



# Control Simulation Experiments with the Lorenz-96 Model

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# Outline

- **Lorenz-96 model**
- **Extreme events**
- **Control Simulation Experiments**
- **Full control**
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- **Summary**

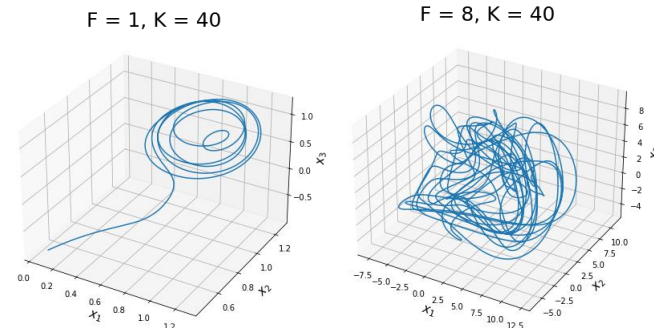
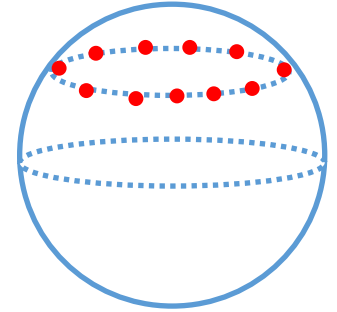
# Lorenz-96 Model

- A dynamical system defined by

$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_k + F$$

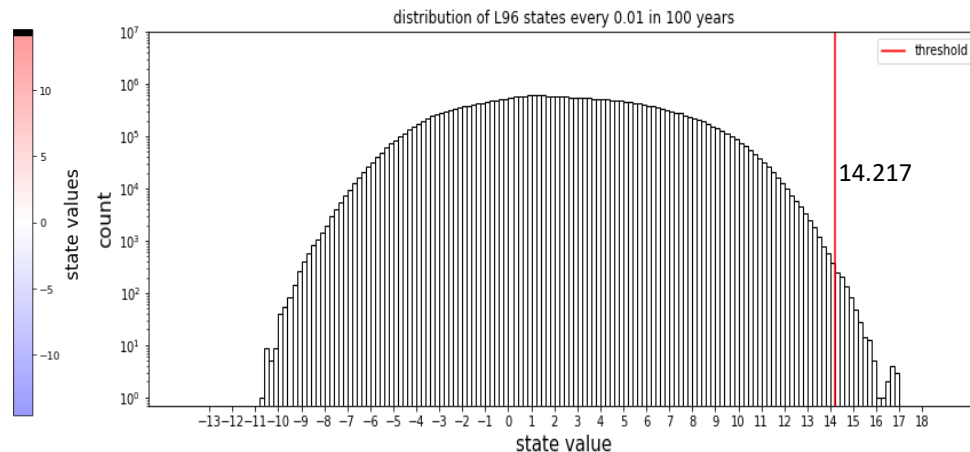
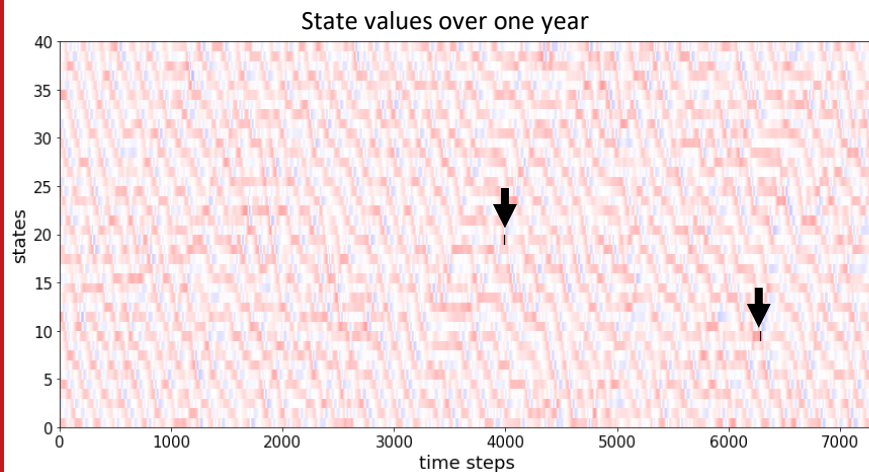
where  $k = 1, \dots, K$ , and  $x_{k-K} = x_{k+K} = x_k$ ,  $K > 3$ <sup>[1]</sup>.

