
$$\begin{cases} y_{i+1} = y_i + \frac{1}{6} (k_1 + 2k_2 + 3k_3 + k_4) \\ k_1 = hf(x_i, y_i) \\ k_2 = hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1\right) \\ k_3 = hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2\right) \\ k_4 = hf(x_i + h, y_i + k_3) \\ y_0 = y(a), i = 0, 1, \dots, n-1 \end{cases}$$

Discretize the equation and solve the boulder transport model by numerical method

Boulder initial and critical condition

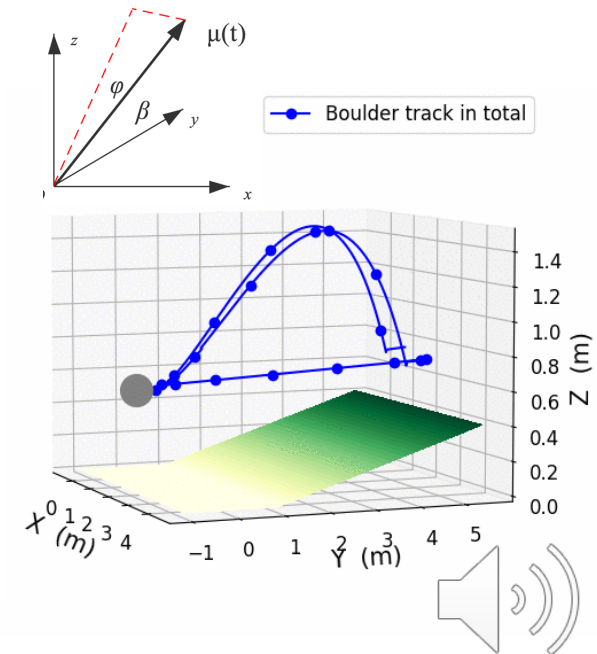
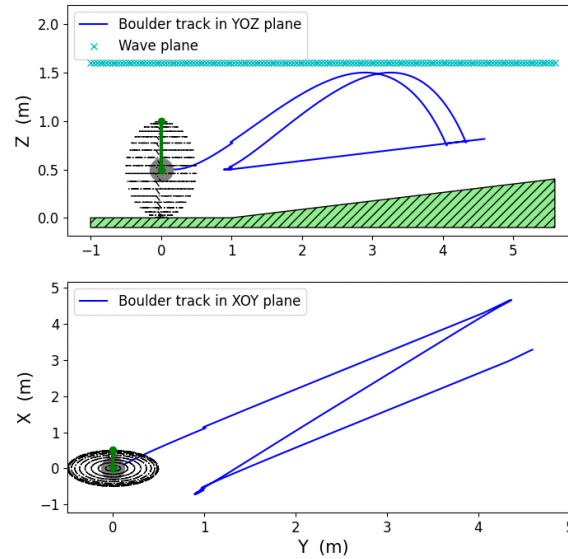
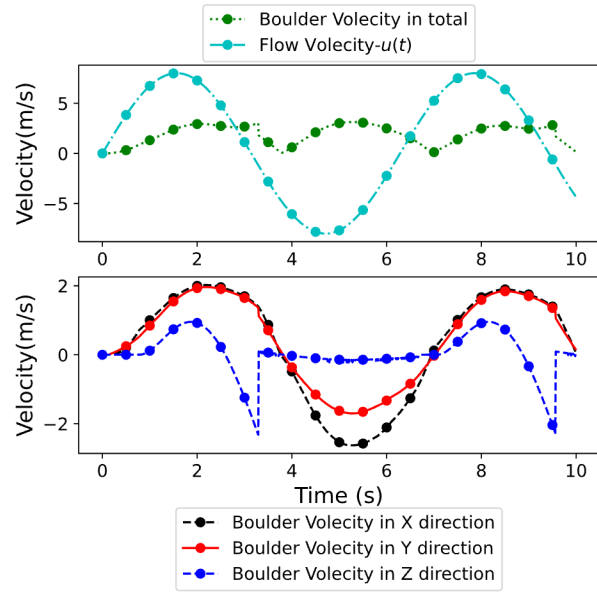


