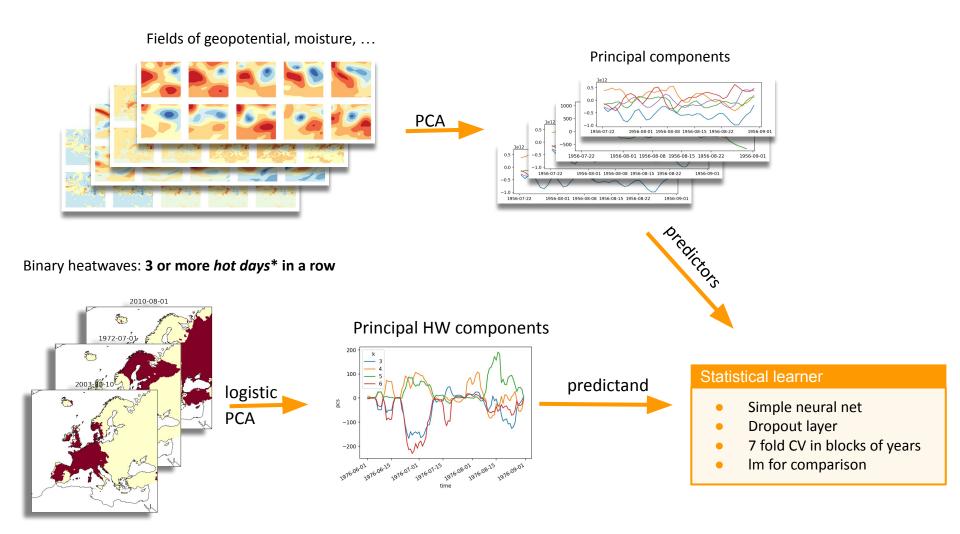
Identifying drivers for heat waves using wavelets and machine learning approaches

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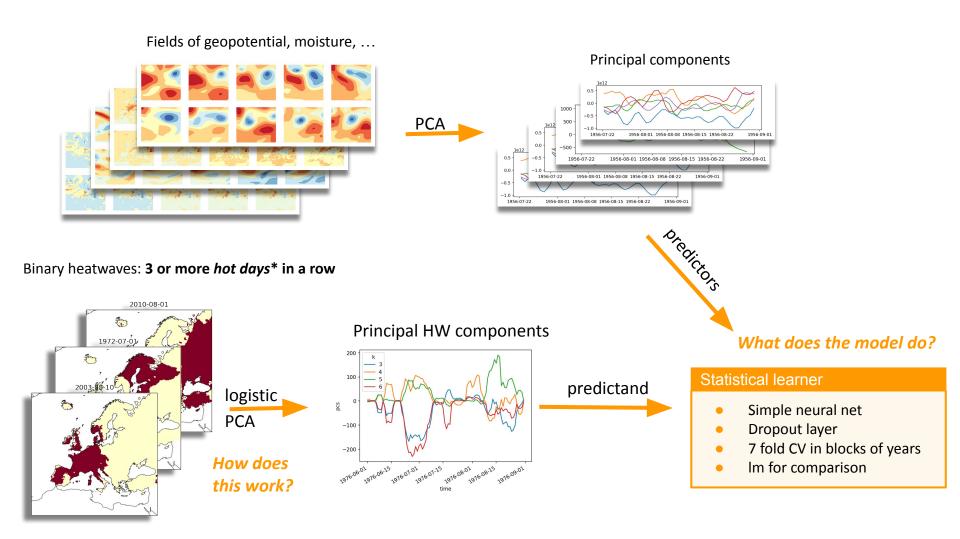
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Data: ERA5 JJA, 1950-2020, EURO-CORDEX domain