- $x_k$ : values of some atmospheric quantity in  $K$  sectors of a latitude circle.
- $F$  and the linear terms simulate the external forcing and internal dissipation.
- Quadratic terms simulate the advection.
- **1 time unit = 5 days** of atmospheric time.
- For the following experiments:  $F = 8$ ,  $K = 40$ .



# CSE - extreme events

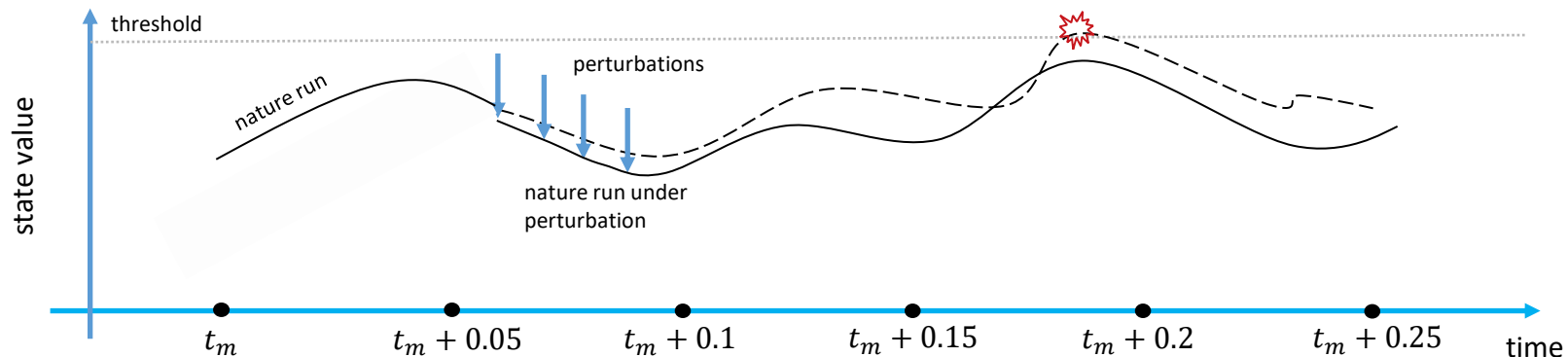
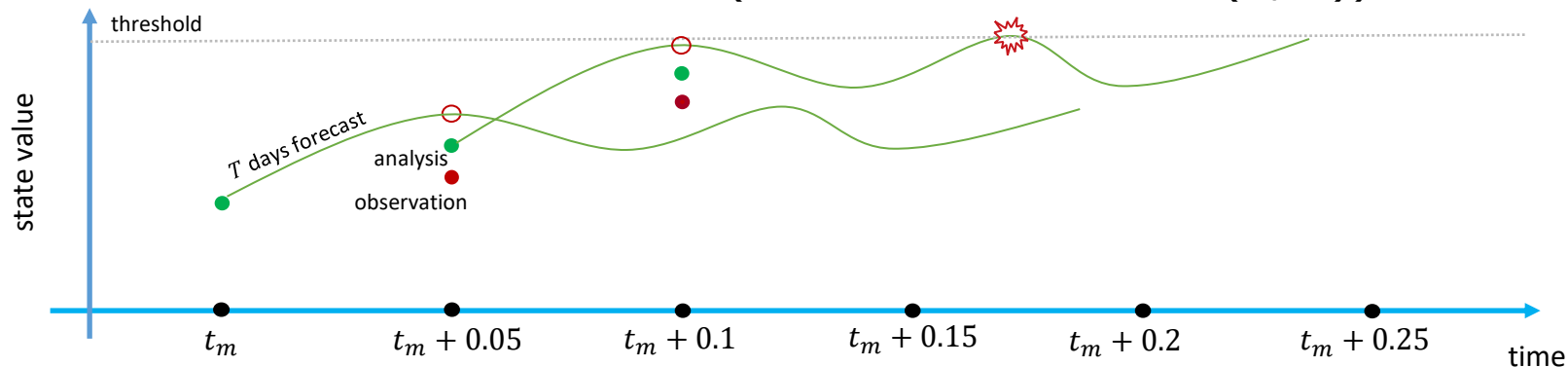
- Aim: to avoid extreme values.
- Integrate L96 over 110 years, keep the record of every  $dt=0.01$  for the last 100 years. Record the maximum value over each 6h period.



