Numerical simulation of directional JONSWAP sea waves taking into account four- and five-wave nonlinear resonances

Anna Kokorina [1] & Alexey Slunyaev [1,2]

[1] Institute of Applied Physics, RAS (Russia)

[2] National Research University-Higher School of Economics (Russia)

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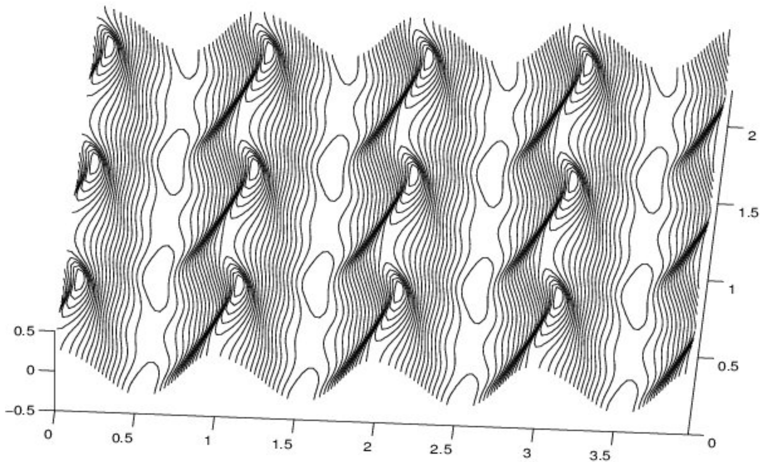
Rogue wave mechanisms

The conventional statistical models take into account the second-order nonlinear effects which are non-resonant (3-wave nonlinear interactions).

The present-day operational wave forecasting is based on the equations for the balance of wave spectral energy (phase-averaged kinetic equations). They take into account 4-wave resonant nonlinear interactions averaged over long time under the assumption of a quasi-stationary state. As a result, they possess a very large time scale $O(\varepsilon^{-4}T_p)$ (where ε is the wave steepness and T_p is its period) and cannot capture faster processes which may occur due to the 4-wave near-resonant nonlinear interactions, such as the modulational instability with the time scale $O(\varepsilon^{-2}T_p)$.

Most of the performed DNS are limited by consideration no higher than 4-wave nonlinear wave resonances (what gives the profit in the simulation time). However, the importance of 5-wave interactions with respect to the rogue wave problem was suggested in some publications [Annenkov & Badulin, 2001].

The 5-wave resonances are forbidden in planar wave geometry. Hence, only the fully-dimensional simulations (of directional waves) are required.



horseshoe pattern caused by 5-wave resonance [Fructus et al, 2004]

Numerical approach (2D example)

Simulation of the evolution in time. The potential Euler equations are solved using the High Order Spectral Method (HOSM, West et al., 1987) which truncates the Taylor expansion for the velocity potential in the vicinity of the rest water level to the order *M*. The depth is assumed infinite, the wind is absent.

1. Fully nonlinear boundary conditions on the free surface $\eta(x,t)$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \varphi}{\partial z}\left[1+\left(\frac{\partial \eta}{\partial x}\right)^2\right]$$

$$\frac{\partial \Phi}{\partial t} = -g\eta - \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \varphi}{\partial z}\right)^2\left[1+\left(\frac{\partial \eta}{\partial x}\right)^2\right]$$

Here $\varphi(x,z,t)$ is the velocity potential in the volume occupied by the fluid, and $\Phi(x,t) = \varphi(x,z=\eta,t)$ is the surface potential. (The formulation suggested by Zakharov, 1968)

2. The problem in the varying volume is approximated through the problem in the volume of a fixed shape.

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

$$\varphi(x,z,t) = \sum_{m=1}^{M} \varphi_m(x,z,t)$$

$$\Phi(x,t) = \sum_{m=1}^{M}\sum_{k=0}^{M-m}\frac{\eta^k}{k!}\frac{\partial^k}{\partial z^k}\varphi_m(x,z=0,t)$$

The velocity potential is decomposed into an asymptotic series with respect to small displacement of the surface. The potentials on the moving surface and at the fixed boundary are related using the nonlinear Taylor expansion up to the order *M*, which may be any. The analytic solution to the Laplace equation + BCs at the bottom and on the fixed upper boundary is used to calculate the vertical velocity on the free surface.

