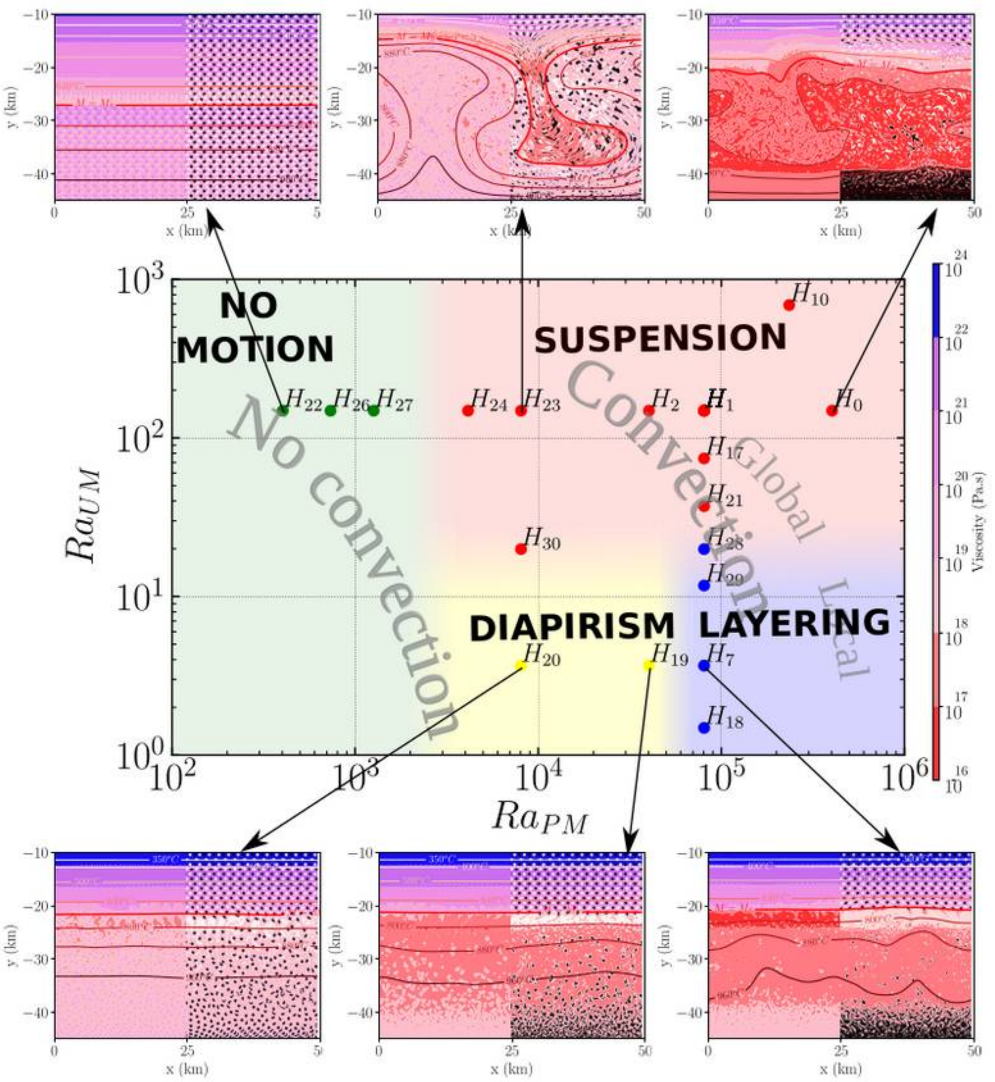
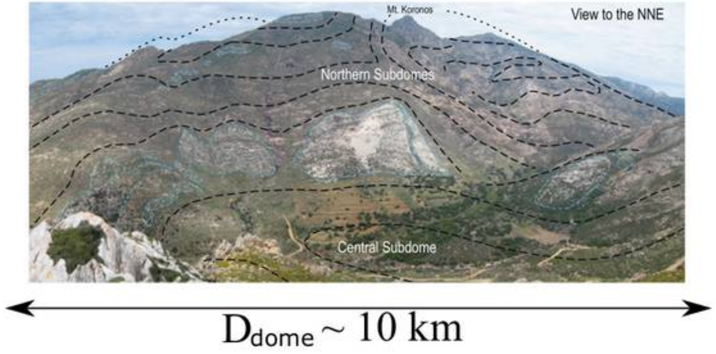
Gravitational instabilities in partially molten crust with a Volume-Of-Fluid method

Louis-Napoléon A., Bonometti T., Gerbault M., Vanderhaeghe O., Roland M., Maury N.

