Towards Canopy parameterization for Multiscale Finite Element Method

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25 May 2022

European Geosciences Union

Universität Hamburg
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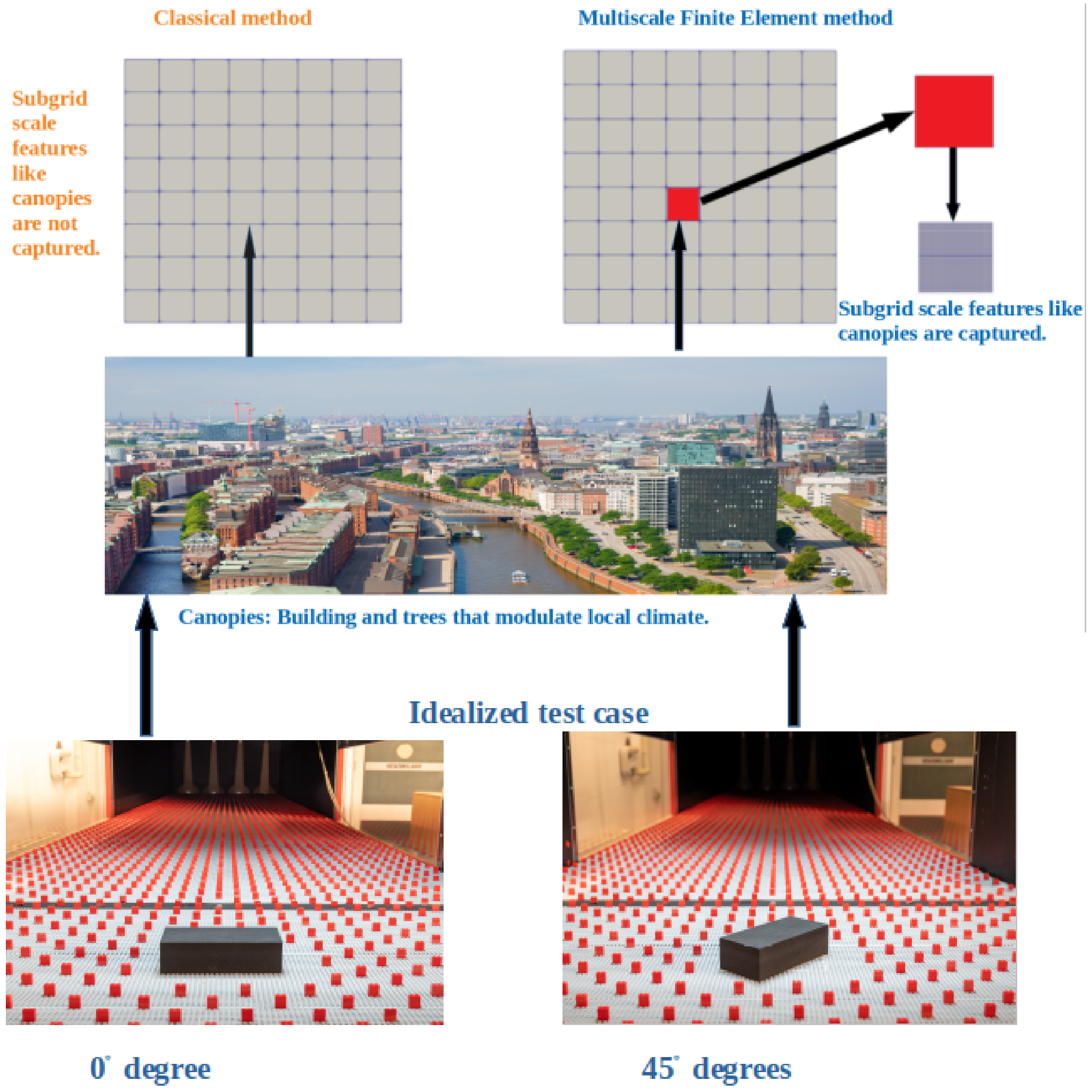
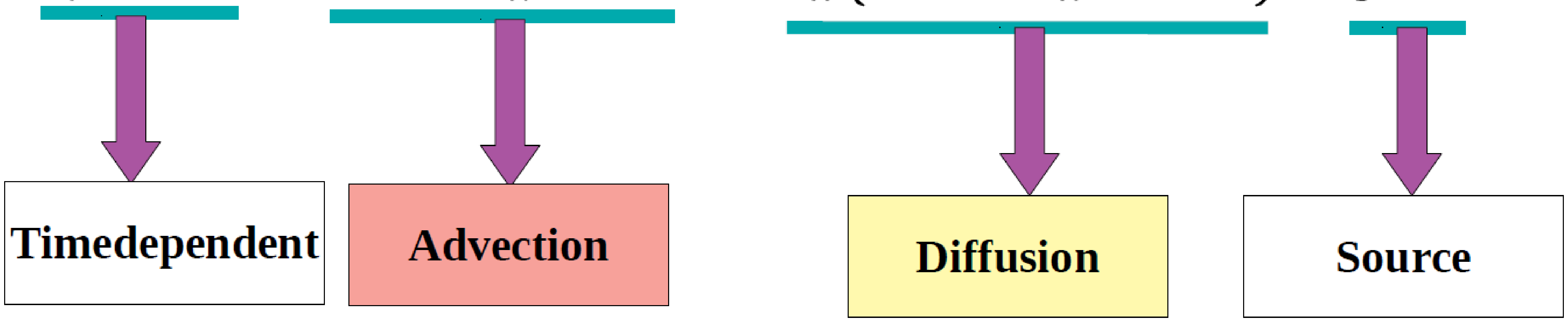
Introduction to Multiscale in canopy