- The first 200 maximum values are extreme values (on average 2 times / year), the threshold for extreme events is 14.217.

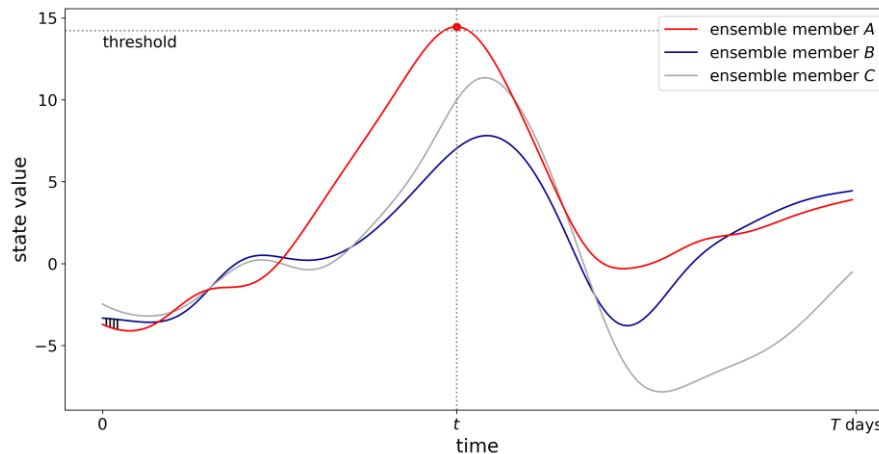
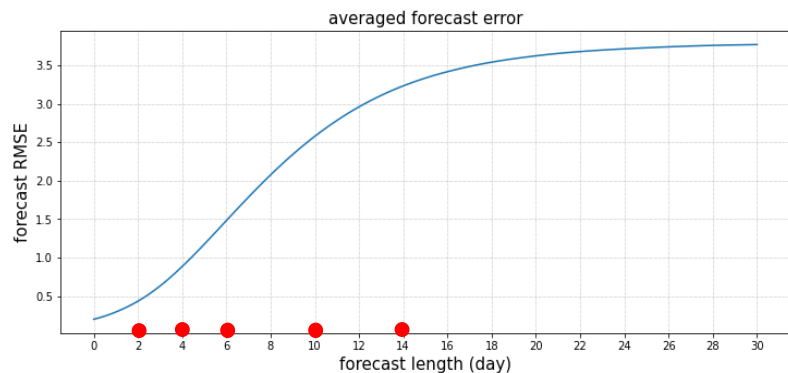
# CSE - procedure

- The observations are noised (nature run +  $\text{Normal}(0, 1)$ ).



