

# Sub-grid modelling of pitch-angle diffusion for ion-scale waves in hybrid-Vlasov simulations with Cartesian velocity space



UNIVERSITY OF HELSINKI  
FACULTY OF SCIENCE



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<sup>2</sup> *Center for Astronomy, Institute for Theoretical Astrophysics, Heidelberg University, Heidelberg, Germany*

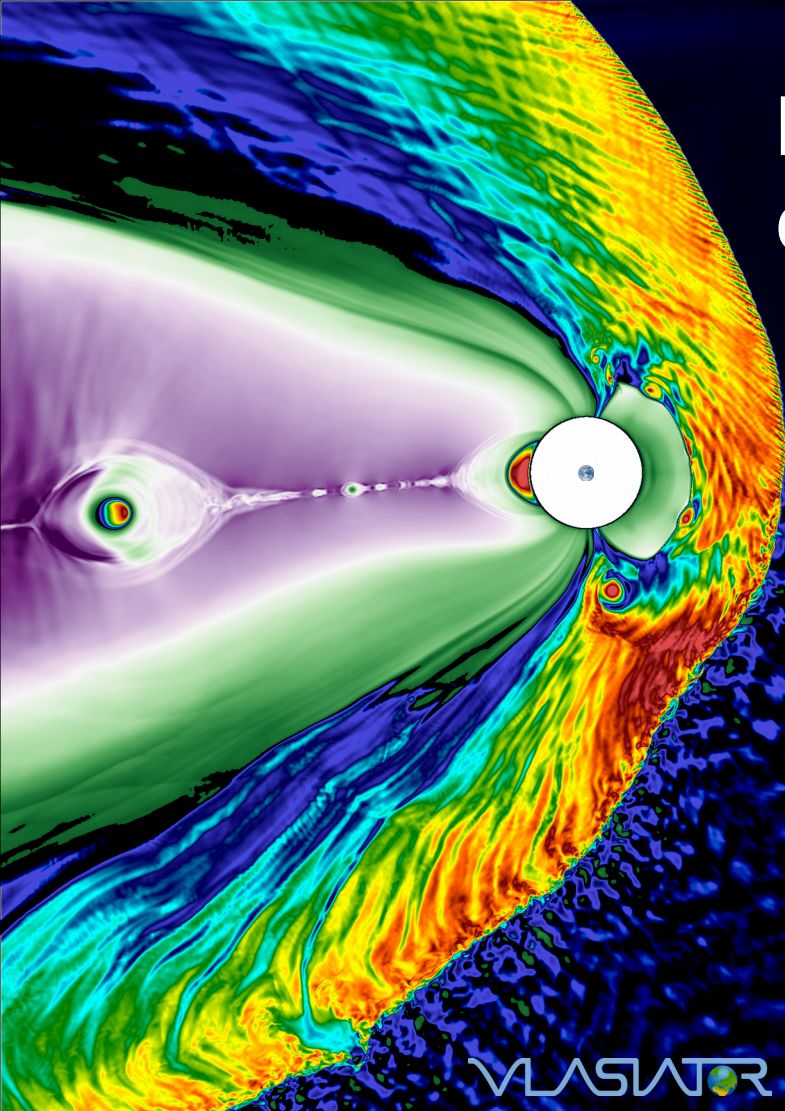
<sup>3</sup> *Swedish Institute of Space Physics, Uppsala, Sweden*

<sup>4</sup> *Space and Earth Observation Centre, Finnish Meteorological Institute, Helsinki, Finland*

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# Numerical simulations play a central role in modern sciences