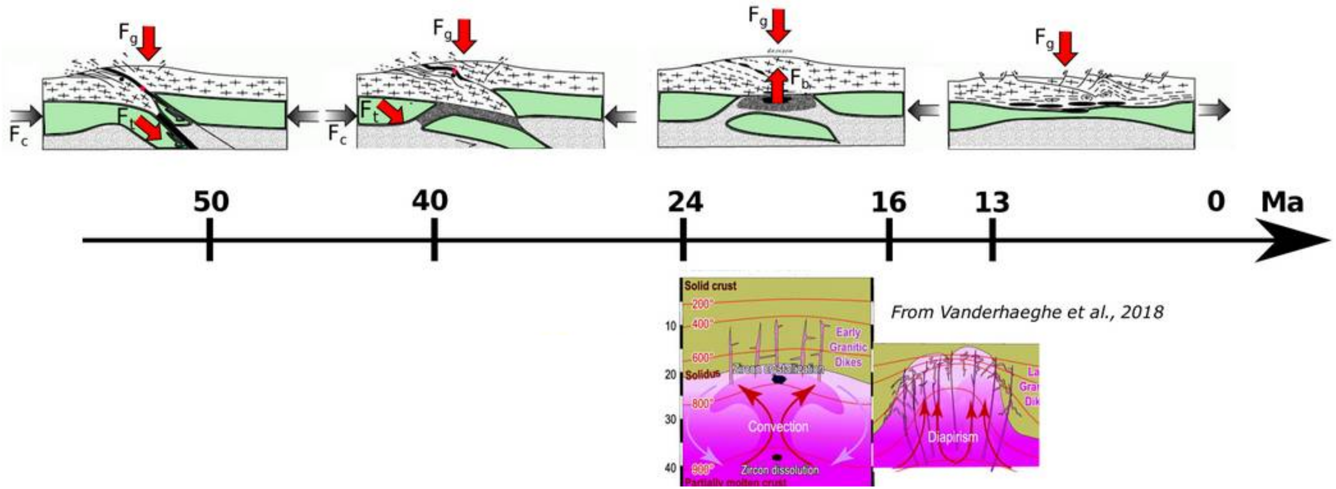




Naxos domes

Louis Napoléon et al., J.G.Int 2020, 2021,
https://doi.org/10.1093/gji/ggab510
https://doi.org/10.1093/gji/ggaa141

Migmatite domes during the Aegean orogeny :
Diapirs ok, convection ? size and timing ?



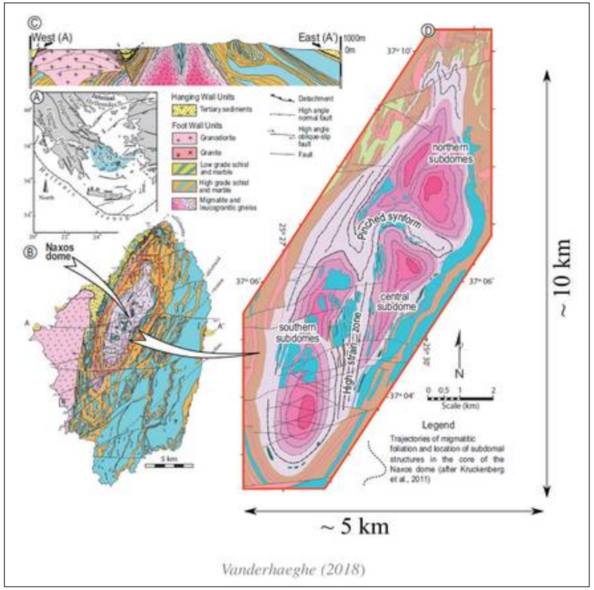
From Vanderhaeghe et al., 2018

Convection ? Diapirism ?

