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CriticalEarth

A minimal SDE model of D-O events with multiplicative noise

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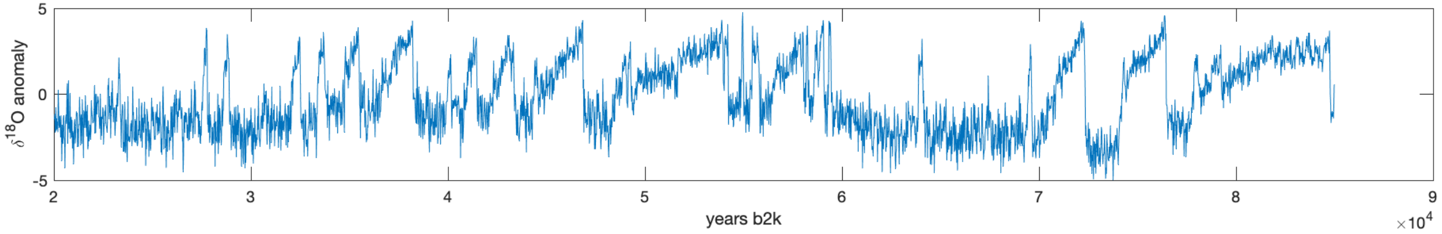
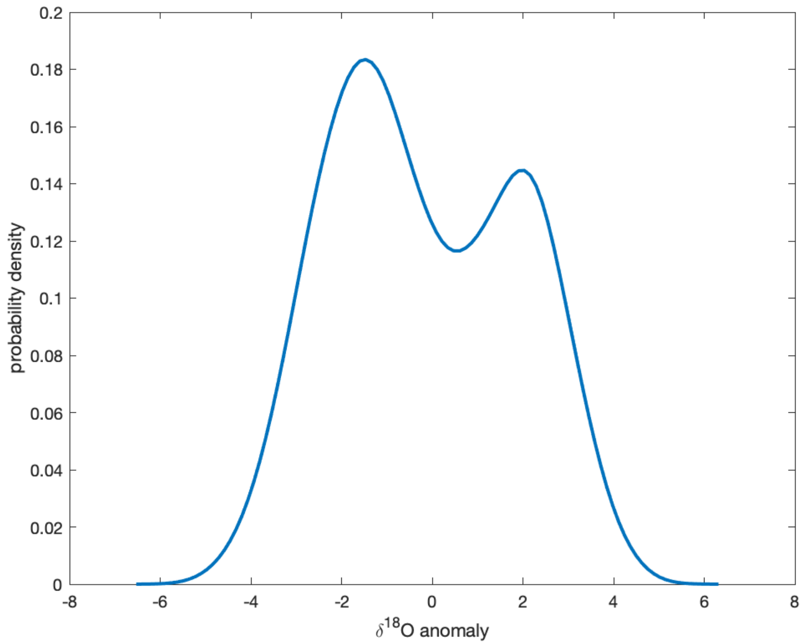
Model from data:

Have this





Want this

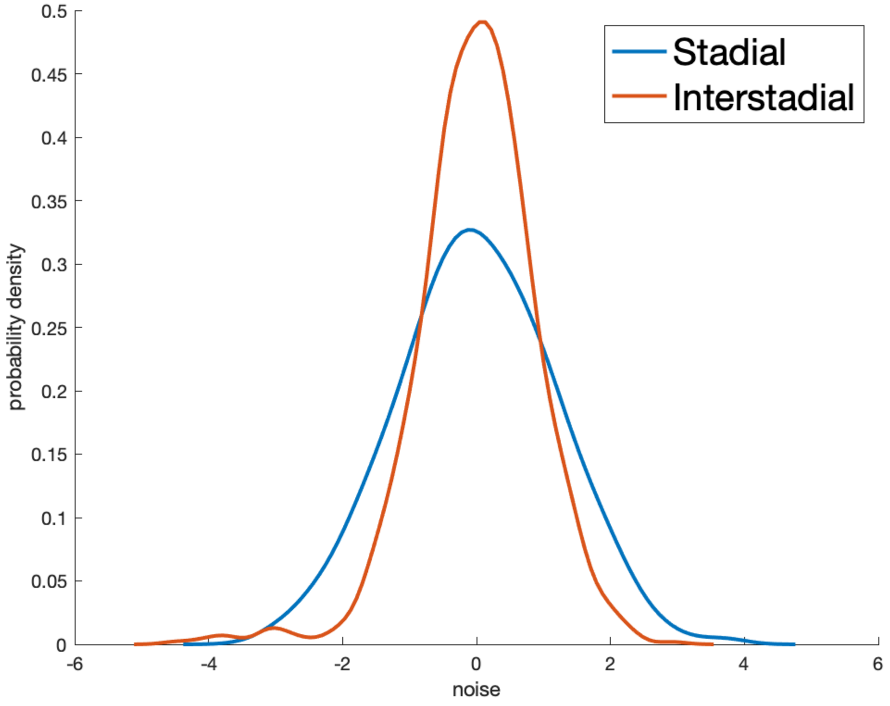
$$dx = -\frac{\partial U}{\partial x}dt + \sigma(x) \cdot dB(t)$$