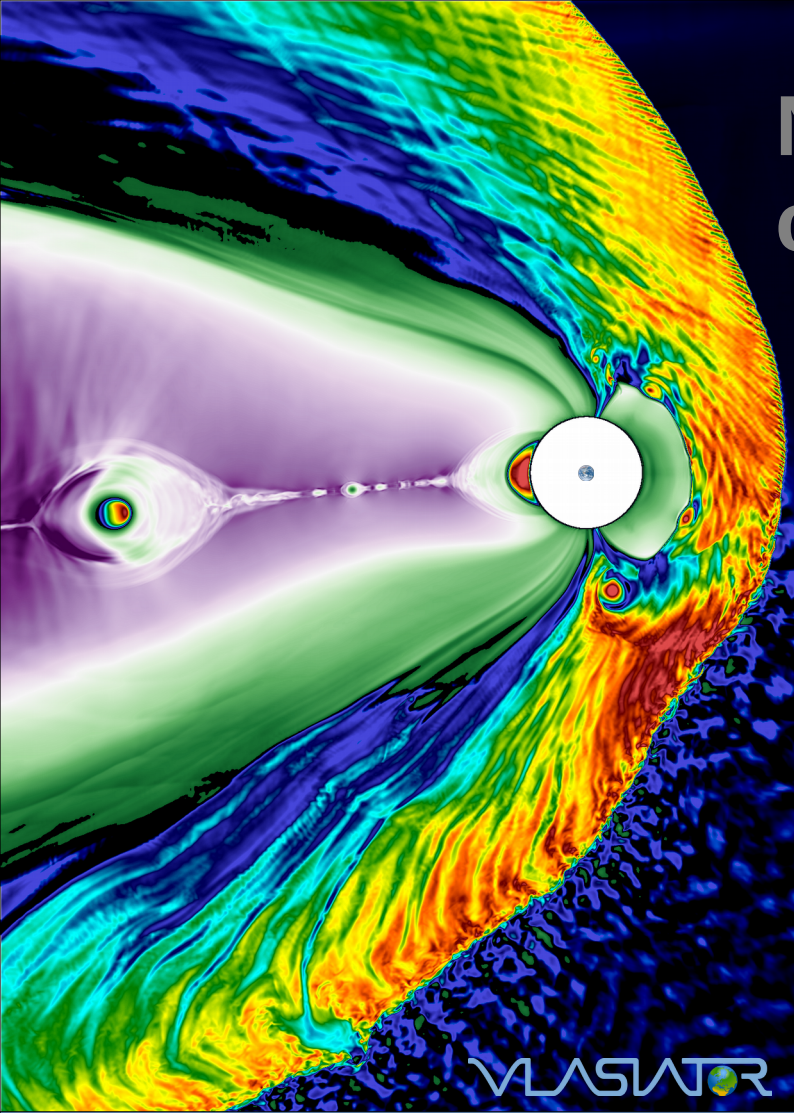
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**MAIN LIMITING FACTOR:  
Resolution of the system**

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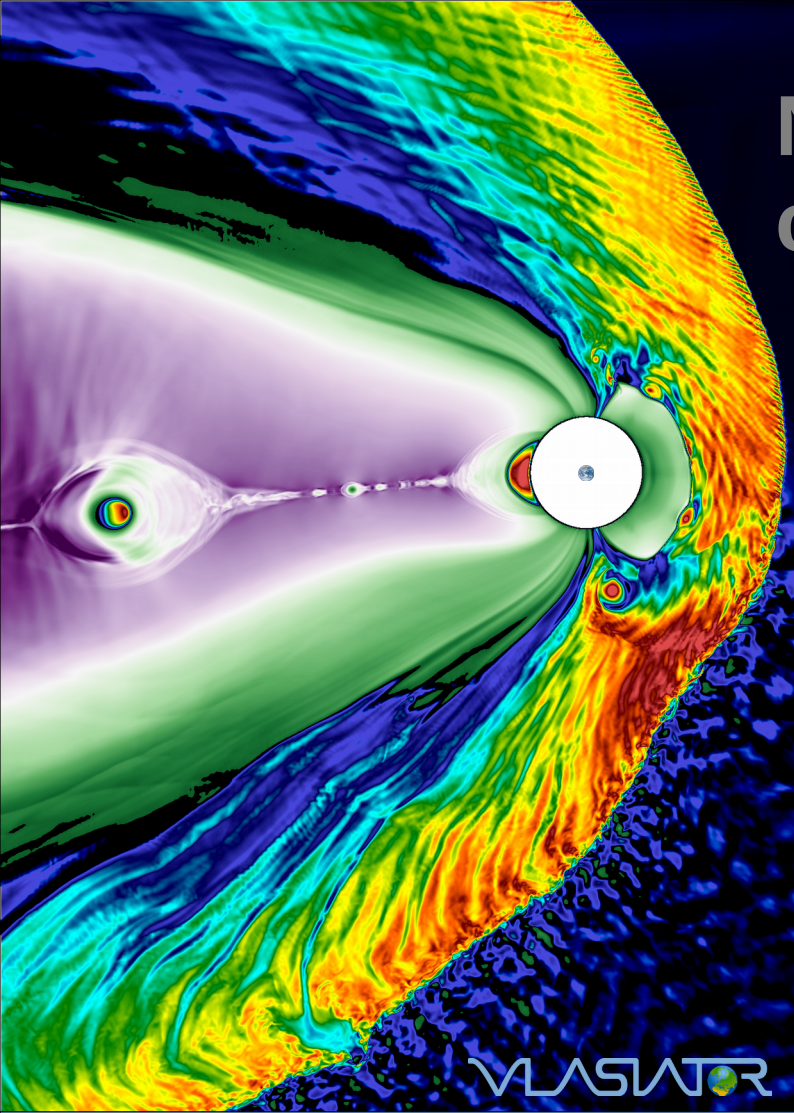