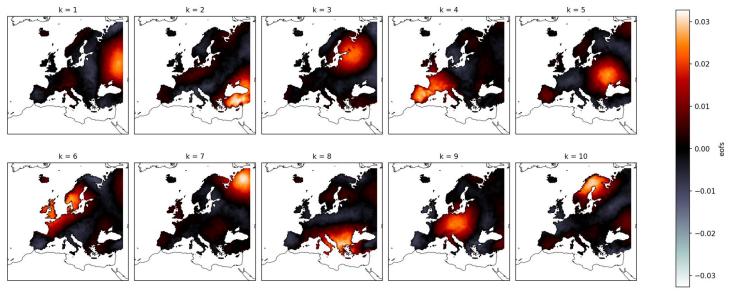
Data: ERA5 JJA, 1950-2020, EURO-CORDEX domain

PCA for binary fields (!)

Regular PCA ("EOFs"): given data X, find vectors U such that $|X - UU^TX|^2$ is minimal \rightarrow minimize *Gauss deviance*, solution: eigenvectors of X^TX

Landgraf and Lee (2020): assume exponential family, compute natural parameters θ \rightarrow iteratively search a projection θUU^T that minimizes the relevant deviance

Binary data: Bernoulli distribution, $\theta = \log(p/(1-p))$



10 rotated "logistic EOFs" for European heatwaves

Predictand: 10 rotated logistic PCs of heatwaves

Predictors: 20 PCs of soil moisture and geopotential at 1000, 800, 500, 300 and 100 hPa

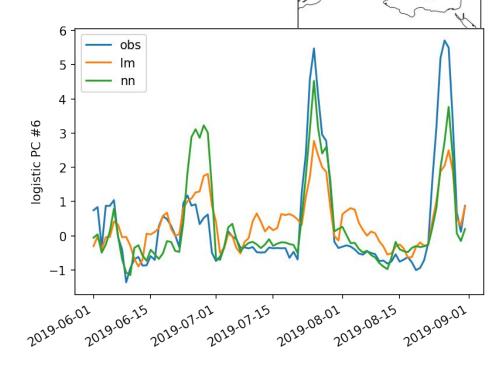
Multivariate linear regression

- $10 + 20 \times 10 = 210$ parameters
- Least squares fit
- $R^2=0.46$

vs.

Simple feed forward neural net

- One hidden layer with 40 nodes
 - \rightarrow (20+1) × 40 + (40+1) × 10
 - = 1250 parameters ReLu activation, 20% dropout
- Optimized with Adam
- $R^2 = 0.75$



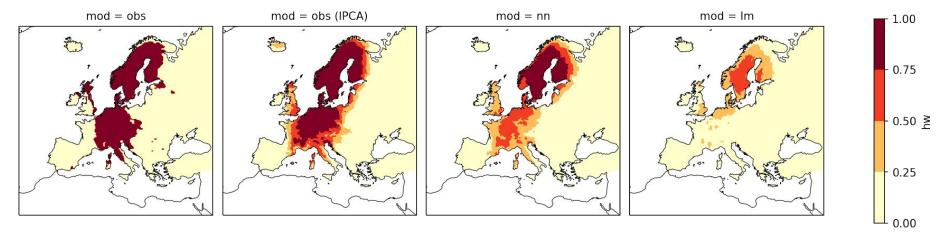
k = 6

Observed and modelled heatwave PC # 6 in summer 2019

Modelling heatwaves in the reduced space

Predictand: 10 rotated logistic PCs of heatwaves

Predictors: 20 PCs of soil moisture and geopotential at 1000, 800, 500, 300 and 100 hPa

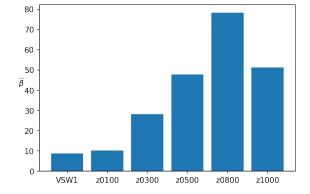


Observed, PCA reduced, and simulated heatwaves on 2019-07-26

Variable importance

Models are not bad, but how do they identify heatwaves? Linear model: just look at coefficients (?)

What to do for the neural net? The coefficients tell us nothing!



