Daily precipitation with stretched-exponential tails could explain the statistics of observed annual maxima

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Parent distribution of daily precipitation: $F(x)$

↓

Distribution of maxima from samples of size n: $F(x)^n$

→

Extreme value theorem (Fischer & Tippet 1928):

Asymptotic assumption → $\lim_{n\to\infty} F(x)^n = F_{\mathrm{GEV}}(x)$

$F(x)$ usually unknown (??)

Known:
Generalized Extreme Value distribution

However, $n \ll \infty$

- convergence to GEV can be slow for parent distributions with tails that are not power-type or exponential
- a small number of events is sampled from the tail of $F(x)$ every year

Knowing the tail of $F(x)$ would allow

- decreased uncertainty (more data points are available for estimation)
- relations between physics and statistics of extremes (explicit consideration of events intensity and occurrence)

Can we reasonably assume a tail behavior for $F(x)$??

✓ Theoretical[1] and empirical[2] evidence suggest that daily precipitation could have Weibull tails

(i.e. the probability of exceeding large values decreases as a stretched-exponential)

✓ Distributions with stretched exponential tails are used successfully to generate rainfall and its extremes[3]

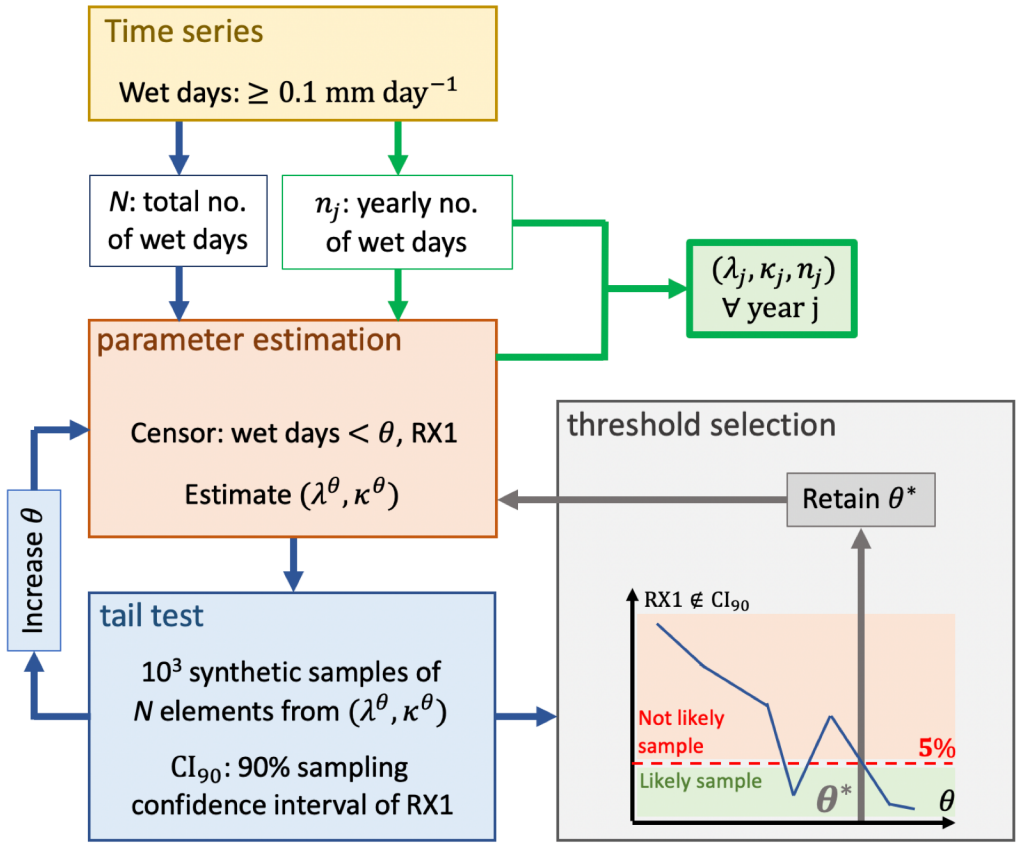
⁉ Here we check whether Weibull tails can describe the reality we observe globally