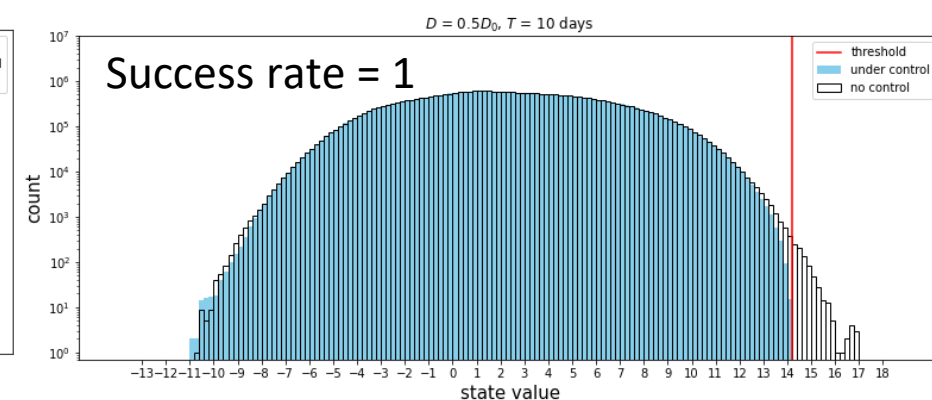
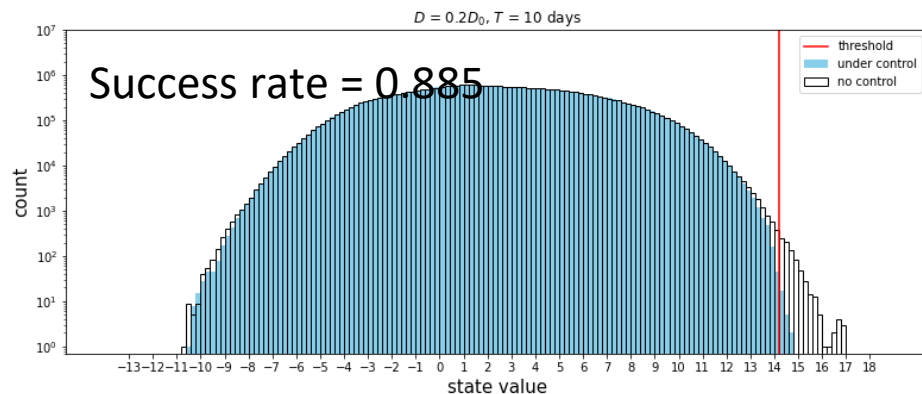
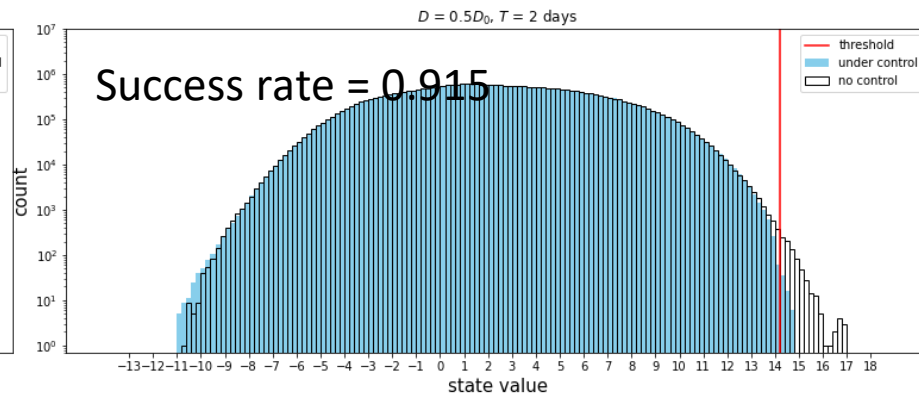
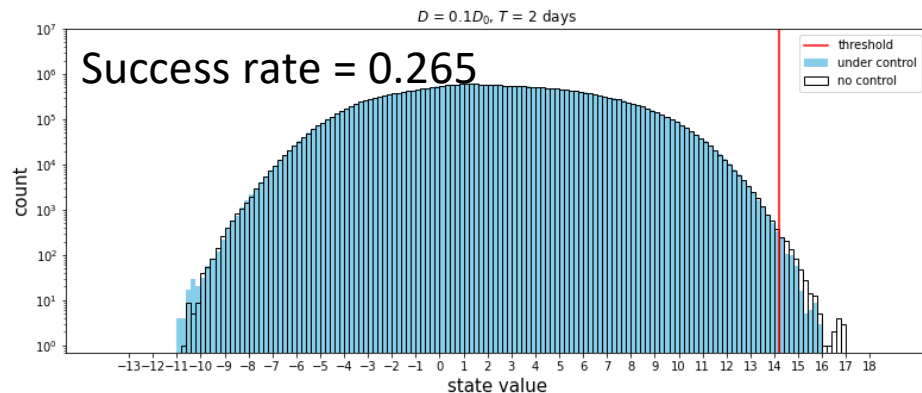
# CSE - LETKF

- We use LETKF with 10 ensemble members,  $\rho = 1.06$ , R-Localization (cut-off radius  $2\sqrt{\frac{10}{3}} \times 5.45$ ) : analysis RMSE  $\approx 0.19890$  ...
- Forecast length  $T$ .