Case of Naxos

Modeling time window

Naxos domes

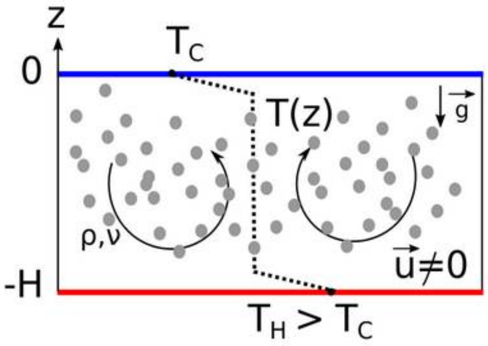


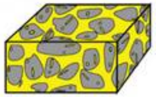
Vanderhaeghe et al., 2018,
doi.org/10.1016/j.tecto.2018.03.007

Model requirement : follow spatial heterogeneities in a convecting medium

- Structural evolution depends on *Ra(…)* and $B = \frac{\rho_p - \rho_f}{\rho_f \, \alpha \Delta T}$

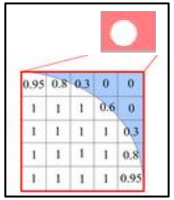
- Two possible regimes : suspension and sedimentation
  (*Hoïnk, 2005; Lavorel & LeBars, 2009, Patočka et al., 2020*)





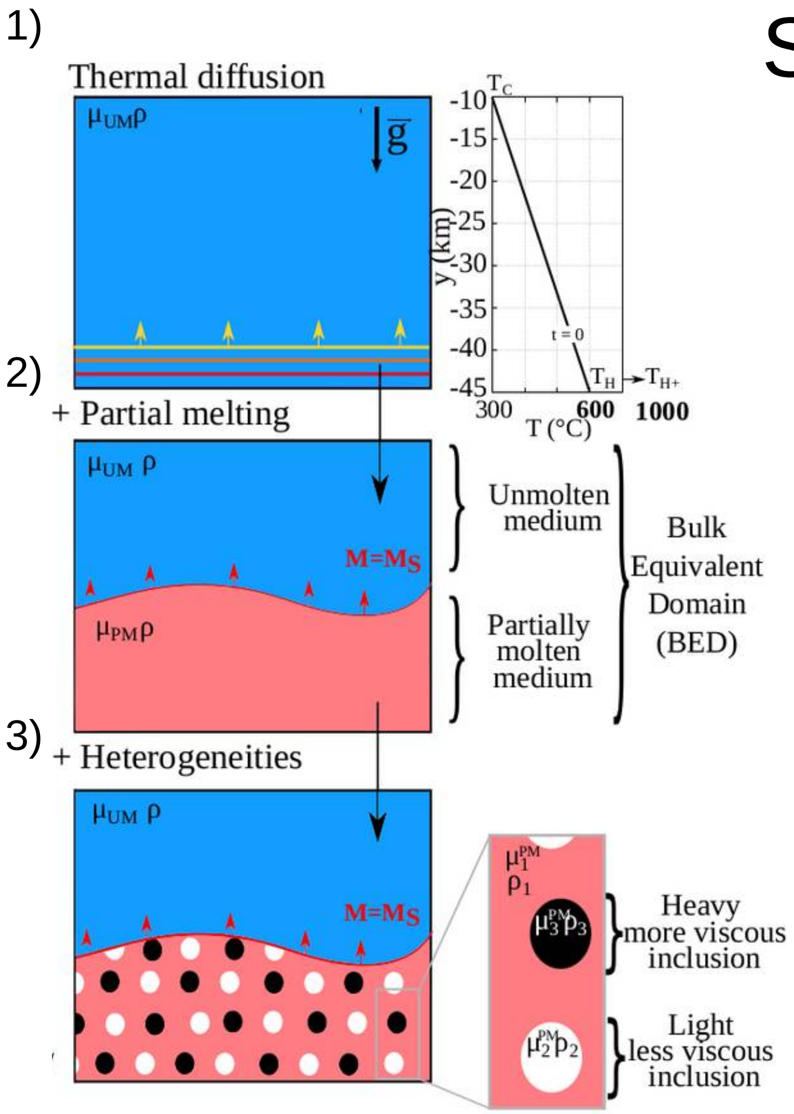
Open∇Foam and the VOF Method, equations

Equations of Phase transport C_i ,

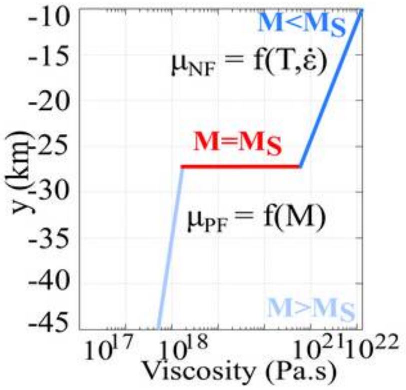


Mass and Momentum conservation, Heat Equation.