[1]Wilson & Toumi 2005 10.1029/2005GL022465;
[2]Zorzetto & al 2016 10.1002/2016GL069445; Marra & al 2019 10.1016/j.advwatres.2019.04.002; and many others
[3]Papalexiou 2022 10.1029/2021WR031641

A test to assess the tail behavior

- Directly checks if **observed annual maxima** are **likely samples from the assumed tail**

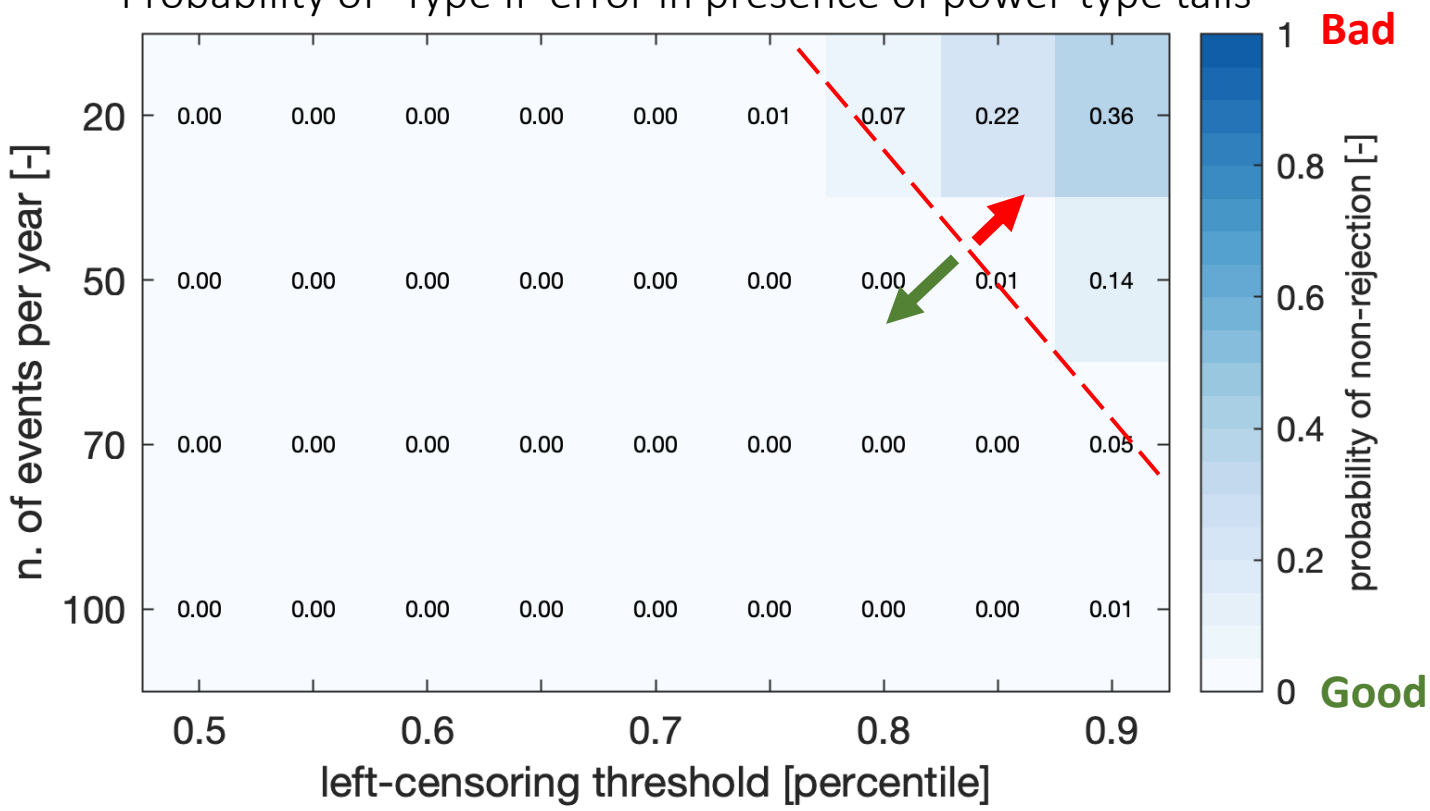
In our case: Weibull



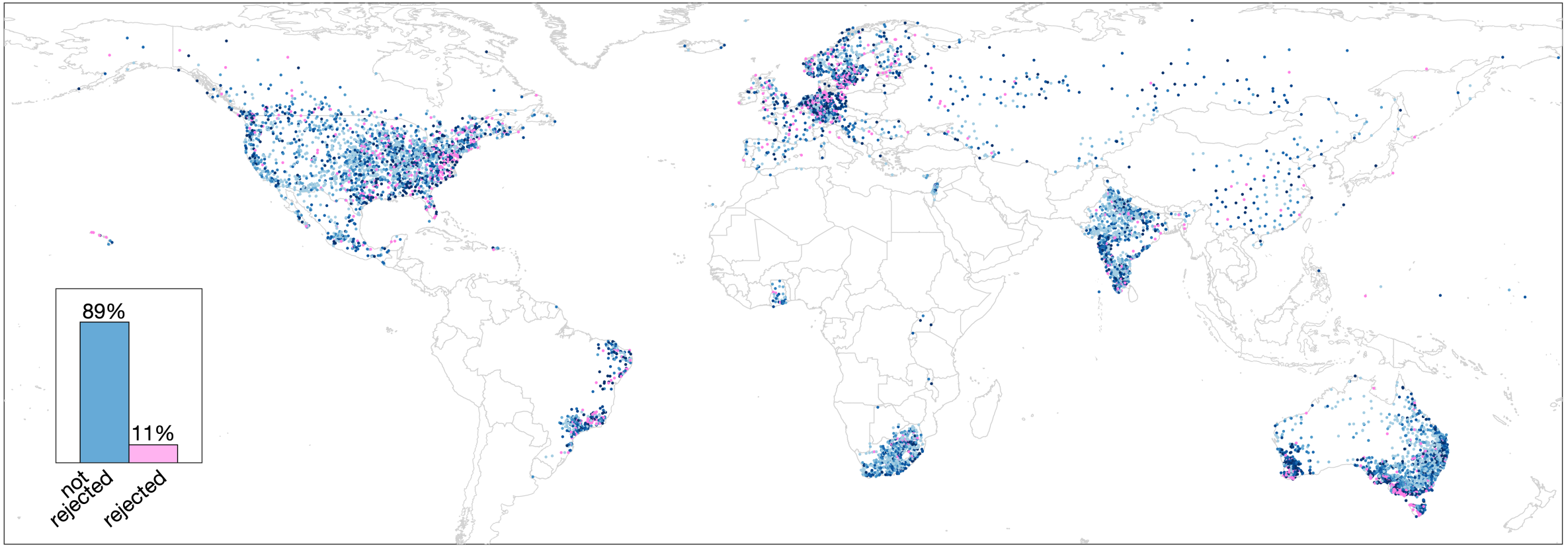
Probability of 'Type I' error ≃ 3-7%

Most supported competitor (heavier)