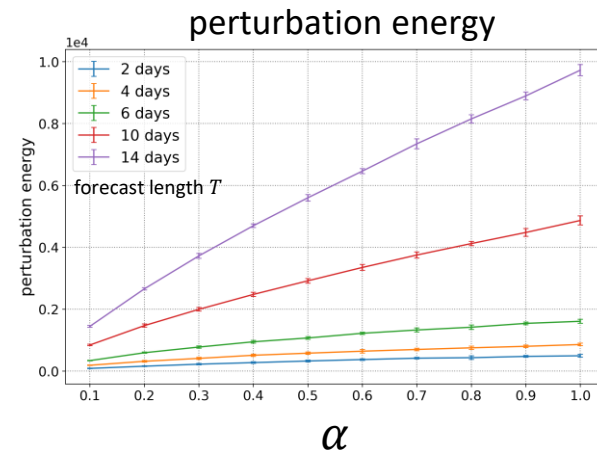
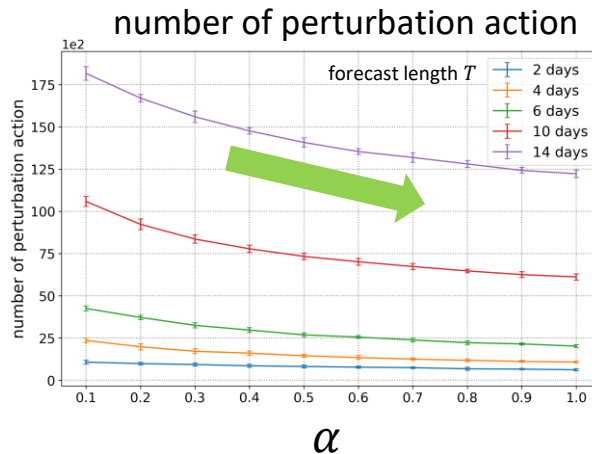
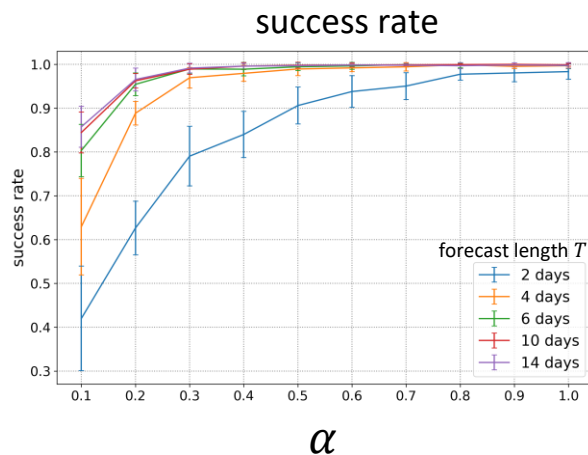
- Perturbation vector: the difference between two proper ensemble members at appropriate time points rescaled to a fixed norm.
- Norm of perturbation vectors :  $D = \alpha D_0$  where  $D_0$  is equal to the analysis RMSE.

# Full control: results for 100 years



Success rate :=  $1 - (\text{\#extreme events in 100 years}) / 200$

# Full control – efficiency and perturbation energy



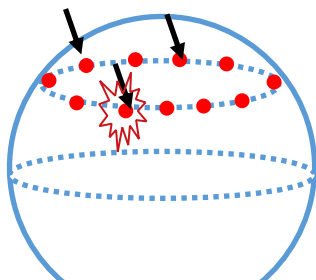
❖ norm of perturbation vectors  $D = \alpha D_0$ ,  $D_0$  = analysis RMSE

perturbation energy  $:=$   
 $4\alpha D_0 \times \#(\text{perturbation actions})$