[Dommermuth&Yue, 1987; West et al, 1987]

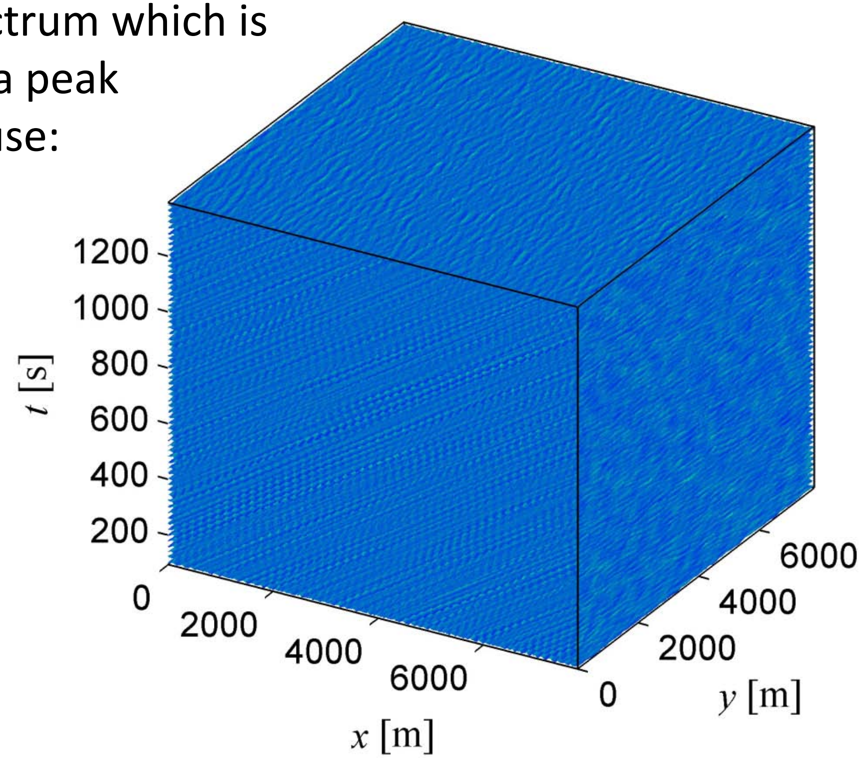
Numerical simulation of directional waves

Sea state conditions

Irregular waves with a given averaged JONSWAP spectrum which is a Pierson-Moskowitz spectrum multiplied by an extra peak enhancement factor γ (peakedness parameter). We use: T_p = 10 s, γ = 3, 6; random Fourier amplitudes and random phases, $H_s \approx 3.5 - 7$ m.

The directional spreading is according to the $\cos^2$ distribution with $\theta = 12°, 62°, 90°, 180°$

$$D(\chi) = \begin{cases} \frac{2}{\Theta}\cos^2\left(\frac{\pi\chi}{\Theta}\right), & |\chi| \le \frac{\Theta}{2} \\ 0, & |\chi| > \frac{\Theta}{2} \end{cases}$$



Simulated by the HOSM (West et al, 1987) with different orders of nonlinearity of the code:

M = 3 (4-wave interactions are resolved) and

M = 4 (5-wave interactions are resolved).

The double-periodic in space domain of the size 50 × 50 dominant wave lengths is simulated for 20 minutes, what corresponds to about 120 dominant wave periods. The data is stored with high resolution in space and time.