Probability of 'Type II' error in presence of power-type tails

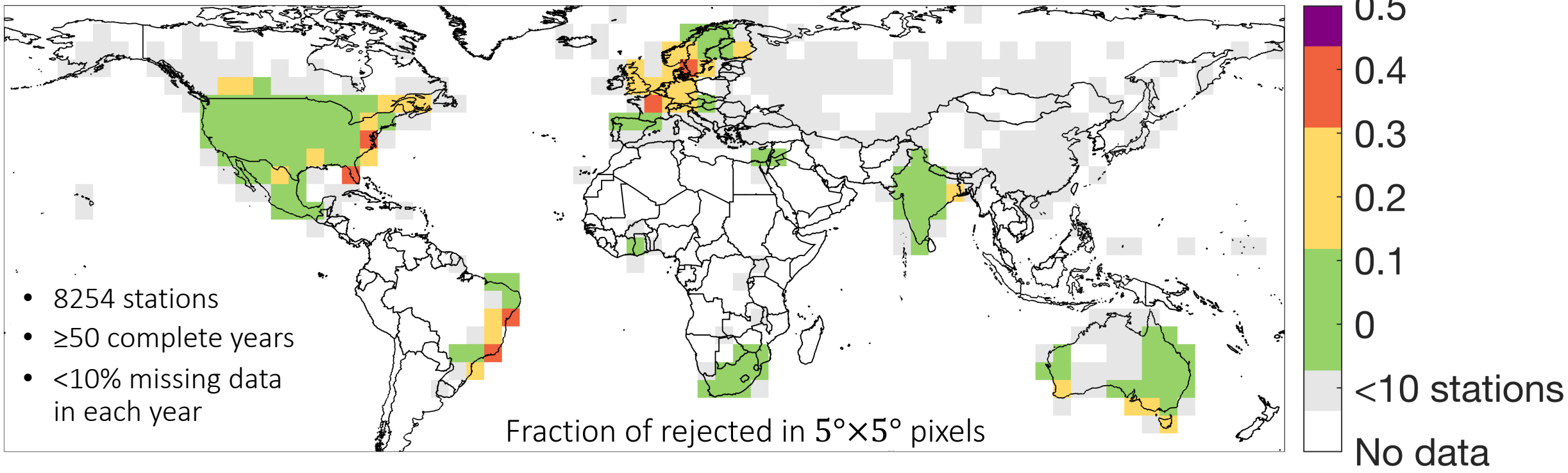


Weibull tails of daily precipitation: global definition

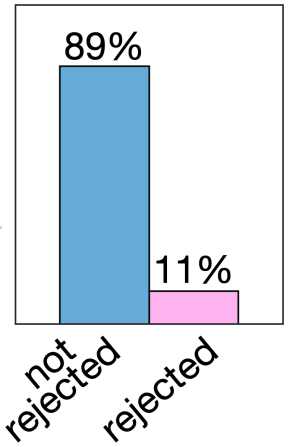


- Tested tails defined with different left-censoring thresholds
- Definition of the tails shows consistent geographic patterns

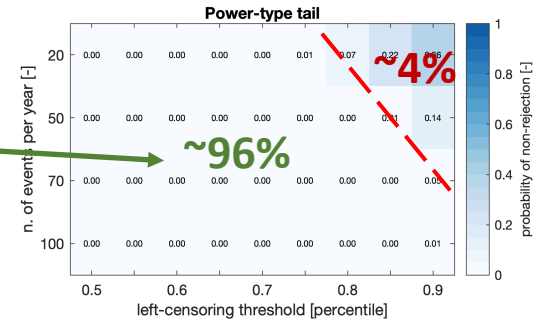
Weibull tails of daily precipitation: global definition