Boulder transport parameter study

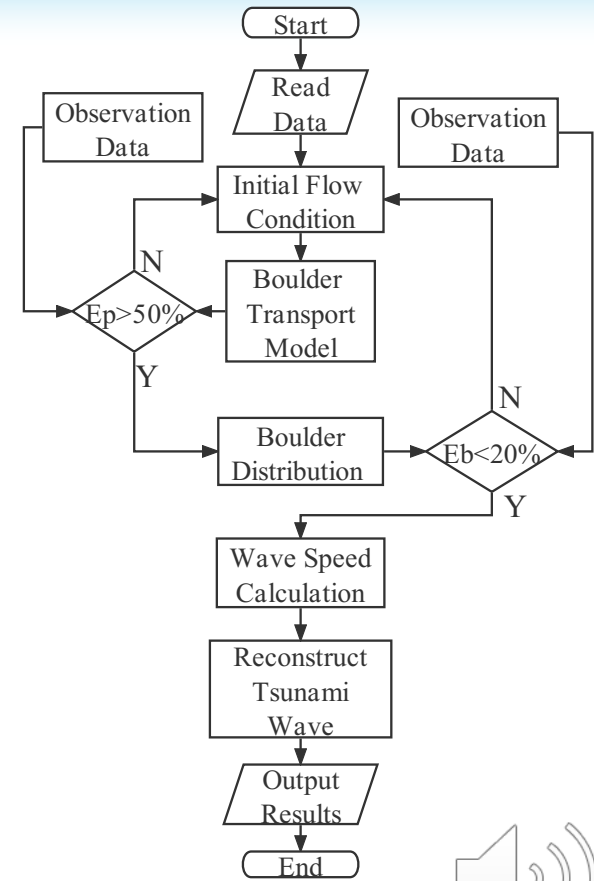
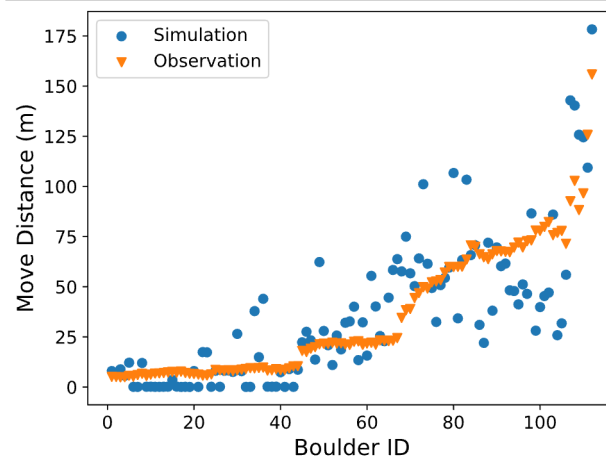
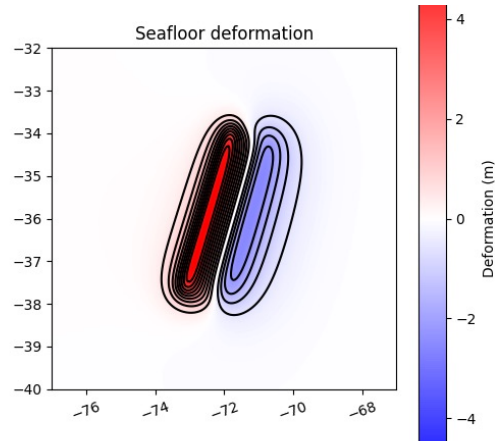
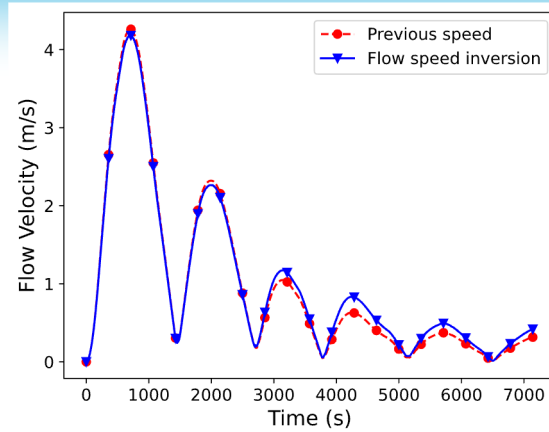
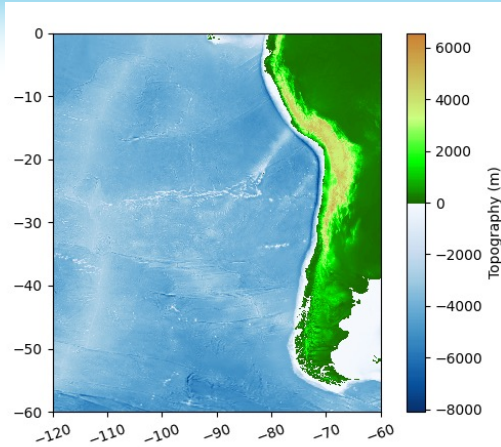


4. Boulder movement in 3D mode

Boulder diameter— d (m)	1.0	Slope angle— α (°)	5	Impulsive coefficient— k	0.5
Boulder density— ρ_s (Kg/m ³)	2500	Fluid direction angle— ϕ/β (°)	45/45	Water depth— h (m)	1.6
Water density— ρ_s (Kg/m ³)	1030	Friction coefficient— λ	0.5	Flow velocity— $u(t)$ (m/s)	4*sin(t)
Drag coefficient— C_d	1.0	Froude number— $u_0/(gh)^{0.5}$	0~1	Flow acceleration— $a(t)$ (m/s ²)	4*cos(t)



5. Validation and inversion in field observation data



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