- Finite Element Method needs more computation in order to capture subgrid feature like canopies. Whereas Multiscale Finite Element needs less computation because of the basis calculated in parallel.
- Canopy consist of bulidings or trees that represent local climate in regional or global climate models.
- In present study canopy is considered as an idealized single building obstacle in domain.
- The idea is to use canopy parametrization as diffusion coefficent that is used to solve advection-diffusion equation.



Advection-Diffusion Equation

$$\partial_t u(x,t) + c(x,t)\partial_x u(x,t) = \partial_x\big(\mu(x,t)\partial_x u(x,t)\big) + g(x), \qquad (x,t)\epsilon(0,T]$$



$$u(0,t) = u(1,t) \qquad t\epsilon(0,T]$$

$$u(x,0) = f(x) \qquad x\epsilon I$$

Methods

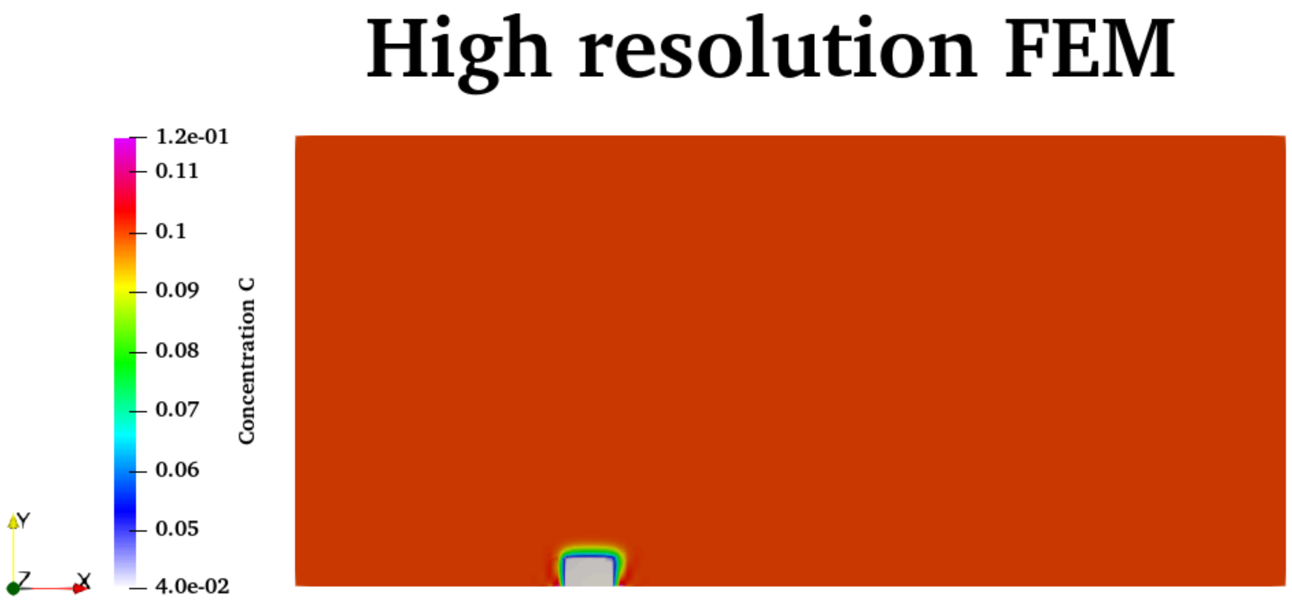
Algorithm 1 Algorithm for Finite Element Method