- 8254 stations
- ≥50 complete years
- <10% missing data in each year



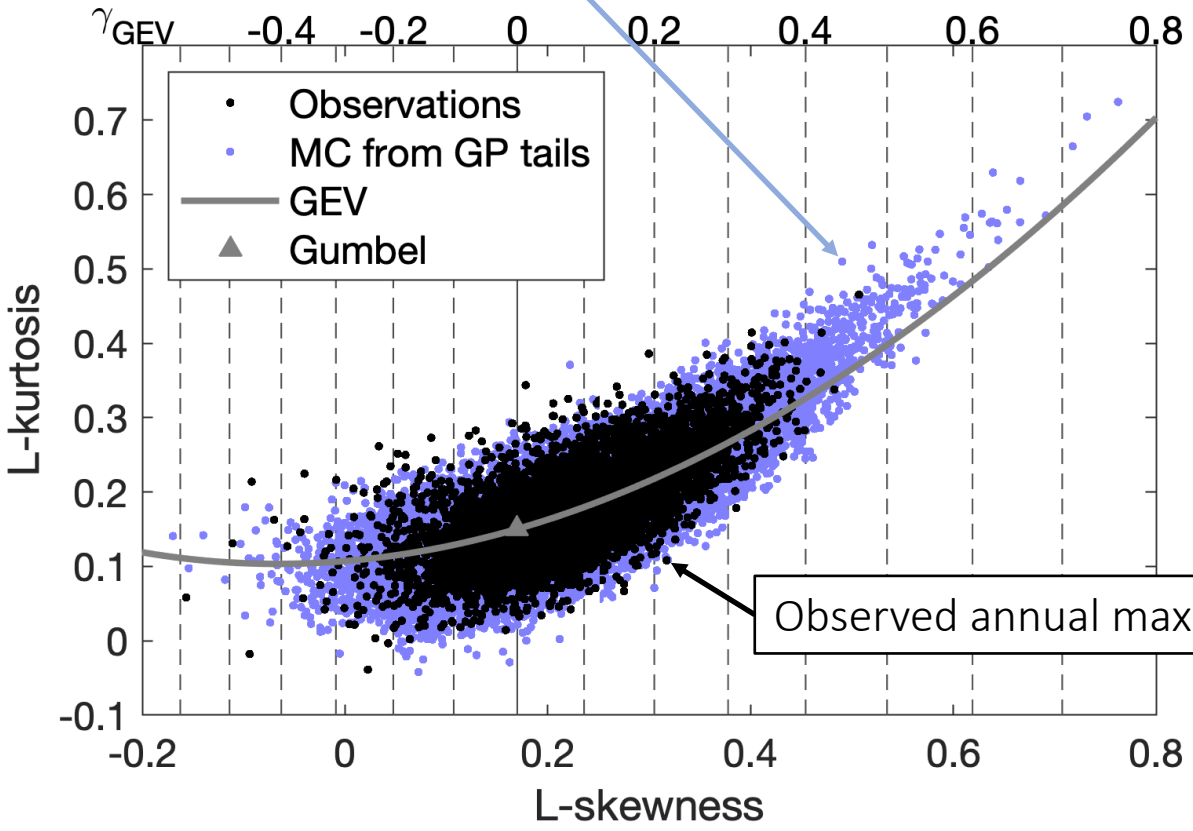
- Tested tails defined with different left-censoring thresholds
- Weibull is rejected in ~11% of the stations
- ~96% of not-rejected are in the '0% Type II' area