$$\partial_t p(x,t) = \partial_x \left[\frac{\partial U}{\partial x} p(x,t)\right] + \partial_x^2 \left[\frac{\sigma(x)^2}{2} p(x,t)\right]$$

$$\partial_t p(x,t) = 0$$

$$\boxed{p(x,t)} \propto \frac{1}{\sigma(x)^2} \exp\left[-\int^x \frac{2}{\sigma(x')^2} \frac{\partial U(x')}{\partial x'} dx'\right]$$

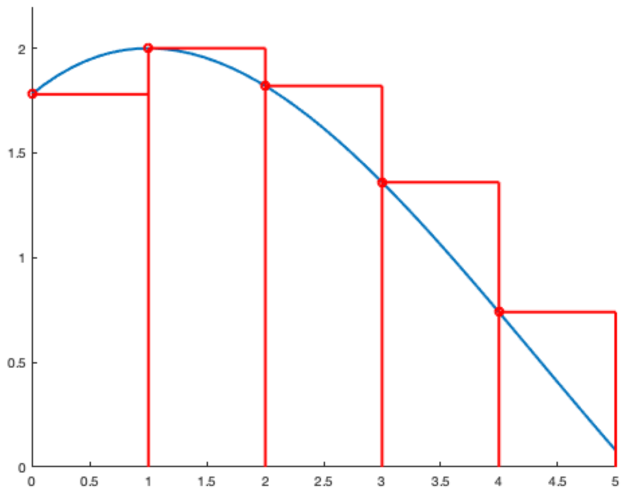
Why multiplicative noise?



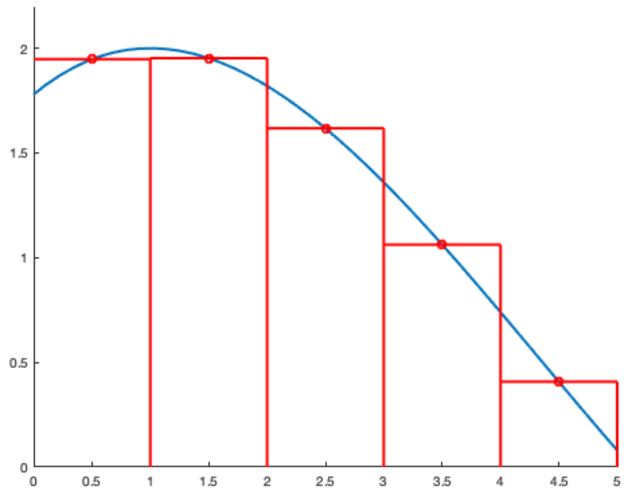
Greater in stadial than interstadial

Linearly decreasing $\sigma(x)$

BUT: Issue when integrating!



$$\tau_k = t_k$$



$$\tau_k = \frac{1}{2}(t_{k+1} + t_k)$$

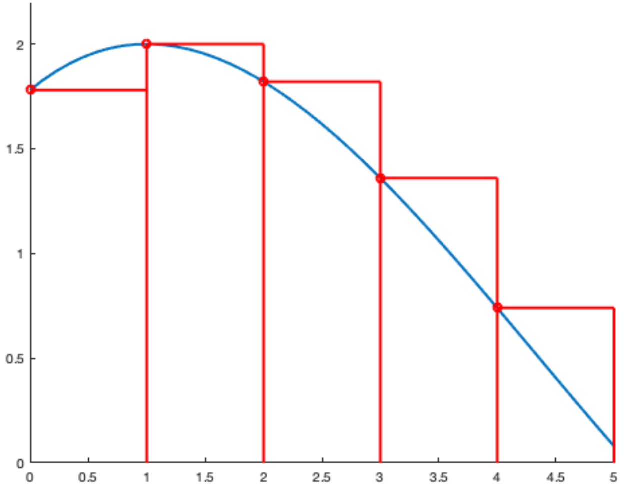
$$\mathbb{E}\left(\int_0^T B_t dB_t\right) = \sum_k (\tau_k - t_k)$$