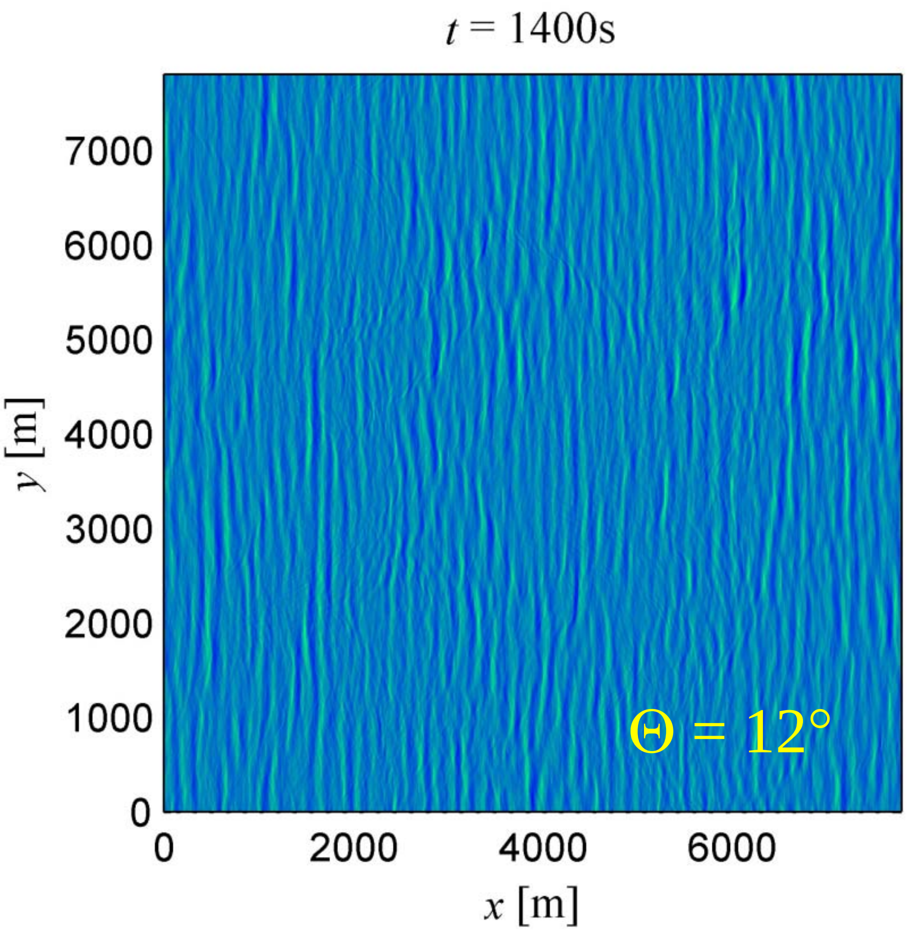
One simulation consists of $1024\times1024\times2400 = 2.1\cdot10^9$ "measurements".

Each sea state is simulated at least 7 times ~ 500 km^2

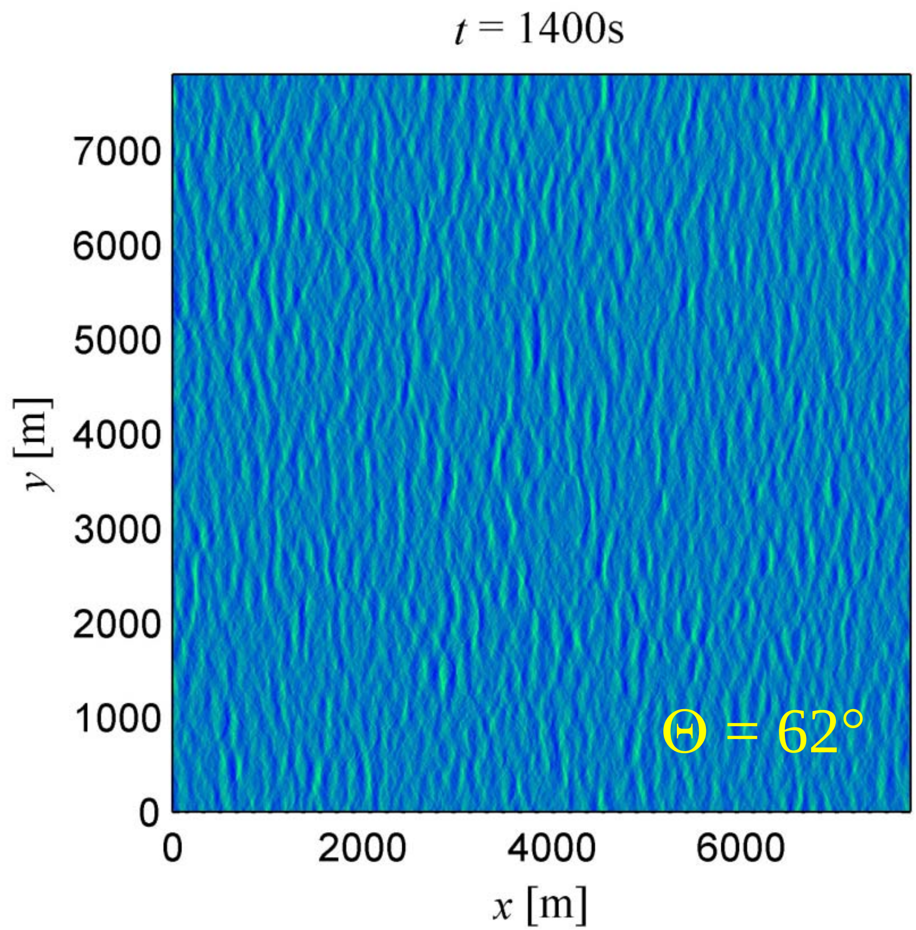
[Slunyaev & Kokorina, WaterWaves'19, IzvAOP'20]