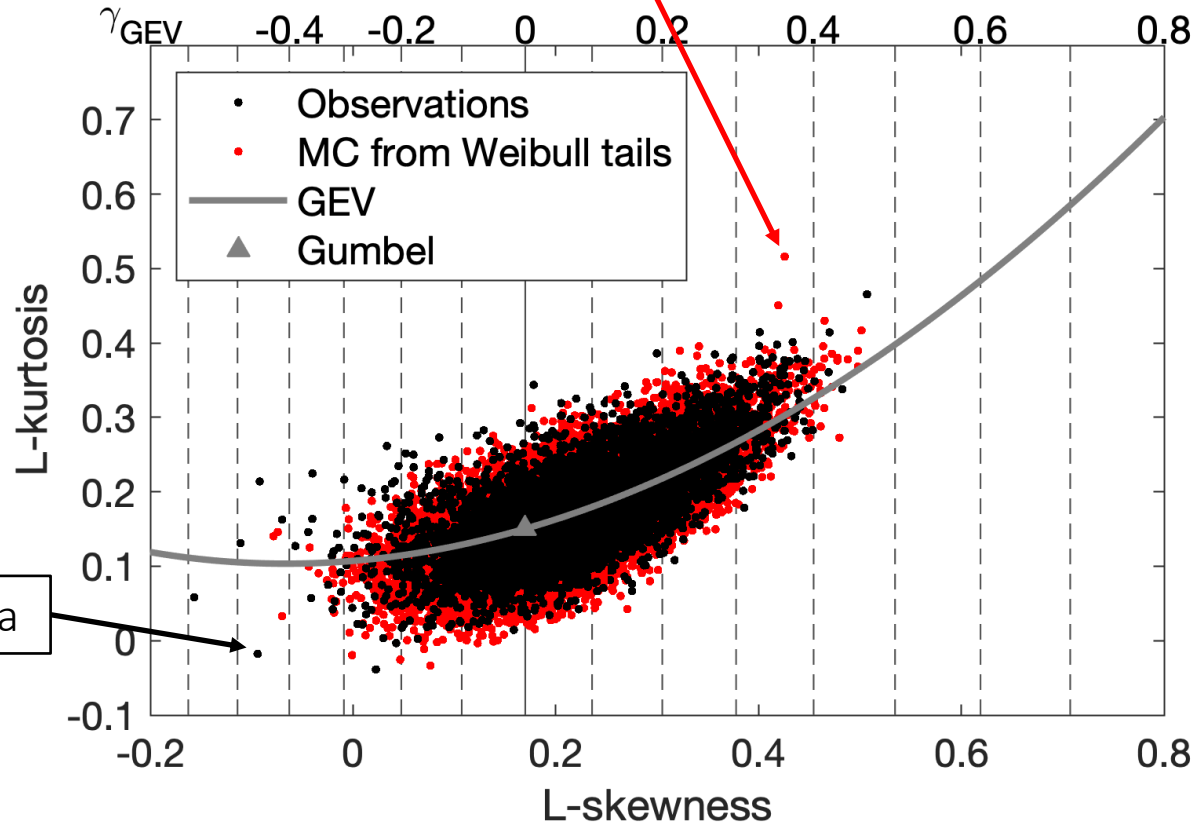
Non-asymptotic Weibull tails reproduce the L-moments of annual maxima

Generalized Pareto tails (GP tails)
threshold = 95th percentile
5 repetitions for each station, same record length

Weibull tails
threshold defined using the test
5 repetitions for each station, same record length