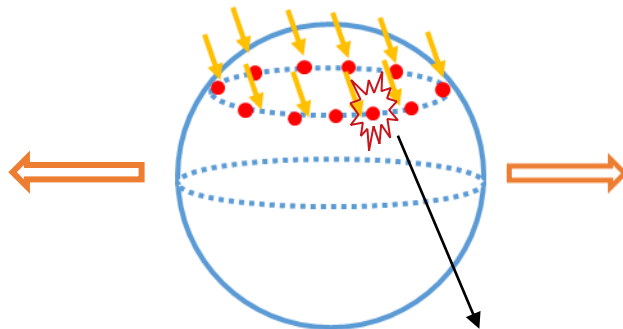
- When the norm of perturbation vectors is small and the forecast length is short, the perturbation vectors are less efficient in terms of avoiding extreme events.
- In contrast, big norm and longer forecast require more energy of perturbations.



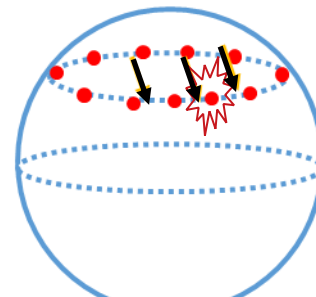
# Partial control



Randomly select  $m$  states.  
Perturb these states  
when necessary.

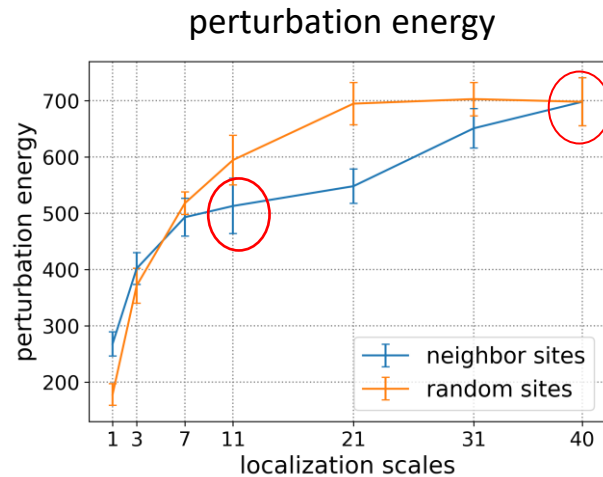
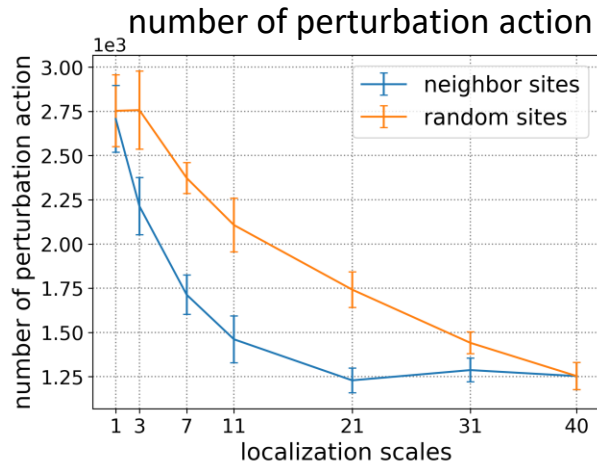
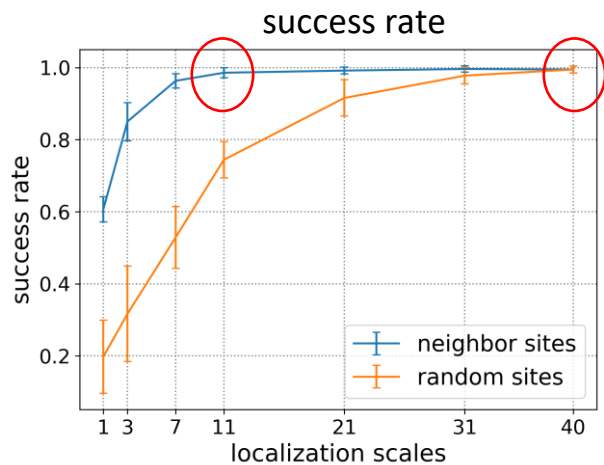