Mean absolute regression coefficients \rightarrow is Φ_{800} the most important predictor?

Idea (Shapley 1952, Lipovetsky and Conklin 2001): Split up the overall R² as follows:

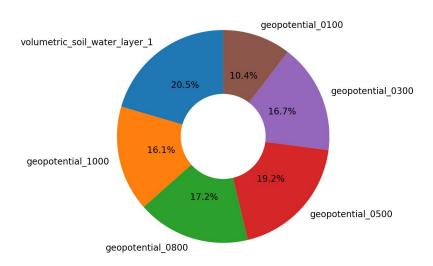
$$R^{2} = f(X_{1}, ..., X_{n}) = \varphi_{1} + \varphi_{2} + ... + \varphi_{n}$$

$$\varphi_{i} = n^{-1} \sum_{j=0}^{n-1} \frac{\binom{n-1}{j}^{-1}}{\sum_{\text{all } S \text{ with } |S|=j, \ X_{i} \notin S}} f(S \cup X_{i}) - f(S)$$
mean over set sizes mean over sets of size j missing X_{i} change if j were added

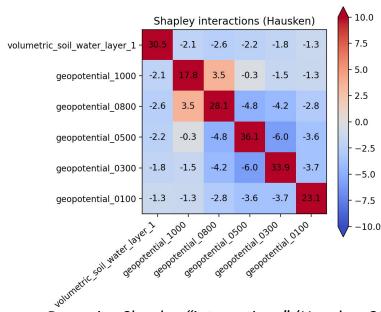
 \rightarrow train all possible 2⁶ models, compare their R² to get the Shapley values!

Variable importance: Shapley values

$$\varphi_{i} = n^{-1} \sum_{j=0}^{n-1} \frac{\binom{n-1}{j}^{-1}}{\text{all } S \text{ with } |S|=j, X_{i} \notin S} \underbrace{f(S \cup X_{i}) - f(S)}_{\text{change if } i \text{ were added}}$$
mean over set sizes mean over sets of size j missing X_{i} change if j were added



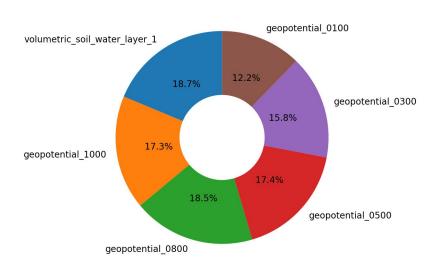
Percentage contributions to the overall model performance R^2 for the neural net



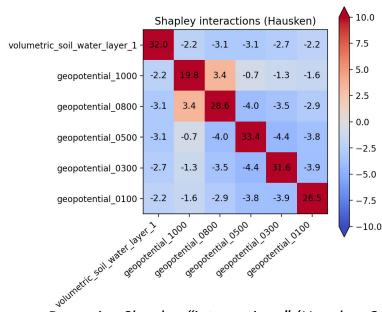
Recursive Shapley "interactions" (Hausken 2001) for the neural net

Variable importance: Shapley values

$$\varphi_{i} = n^{-1} \sum_{j=0}^{n-1} \binom{n-1}{j}^{-1} \sum_{\substack{\text{all } S \text{ with } |S|=j, \ X_{i} \notin S \\ \text{mean over set sizes}}} f(S \cup X_{i}) - f(S)$$



Percentage contributions to the overall model performance R^2 for the linear model



Recursive Shapley "interactions" (Hausken 2001) for the linear model

Summary

- There is a PCA for binary variables → reduced version of any binary event you want!
- A simple neural net can explain 75% of the reduced heatwave variability
- Shapley values and interactions reveal how much can be learned from each predictor, Im and neural net are not so different after all
- It seems that the model has learned Φ_{800} Φ_{1000} $^{\circ}$ T_{900} (hydrostatic relation)

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Heatwave definition and basic statistics