Itô vs Stratonovich

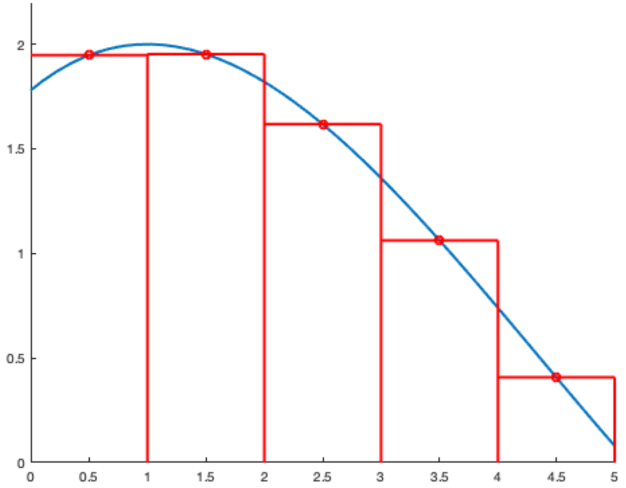
$$\int_0^t f(s)\cdot dg(s) = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{k-1}{n}t\right)\left[g\left(\frac{k}{n}t\right) - g\left(\frac{k-1}{n}t\right)\right]$$

$$\int_0^t f(s)\circ dg(s) = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{1}{2}\left[\frac{k-1}{n}t + \frac{k}{n}t\right]\right)\left[g\left(\frac{k}{n}t\right) - g\left(\frac{k-1}{n}t\right)\right]$$

Itô integral: better suited to discrete processes



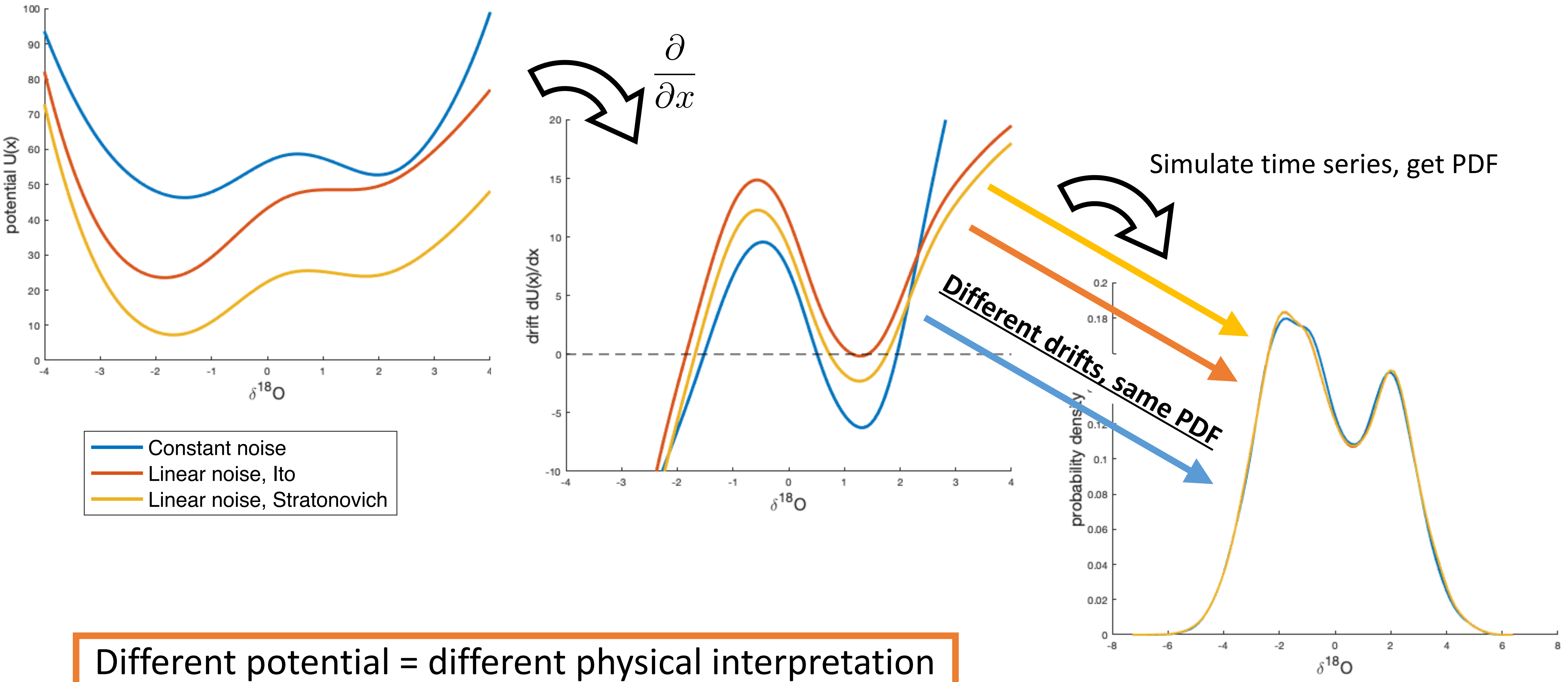
Stratonovich integral: better suited to continuous processes