Setup : melting and heterogeneities



Melt fraction $M = \dfrac{T - T_{sol}}{T_{liq} - T_{sol}}$



Densities

$$\tilde{\rho} = \rho_{ref} \times [1 - \alpha(T - T_{ref})]$$
$$\rho_{ref} = [\rho^{S0}(1 - M) + \rho^{L0} M]$$

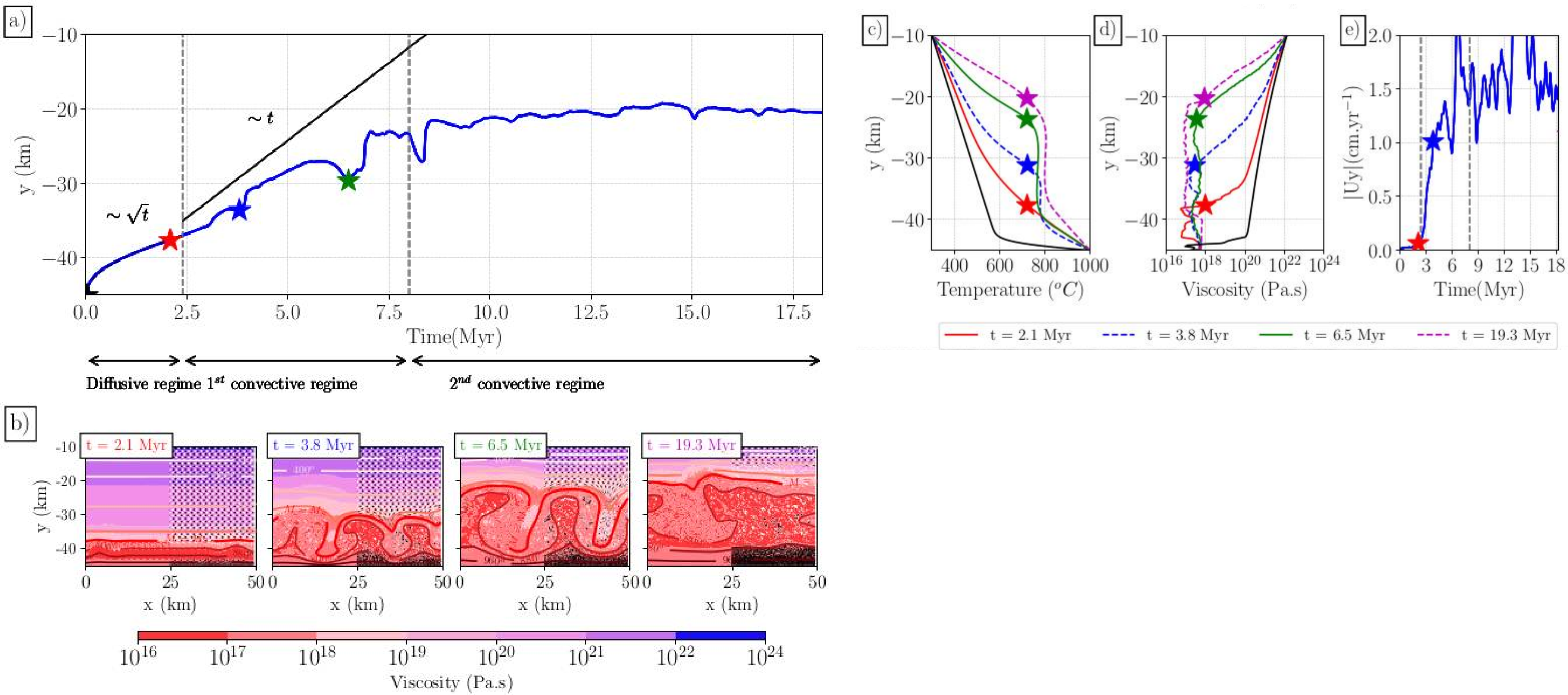
Viscosities