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**MAIN LIMITING FACTOR:  
Resolution of the system**

Trade-off between precision and realistic computational resources

**What do we do when we cannot meet this trade-off?**

# Absence of proton cyclotron instability at low spatial resolution



- Global hybrid-Vlasov simulation of near-Earth space
- 1D, 2D, 3D Cartesian spatial grid
- 3D Cartesian Velocity grid
- Protons described as Velocity Distribution Functions
- Electrons cold, massless, charge-neutralising fluid

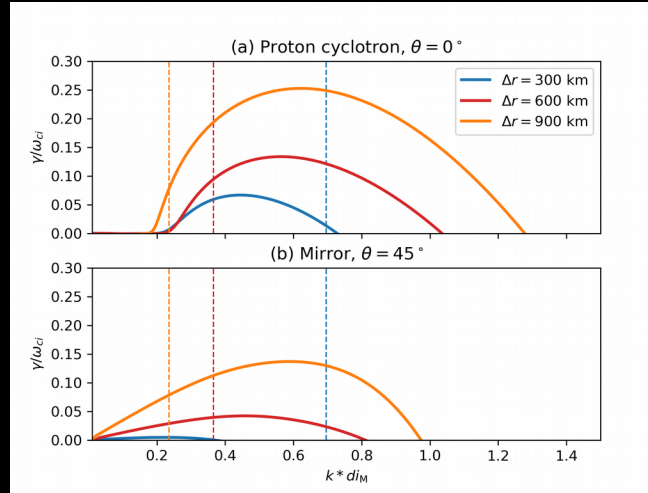
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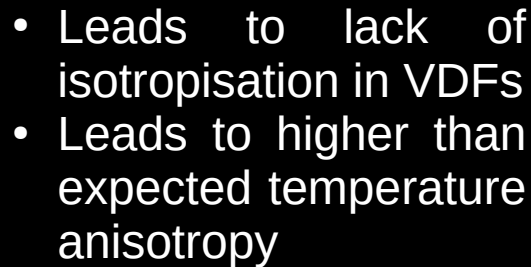
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# Sub-grid modelling of pitch-angle diffusion

$$\frac{\partial}{\partial \mu} \left[ (1 - \mu^2) D_{\mu\mu} \frac{\partial f_{\mu v}^{2D}(\mu, v, t)}{\partial \mu} \right] = \frac{\partial f_{\mu v}^{2D}(\mu, v, t)}{\partial t}$$