Numerical simulation of directional waves

long-crested sea



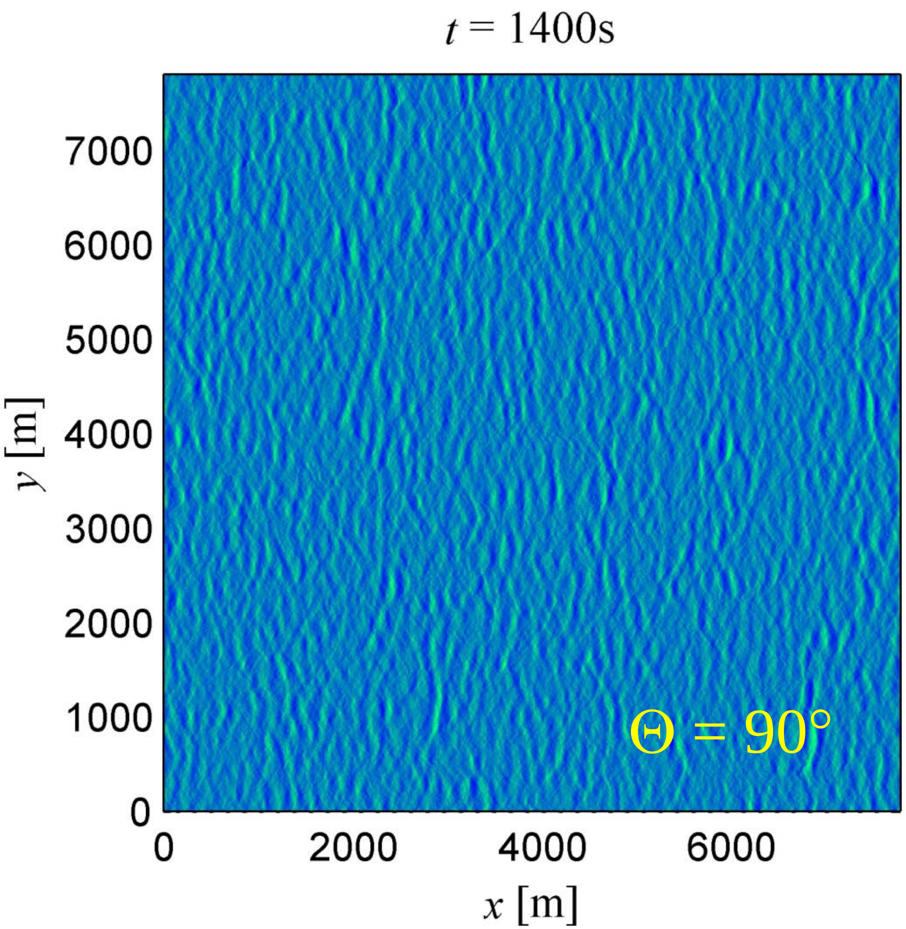
short-crested sea



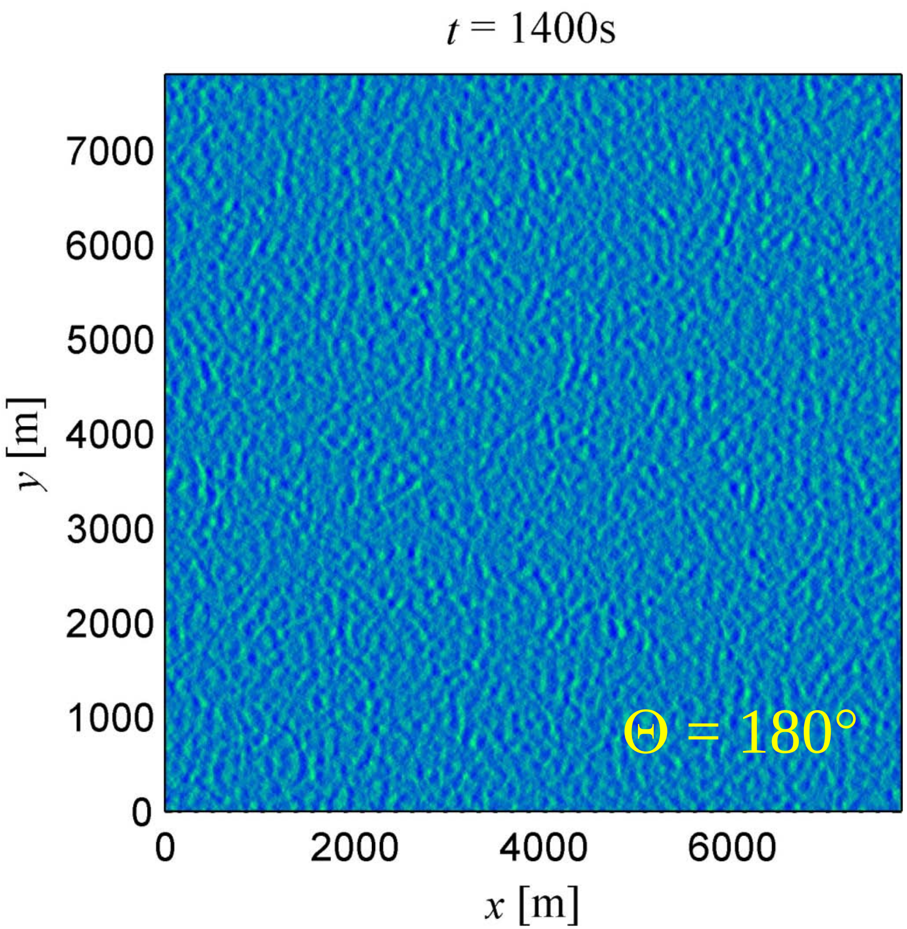
No wind. Almost no dissipation. T_p = 10 s, H_s = 7 m, $\cos^2$ directional spread Θ