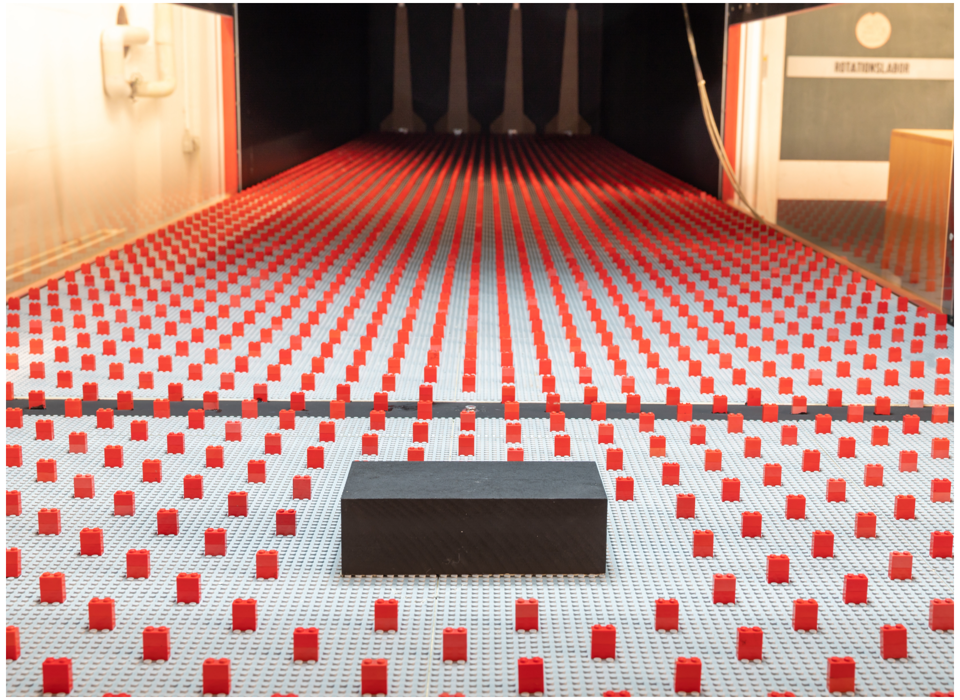
1. Setup mesh
2. Setup system and constraints (set n =0)
3. **while** $t^{n+1} \leq T$ **do**
   - Assemble system
   - Solve for u^{n+1}
   - Set n=n+1

Algorithm 2 Algorithm for Multiscale Finite Element Method

1. Setup mesh (coarse).
2. Setup system and constraints.
   Compute Ms-basis at t=0 (set n =0)
3. **while** $t^{n+1} \leq T$ **do**
   compute basis at t^{n+1}
   - Assemble system
   - Solve for u^{n+1}
   - Set n=n+1

Reference Solution





Case :Boundary conditions

Domain is $-300 \leq x \leq 800$ and $0 \leq y \leq 500$

Initial condition= Dirichlet condition

$$c(z) = \begin{cases} 0 & -25 \leq x \leq 25, \quad 0 \leq y \leq 30 \quad \textbf{Building} \\ \\ 0.1 & else, \end{cases}$$