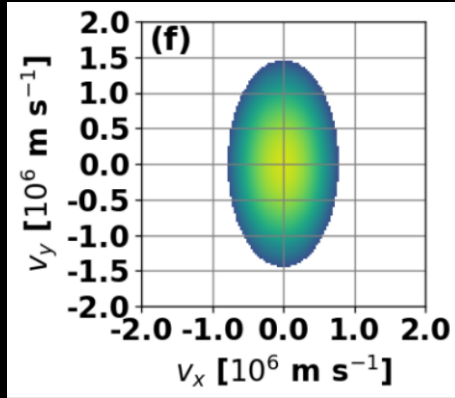
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3D Cartesian velocity space



$$f_{\text{cart}}^{3D}(v_x, v_y, v_z)$$

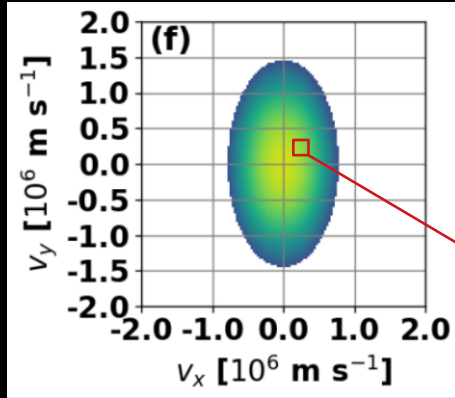
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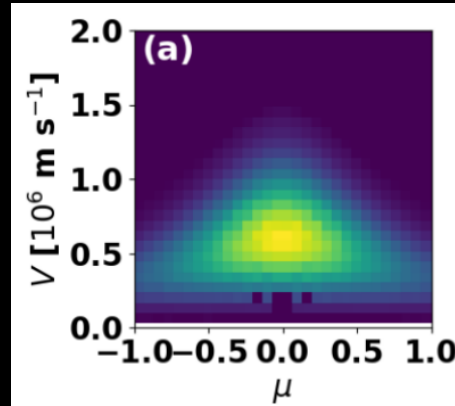
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$$f_{\text{cart}}^{3D}(v\mu, v\sqrt{1-\mu^2}) = \frac{f_{\mu v}^{2D}(\mu, v)}{2\pi v^2}$$

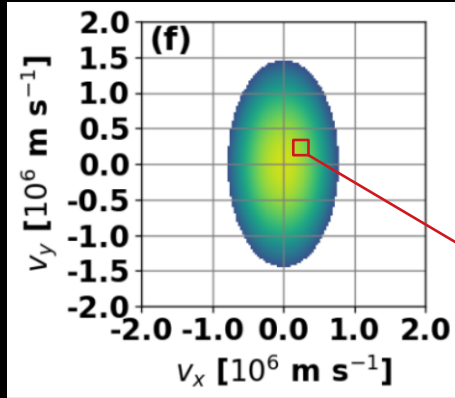
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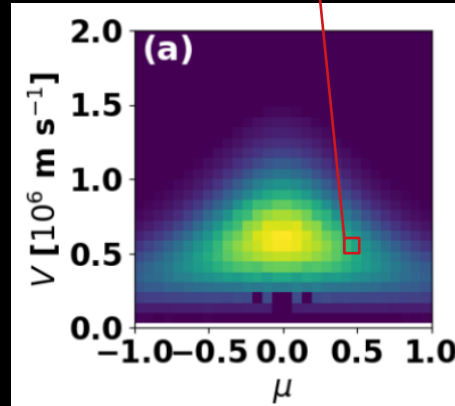
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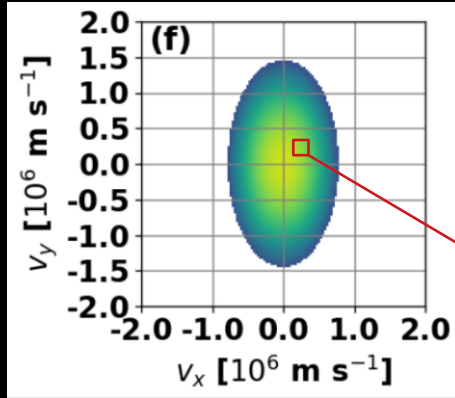
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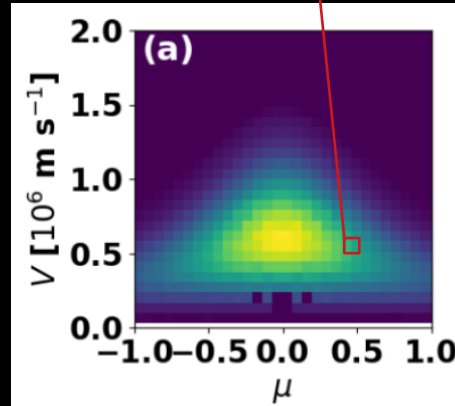
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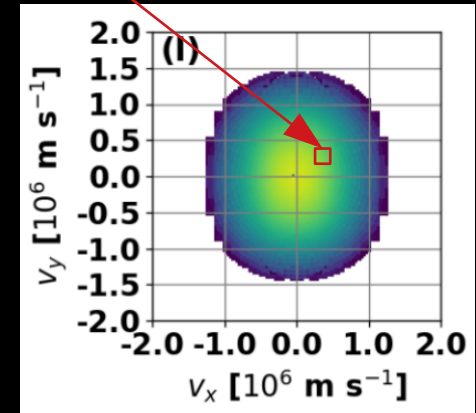