Numerical simulation of directional waves

shorter-crested sea



confused sea

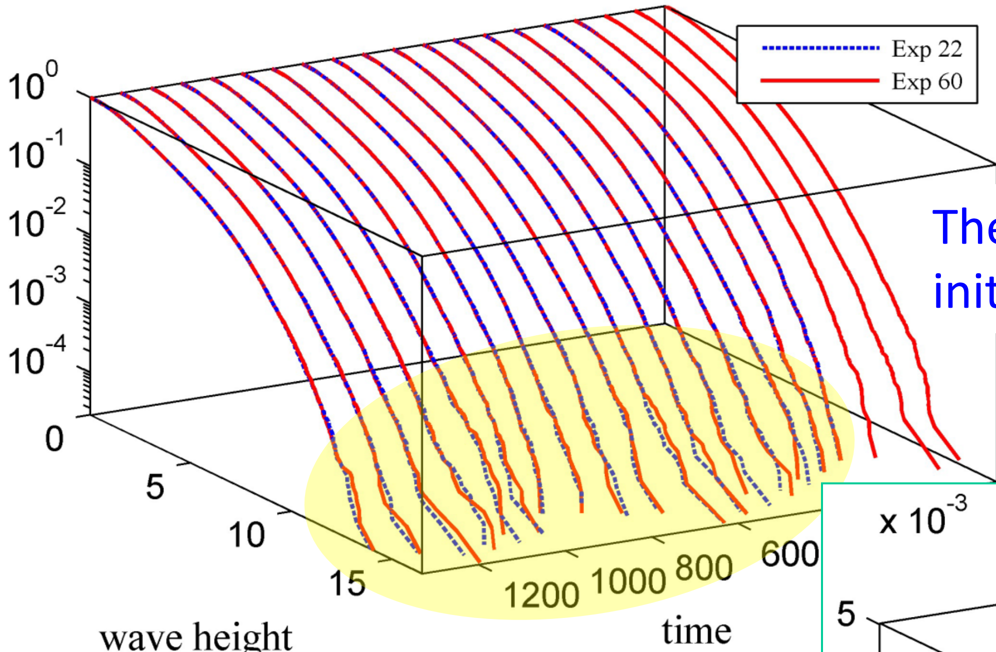


No wind. Almost no dissipation. T_p = 10 s, H_s = 7 m, $\cos^2$ directional spread Θ

