Secular changes in length of day induced by the redistribution potential (display materials)

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This talk is based on the paper ***Secular changes in length of day: Effect of the mass redistribution*** by Baenas, Escapa, & Ferrándiz (Astronomy & Astrophysics, 648, A89, 9 pp., 2021, https://doi.org/10.1051/0004-6361/202140356)

We refer the reader to that publication in order to find the precise definition of the notations and symbols used in the next slides, as well as a comprehensive discussion on this topic.

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Context

- The tidal action of the Moon and the Sun induces an Earth mass redistribution
- Such redistribution gives raise an additional term of the gravitational potential of the Earth —the redistribution tidal potential— that affects the Earth's rotation
- In previous research (Baenas et al. 2019, 2020), we derived the tidal redistribution contributions to the precession and nutation of the Earth
- Those results were computed within the Hamiltonian framework of the rotation of the non-rigid Earth, providing consistent analytical formulae that can be evaluated for different Earth parameters like, e.g., Love number sets
- In this communication we extend the above formulation to study the secular variation of the Earth's rotation rate due to redistribution tidal effects

Context

- A change of the Earth's rotation rate can be alternatively expressed in terms of a change of length of day (LOD)

$$\delta\mathrm{LOD} = -\overline{\mathrm{LOD}}\frac{\delta\omega_z}{\omega_E}$$

- So, the secular acceleration of the Earth about its spin axis is also described by

$$\delta\left(\frac{d\mathrm{LOD}}{dt}\right) = -\frac{\overline{\mathrm{LOD}}}{\omega_E}\delta\left(\frac{d\omega_z}{dt}\right)$$

- Observationally, the tidal part of LOD rate is about 2.3±0.1 ms cy^{-1} (Stepheson et al. 2016), including a contribution of atmospheric tides that amounts -0.1 ms cy^{-1} (Ray et al. 1999)
- There are other significant contributions to LOD rate of non-tidal origin related to changes of the Earth's inertia matrix, e.g., glacial isostatic adjustment (GIA), and other mechanisms (Gross 2015)

Context

- Other works have also computed the tidal secular deceleration

Authors	LOD rate (ms cy^{-1})	Mass redistr.	Earth model	Analyt.
Getino & Ferrándiz (1991)	2.10	AE	1L	Yes
Krasinsky (1999)	2.13	AE	1L	Yes
Ray et al. (1999)	2.38	OT	1L (atm.)	No
Lambert & Mathews (2008)	2.34	AE+OT	2L	No
Mathews & Lambert (2009)	2.50	AE+OT	2L	No
Williams & Boggs (2016)	2.40	AE+OT	2L	Yes

AE: anelasticity of the mantle; OT: oceanic tide; 1L/2L: one-layer/two-layer Earth model

- That research was based on the torque-approach, but Getino & Ferrándiz (1991), not always providing analytical formulae
- Mathews & Lambert (2009) and Williams & Boggs (2016) considered quite similar Earth models (two-layer, coupled core-mantle), although the underlying oceanic model is different (CSR4.0 *vs* FES2004) as well as the Love frequency dependence

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Method and results

- Our baseline Earth model consists of a two-layer structure with a deformable mantle and a fluid core
- The gravitational interaction of the Moon and the Sun is described from Kinoshita (1977) orbital functions $B_{i;p}$, $C_{i;p}$, and $D_{i;p}$ computed from ELP2000 and VSOP82 ephemeris
- The redistribution is characterized by introducing the Love number formalism with frequency dependence per tide:
  - IERS Conventions 2010: solid tides
  - Williams & Boggs (2016): ocean direct contribution (FES2004)
- It is also considered a dissipative torque at the core mantle boundary (CMB), governing the degree of coupling of the core and the mantle (parameter R^*)
- All the above features are implemented (Baenas et al. 2021) within a Hamiltonian framework, providing an analytical expression for the tidal secular canges of LOD or acceleration

Method and results

- The resulting formula for the secular angular deceleration is

$$\delta\left(\frac{d\omega_{0z}}{dt}\right) = -\omega_E \frac{C}{C_{\text{eff}}(T)} \sum_{p,q=M,S} f_q k_p \sum_{\substack{i,j;\tau,\epsilon\in\mathcal{I}\\ m=1,2}} \left|\bar{k}_{2m,j}\right| T^{(\omega_z)}_{ijpq,m} \sin\varepsilon_{2m,j}$$

- Here, we have the following dependences (Baenas et al. 2021):

  - Orbital functions

$$T^{(\omega_z)}_{ijpq,m} = 3C_{i;p}C_{j;q}\delta_{m1} + \frac{3}{2}D_{i;p}D_{j;q}\delta_{m2}$$

  - Love numbers

$$\bar{k}_{2m,j} = \left|\bar{k}_{2m,j}\right| \mathrm{e}^{\mathrm{i}\,\varepsilon_{2m,j}}$$

  - Model parameters

$$k_p = \frac{3Gm_p}{\omega_E a_p^2} H_d, \quad f_q = \frac{m_q a_E^2}{3CH_d}\left(\frac{a_E}{a_q}\right)^3$$

  - Effective polar moment of inertia

$$\frac{1}{C_{\text{eff}}(T)} = \frac{1}{C}\left(1 + \frac{C_c^2}{C}\frac{1-e^{-\frac{CR^*T}{C_mC_c}}}{R^*T}\right) \longrightarrow \begin{cases} \lim\limits_{T\to0^+} C_{\text{eff}}(T) = C_m \ (\text{decoup.}) \\ \lim\limits_{T\to+\infty} C_{\text{eff}}(T) = C \ (\text{tot. coup.}) \end{cases}$$

Method and results

- The tidal secular LOD for totally coupled and decoupled cases

Case	Solid tides (IERS 2010)	Ocean tides (WB2016)	Total (ms cy^{-1})
Tot. coupled	0.108	2.310	2.418
Decoupled	0.122	2.607	2.729

- In the totally coupled case, the different contributions of each frequency band and kind of tides can also be isolated

Potential	Solid tides (IERS 2010)	Ocean tides (WB2016)	Total (ms cy^{-1})
Tesseral *(m=1)*	-0.004	0.352	0.348
Sectorial *(m=2)*	0.112	1.959	2.071
Total	0.108	2.310	2.418

- Very good agreement with the Williams & Boggs (2016) value and the observed estimation 2.4 ms cy^{-1} corrected with the atmospheric tide contribution (-0.1 ms cy^{-1}, Ray et al. 1999)

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Summary

- We have derived a closed-analytical formula for the secular tidal change of LOD through the Hamiltonian formalism of the non-rigid Earth

- The model considers a two-layer Earth, with dissipation at the CMB, and the tidal redistribution characterized for two sets of Love numbers related to the solid and oceanic tides

- For the Love numbers sets given by IERS Conventions 2010 —solid tides— and William & Boggs (2016) —oceanic tides—, we obtained a numerical value about 2.42 ms cy^{-1} assuming a totally coupled core-mantle secular evolution

- That value is in very good agreement with the observed estimation (Stepheson et al. 2016) and other theoretical works (William & Boggs 2016)

Summary

- The Hamiltonian approach offers the advantage that all the effects of the redistribution tidal potential are worked out from the **same Hamiltonian function**, so the redistribution contribution to precession, nutation, and secular changes in LOD remains necessarily **consistent**
- In addition, such analytical formulation, leading to literal formulae, allows a direct application to different Earth rheological and oceanic models

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