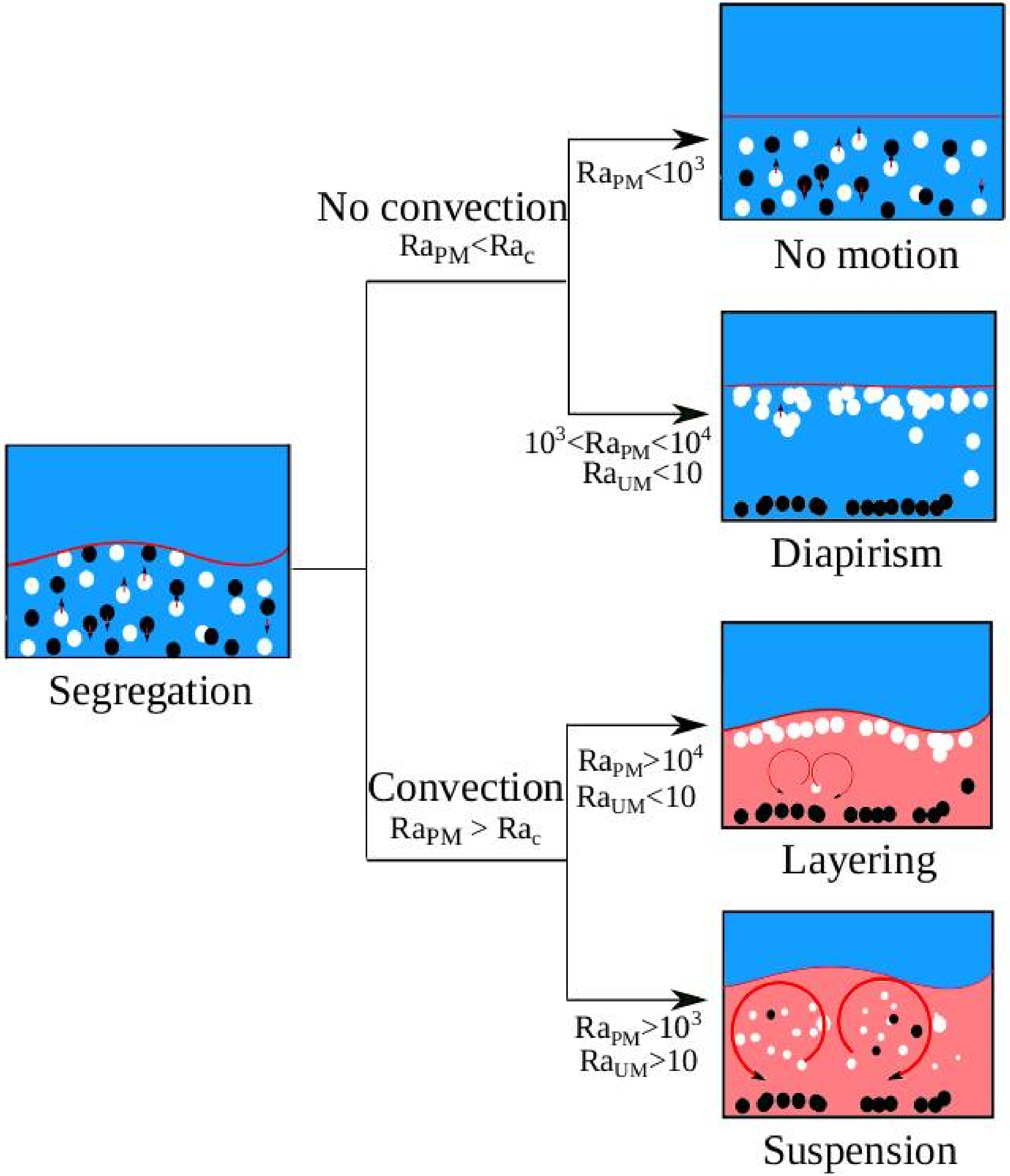
$$\tilde{\mu} = K_{eff}(T)\dot{\varepsilon}^{\frac{1}{n}-1}$$

$$K_{eff}^{UM} = 0.25 \times 10^6 \times (0.75A)^{-\frac{1}{n}} \times exp(\frac{Q}{nRT_{sol}})$$
$$K_{eff}^{PM} = \mu_0 \times exp\left[2.5 + \left(\frac{1 - M_S}{M_S}\right)^{0.48}(1 - M_S)\right]$$