Numerical simulation of directional waves

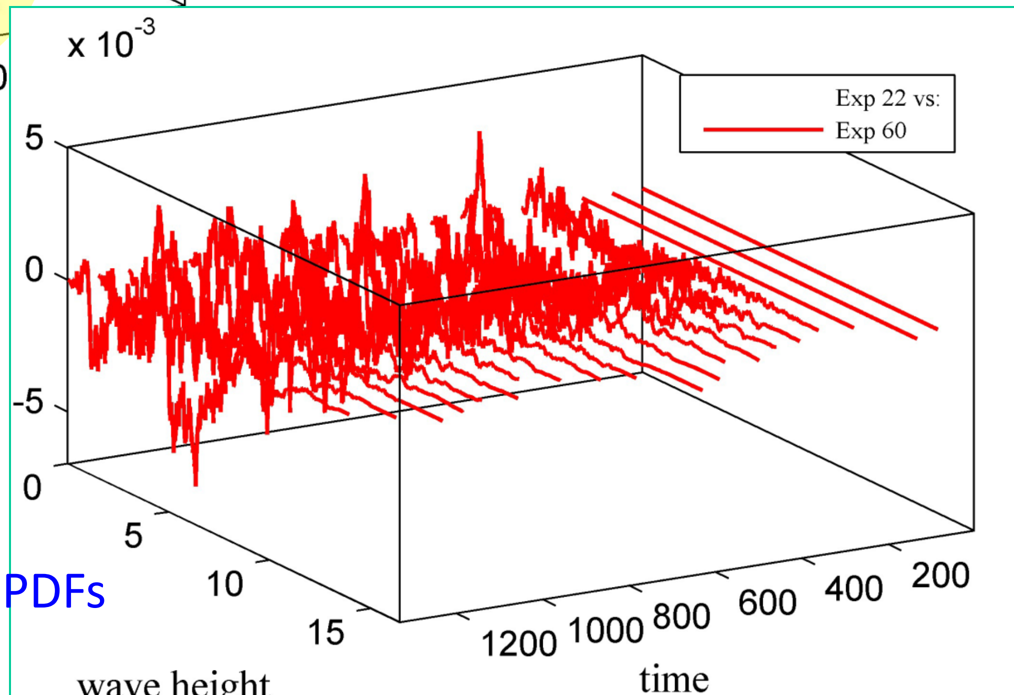
Example of the PDF evolution in two experiments

JONSWAP, H_s = 7 m, γ = 6, Θ = 62°



The PDFs coincide initially

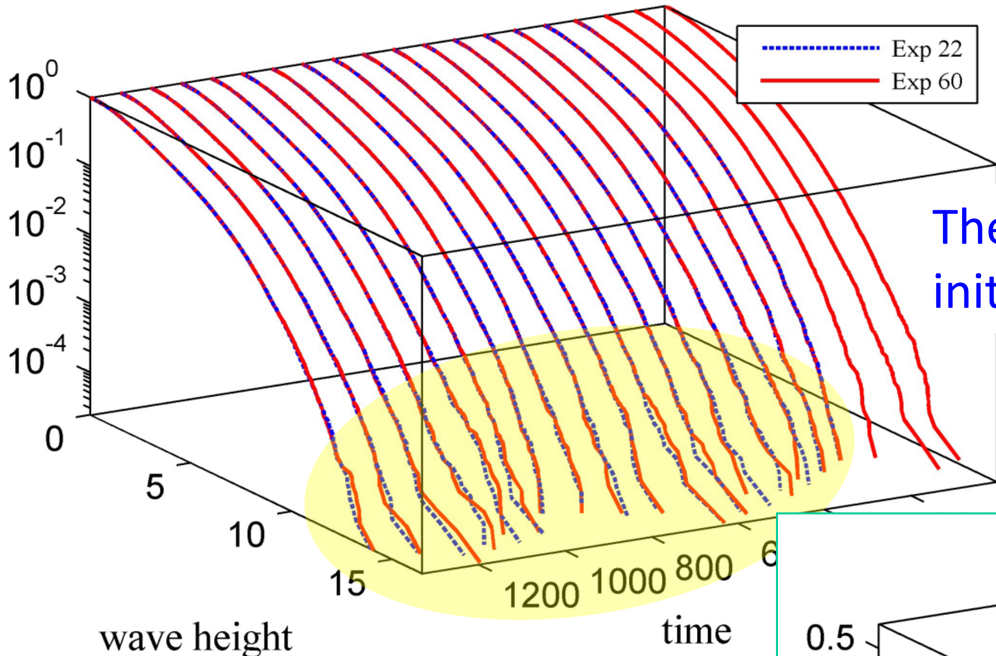
Some difference is observed for the most extreme waves