State shows the most  
extreme value in forecast



Perturb the corresponding  
state (and its neighbors)

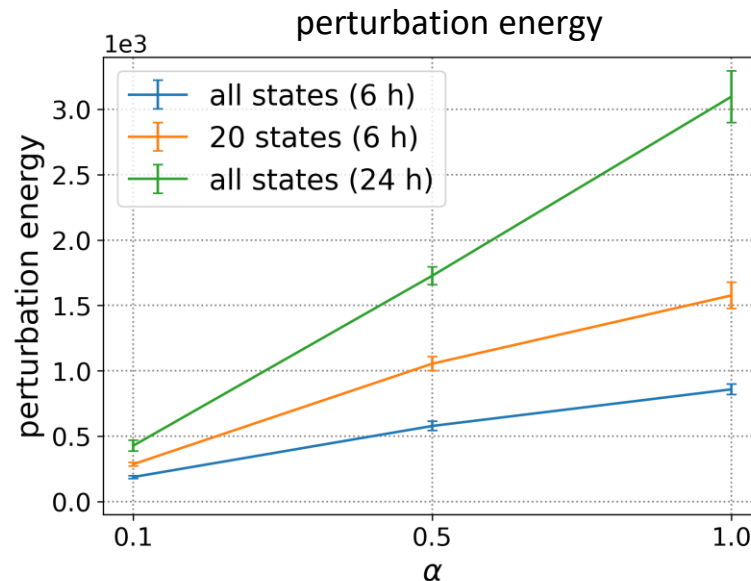
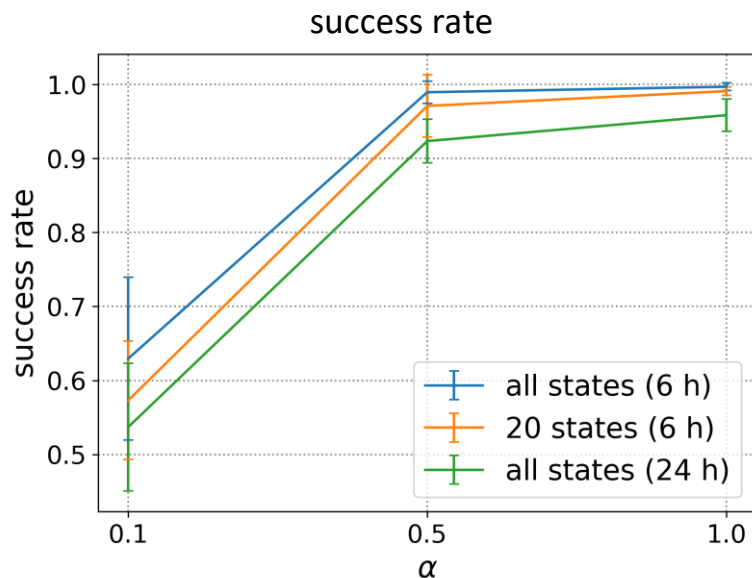
# Partial control (forecast length $T = 4$ days, $\alpha = 0.7$ )



- Perturb random positions generates more actions and cost more energy.
- Perturb random positions is less efficient.

## Partial observations (forecast length $T = 4$ days, $\alpha = 0.7$ )

- Observe 20 states in every 6 hours.
- Observe all states in every 24 hours.



- Partial observations are less efficient compared with observing all states in every 6 hours.

# Summary

- The CSE results show effective control to avoid extreme values.
- Less effective control with
  - 1) perturbations with small norm
  - 2) short forecast length
  - 3) fewer observations, less accurate analysis
- Partial perturbation around the extreme value locations is found effective.

# References

- [1] Lorenz, E., 1996: Seminar on Predictability, Vol. I, ECMWF.
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- [6] Brian R. Hunt, Eric J. Kostelich, Istvan Szunyogh, Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter, Physica D: Nonlinear Phenomena, Volume 230, Issues 1–2, 2007, 112-126, ISSN 0167-2789, 2007.

Thank you for your attention!