Equivalence relation:

$$dx = \left(-\frac{\partial U}{\partial x} + \frac{1}{2}\frac{\partial \sigma(x)}{\partial x}\sigma(x)\right)dt + \sigma(x)\cdot dB(t) \quad \longleftrightarrow \quad dx = -\frac{\partial U}{\partial x}dt + \sigma(x)\circ dB(t)$$

Different potentials under multiplicative noise

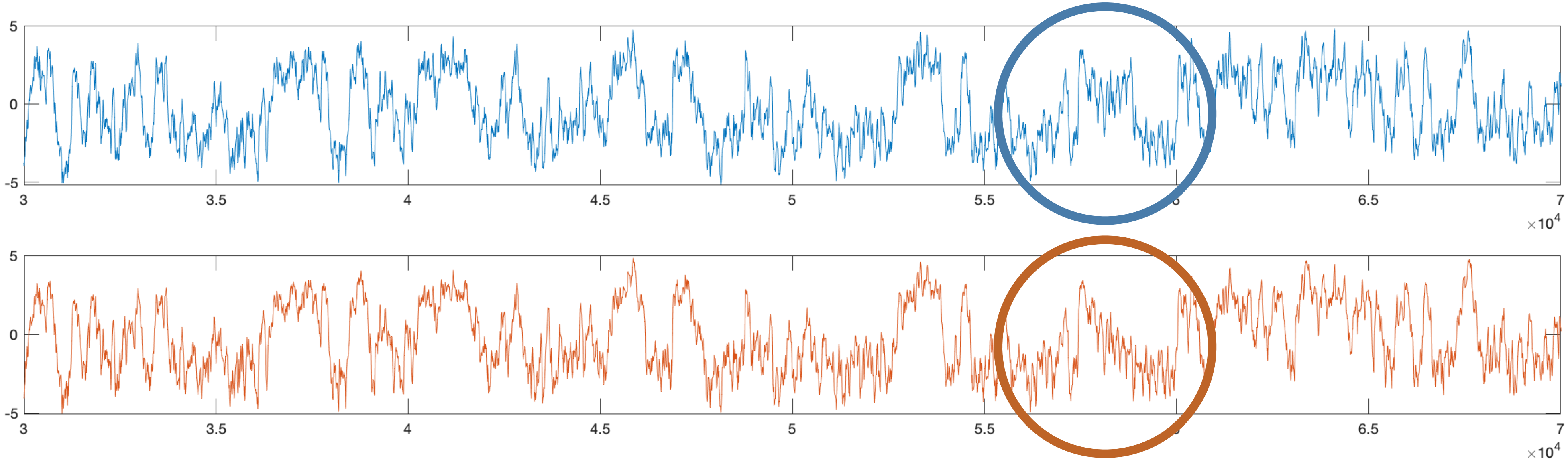


Different potential = different physical interpretation

Example simulated time series

Constant noise

Multiplicative noise



Future work:

Stationary distribution doesn’t tell the whole story

D-O time series not time symmetric

Apply multiplicative noise to other dynamical models

Thank you!

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