Ray theoretical investigations using matching pursuits

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Traveltime tomography and ray theory

The problem of traveltime tomography consists basically of inverting traveltimes of seismic waves for anomalies of the slowness/velocity.

We concentrate here on the infinite frequencies approach: waves are represented as a set of rays, less numerically expensive, adequate up to moderate data coverage.

$$\int_{\mathrm{ray}_k} \delta S(x) = \delta T_k, \quad k = 1, \ldots, N,$$

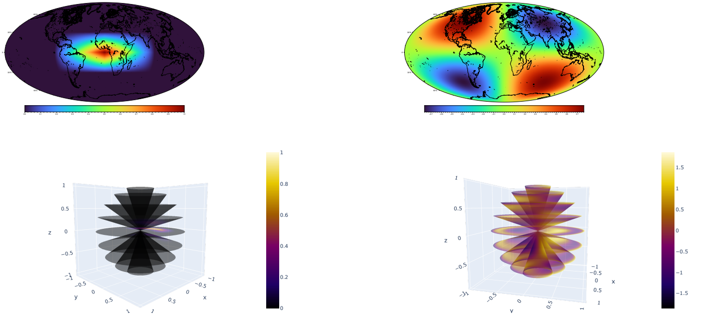
$\delta S = S - S_{\mathrm{ref}}$: deviation in the slowness, $\delta T_k = T_k - T_{\mathrm{ref},k}$ traveltime delays

Why do we want to talk about the choice of the numerical method?

- There are uncertainties about regions inside the Earth, though we have hugh amounts of data and many studies have inverted them.
- The inverse problem of traveltime tomography is ill-posed: the solution is not unique and it is, in particular, also unstable (even small noise can cause severe perturbations of the calculated result).
- The ill-posedness requires a regularization, but different regularizations yield different results.
- There is a multitude of basis functions available, all with their pros and cons.
- Can differences in competing inversions come from different choices of basis functions? In other words, to which extent can basis functions cause artefacts?

Embarras de richesses

Amongst the multitude of basis functions available for geophysical inverse problems, there are, in particular, two types: local and global basis functions.



finite element based on a tesseroid   orthogonal polynomial on a ball

Figure: Local basis functions (left) are appropriate for approximating localized anomalies, but it becomes expensive to use them for covering global structures. Global basis functions (right) work well for global structures, but using them for local corrections of a model deteriorates the model everywhere else.

Therefore, it would be good to **combine different types of trial functions within one inversion**.

Our approach

Based on a matching pursuit approach by Mallat & Zhang 1993 and Vincent & Bengio 2002 designed for problems of signal analysis, the Geomathematics Group Siegen (Fischer & Gutting & Kontak & Kretz & Leweke/Orzlowski & M. & Schneider & Telschow, 2011-present) has developed the **Regularized Functional Matching Pursuit (RFMP)** with variants including the **Learning RFMP** (Schneider & M., 2020-present) as a best-basis algorithm for regularizing ill-posed inverse problems.

The basic idea: We want to minimize ‘data misfit + regularizing penalty term’. For this purpose, we iteratively calculate optimal trial functions $d_1, d_2, \ldots$ and coefficients $\alpha_1, \alpha_2, \ldots \in \mathbb{R}$.

$$\alpha_1 d_1$$
$$\alpha_1 d_1 + \alpha_2 d_2$$
$$\alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3$$
$$\ldots$$

We have already successfully applied this methodology to gravitational field modelling and medical imaging. Therefore, we expect that it will also be useful for seismic imaging.

References

V. Michel: Geomathematics — Modelling and Solving Mathematical Problems in Geodesy and Geophysics, *Cambridge University Press*, 2022.

V. Michel, N. Schneider: A first approach to learning a best basis for gravitational field modelling, *GEM: International Journal on Geomathematics*, **11** (2020), article 9.

N. Schneider, V. Michel: A dictionary learning add-on for spherical downward continuation, *Journal of Geodesy*, **96**:21 (2022).

A publication by V.M., N. Schneider, K. Sigloch, and E. Totten is work in progress.