Neumann condition

$$g = \sum_{i=1}^{2} exp(-(x-250))$$

Diffusion Coefficient

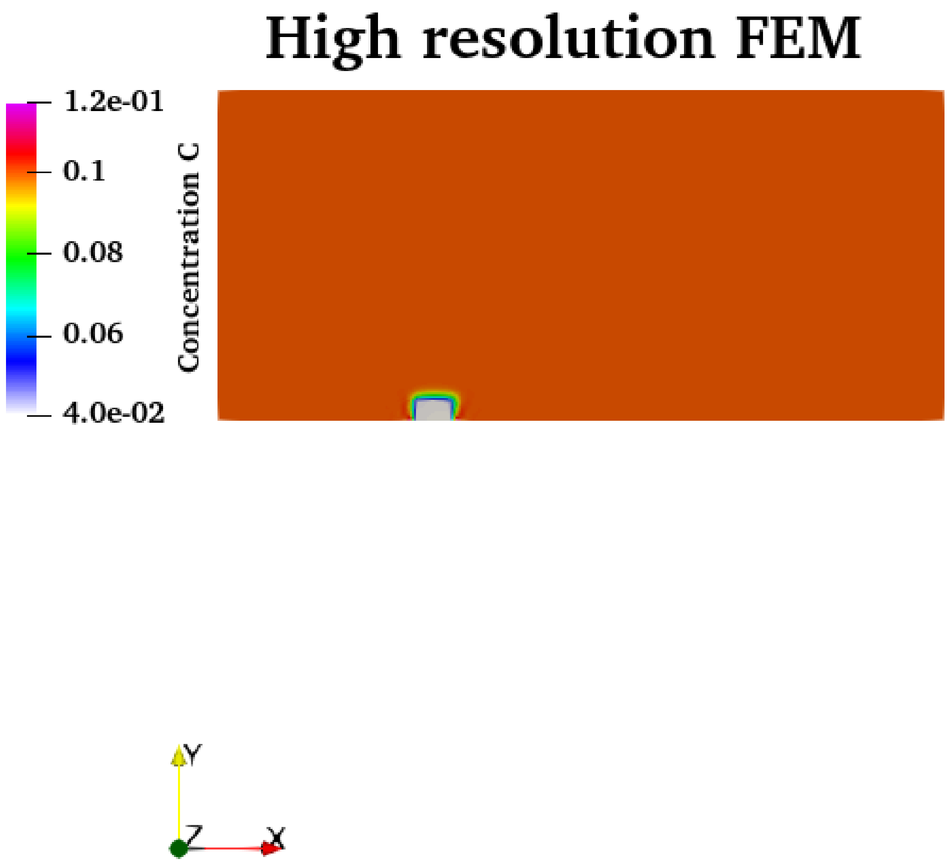
$$K(z) = \begin{cases} 10000 & -25 \leq x \leq 25, \quad 0 \leq y \leq 30 \quad \textbf{Building} \\ \\ x & \quad else \end{cases}$$

Case :Advection

Velocity

$$c(z) = \begin{cases} \begin{pmatrix} 0 \\ 0 \end{pmatrix} & -300 \leq x \leq 800, \quad y = 0 \quad \textbf{Bottom Boundary} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & -25 \leq x \leq 25, \quad 0 \leq y \leq 30 \quad \textbf{Building} \\ \begin{pmatrix} 4 * exp(-(1-(y/30))) \\ 0 \end{pmatrix} & else \quad \textbf{Around the Building} \\ \begin{pmatrix} (0.35/0.4) * log((y+0.5)/0.5) \\ 0 \end{pmatrix} & y \geq 30 \quad \textbf{Above the Building} \end{cases}$$