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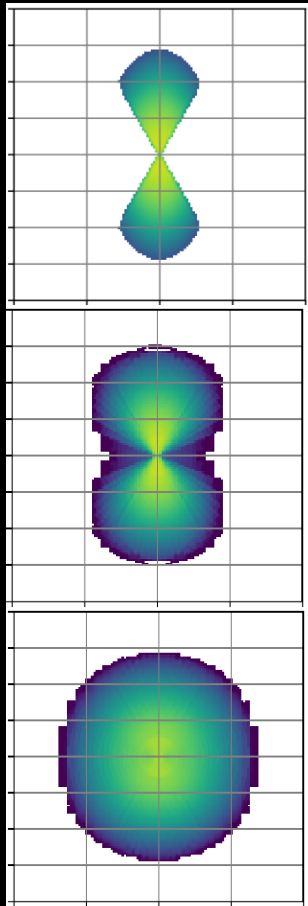
$$f_{\text{cart}}^{3D}(v\mu, v\sqrt{1-\mu^2}) = \frac{f_{\mu v}^{2D}(\mu, v)}{2\pi v^2}$$

$$\frac{\partial f_{\text{cart}}^{3D}(\mathbf{v}, t)}{\partial t} = \frac{1}{2\pi v^2} \frac{\partial f_{\mu v}^{2D}(\mu, v, t)}{\partial t}$$



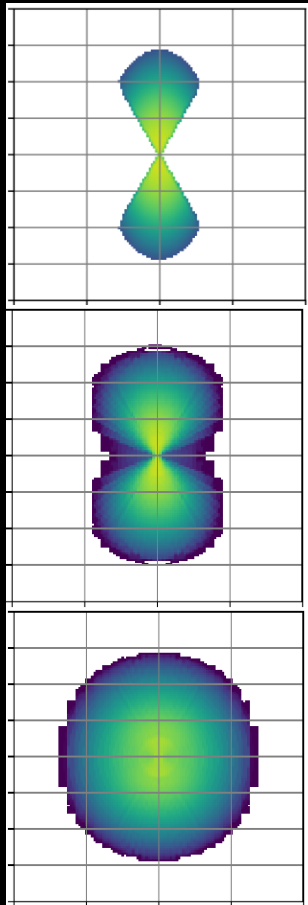
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# Sub-grid modelling of pitch-angle diffusion



- Can be ran with small time sub-steps to maintain numerical stability
- Can be ran with any magnetic field direction
- Isotropises VDFs, reduces temperature anisotropy in the system
- Can be enabled in a small part of the system
- Allows lower resolution runs whilst still modelling accurately pitch-angle diffusion
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<https://www2.helsinki.fi/en/researchgroups/vlasiator>

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