The simple difference between the PDFs exhibits noisy values for small and moderate heights

Numerical simulation of directional waves

Example of the PDF evolution in two experiments

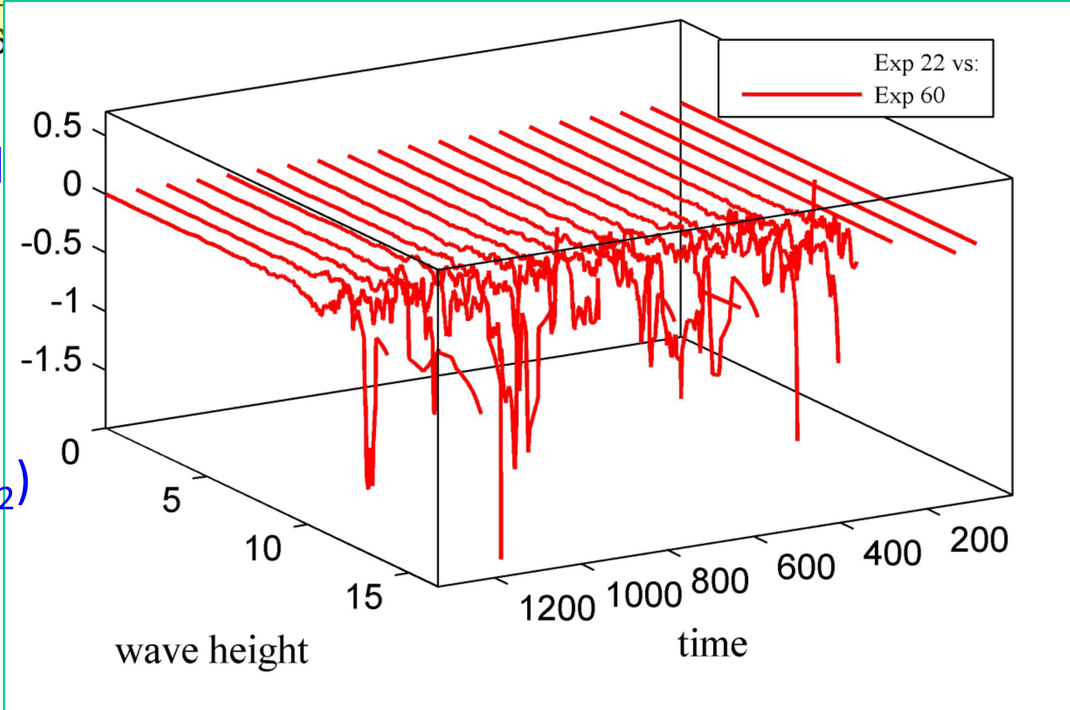


JONSWAP, H_s = 7 m, γ = 6, Θ = 62°

The PDFs coincide initially

Some difference is observed for the most extreme waves

The relative difference $2(P_1 - P_2)/(P_1 + P_2)$ reveals large probability of the very extreme waves in the simulation with higher nonlinearity (Exp 60).



Conclusions

Account of the 5-wave nonlinear interactions **does not change the shape** of the PDF, but leads to the occurrence of even **more extreme waves** in the population of the most extreme ones in the series.

The observed effect **does not show** any reliable **dependence on the wave direction spread**.

Most likely, the effect is related to the **non-resonant wave interactions** (**sharper shapes** of the Stokes waves).

The conclusion is confined by **single-mode** wave configurations with the **JONSWAP** frequency spectra and the **$\cos^2$ directional spread**. Over types of sea states may exhibit different dynamics.