Case : Result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Concentration C
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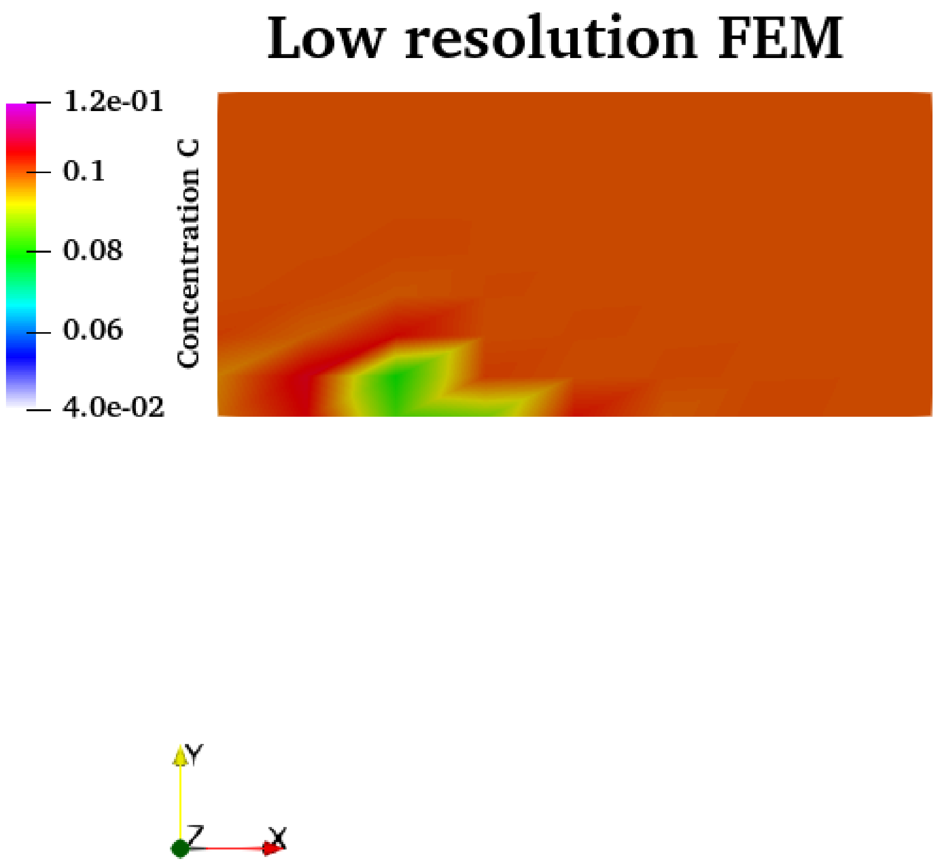
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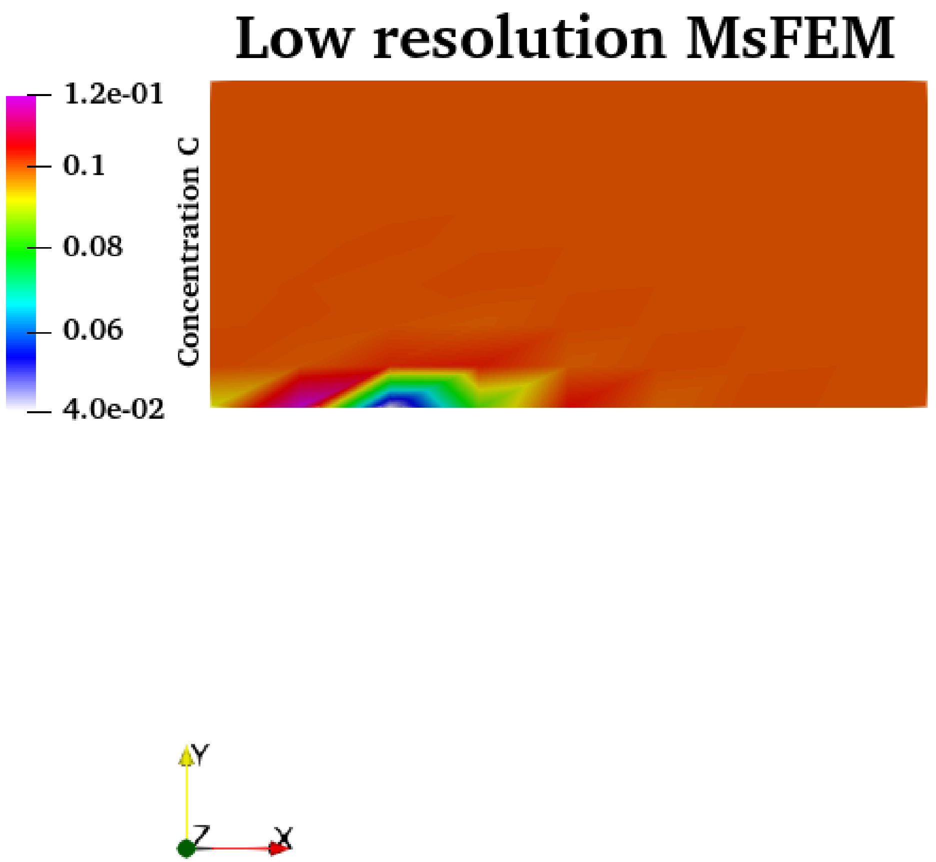
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