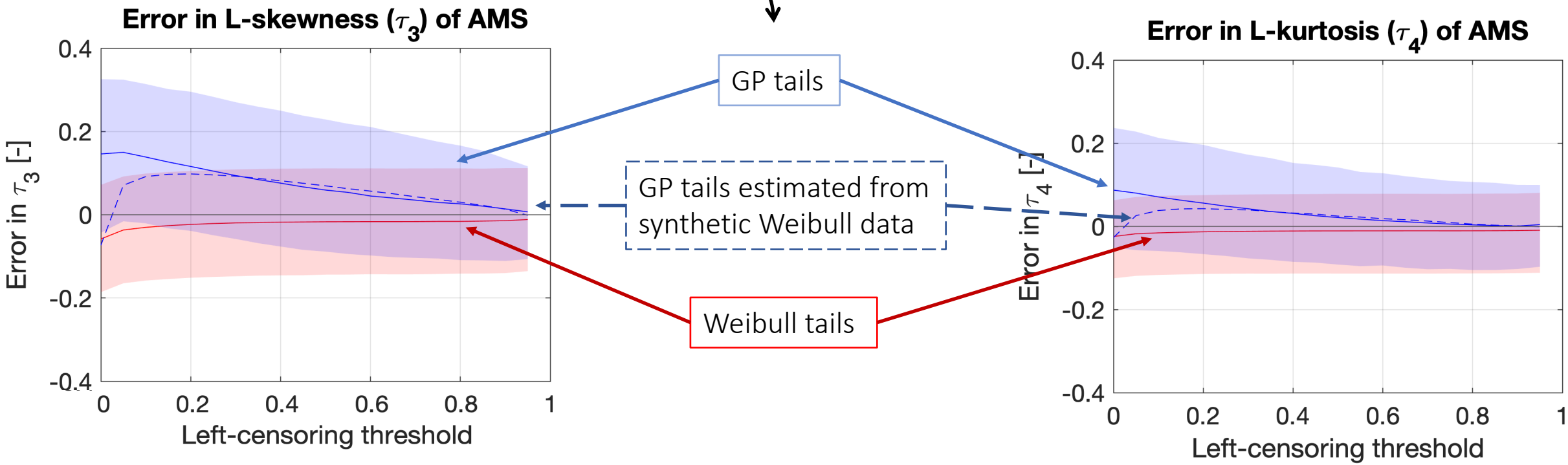


Observed annual maxima



Non-asymptotic Weibull tails can also explain the observed GP tails

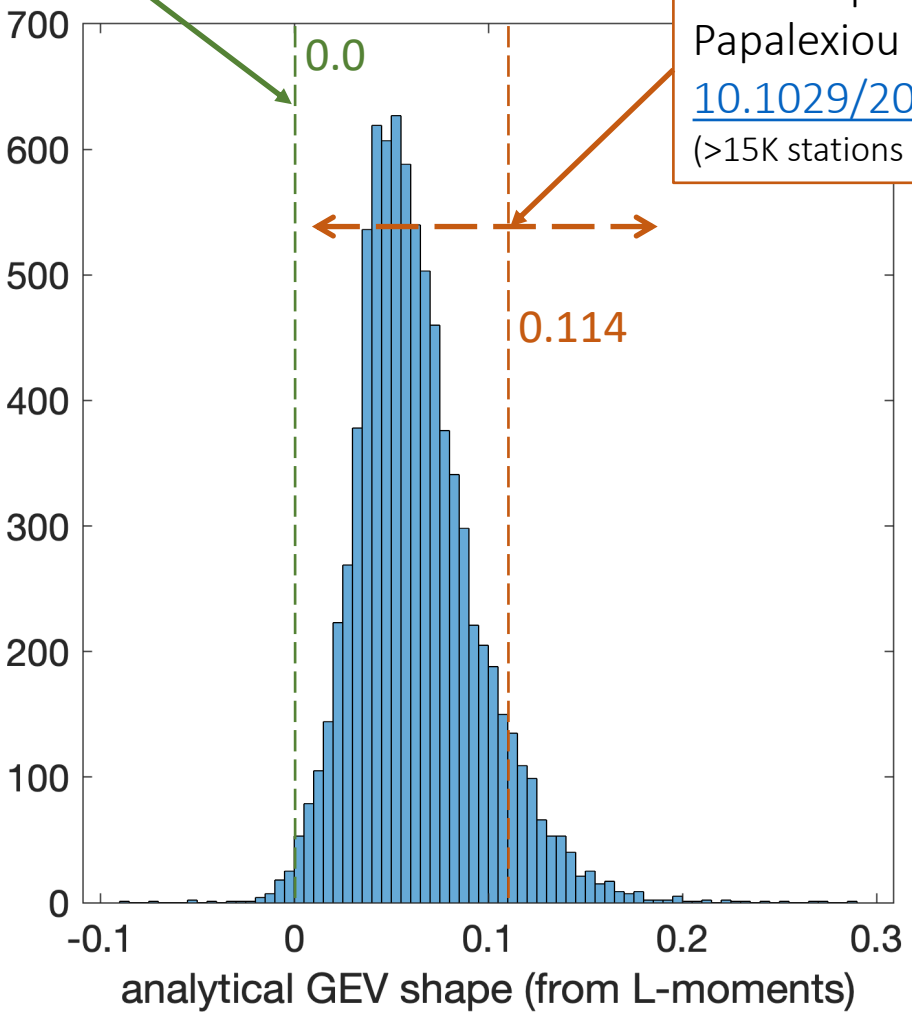
- 10^3-year synthetic records for each station
- *n* events per year (non-asymptotic sampling)
- Different types of tails

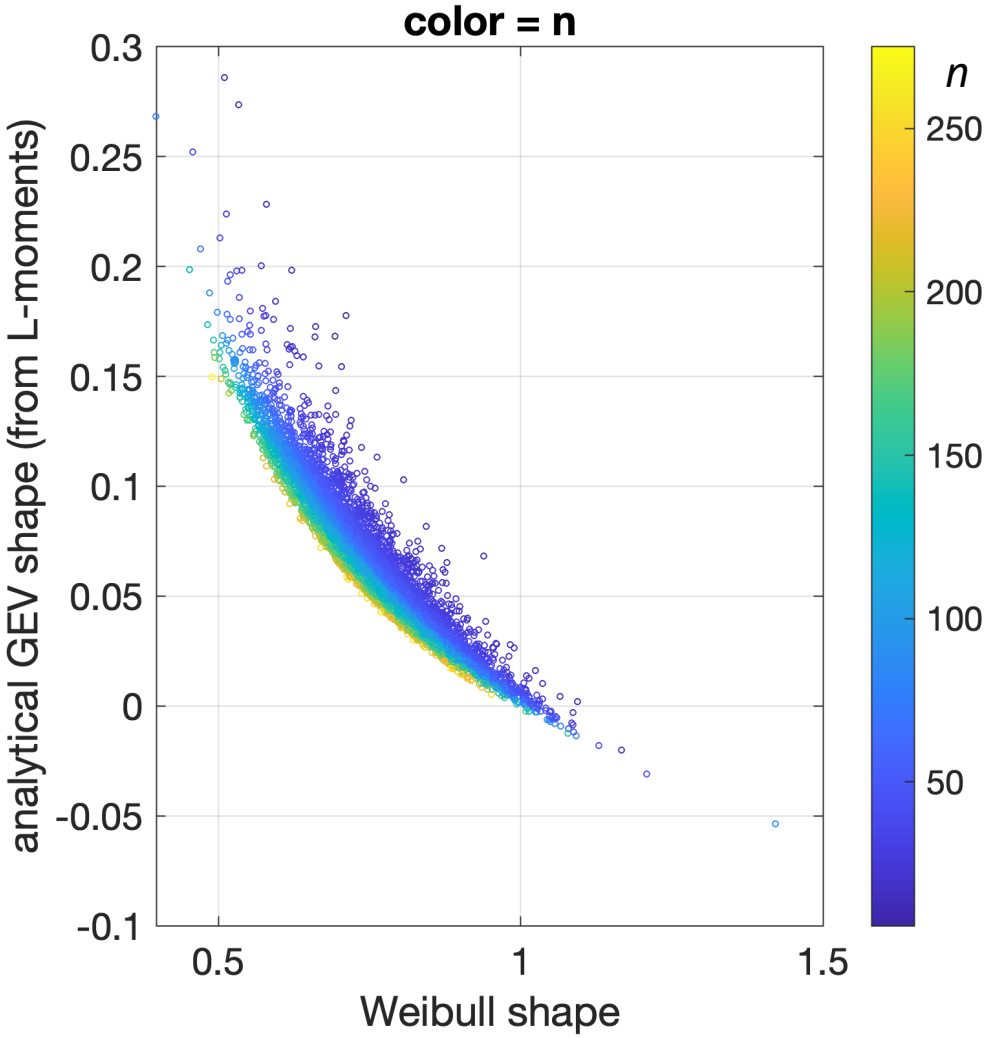


Relation between non-asymptotic Weibull tails and asymptotic GEV distribution

Theoretical GEV shape for **asymptotic** Weibull tails

GEV shape from
Papalexiou & Koutsoyiannis 2013
10.1029/2012WR012557
(>15K stations with >50 years of data)





Take-home

- The hypothesis: *"daily precipitation annual maxima are sampled from Weibull tails"* cannot be rejected for 89% of the stations globally

- Non-asymptotic Weibull tails explain the statistics of observed extremes in terms of
  - L-moments of the annual maxima
  - emerging tails of 'asymptotic' distributions

Thanks :)

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