Chen & Morgan, 1990
Pinkerton & Stevenson,1992

A reference model with Convection and melt heterogeneities



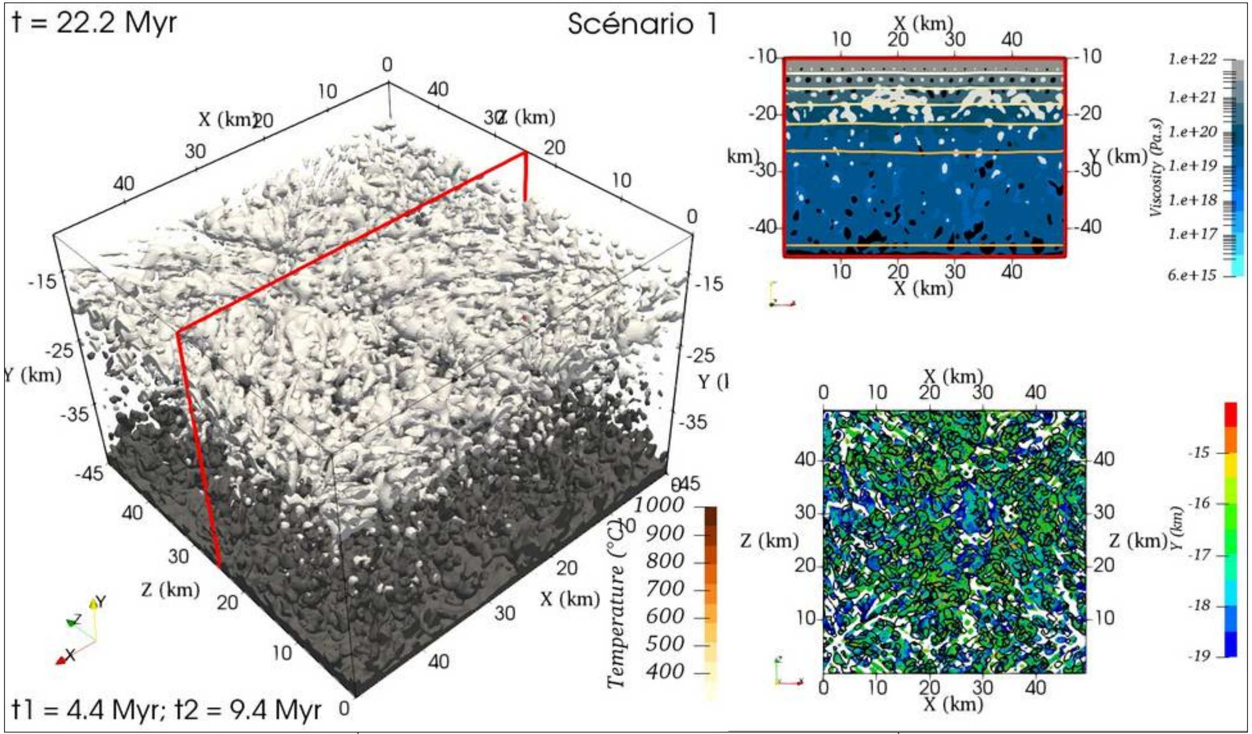


Flow regimes diagram :

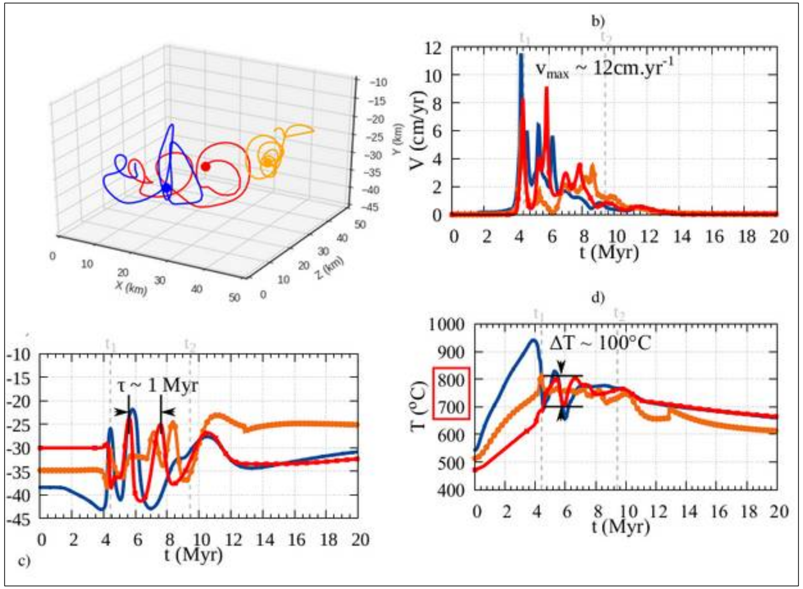
- Diapirism regime,
- Layering regime,
- Suspension regime

function of Ra_{UM} and Ra_{PM}

A 3D model with Convection and melt heterogeneities

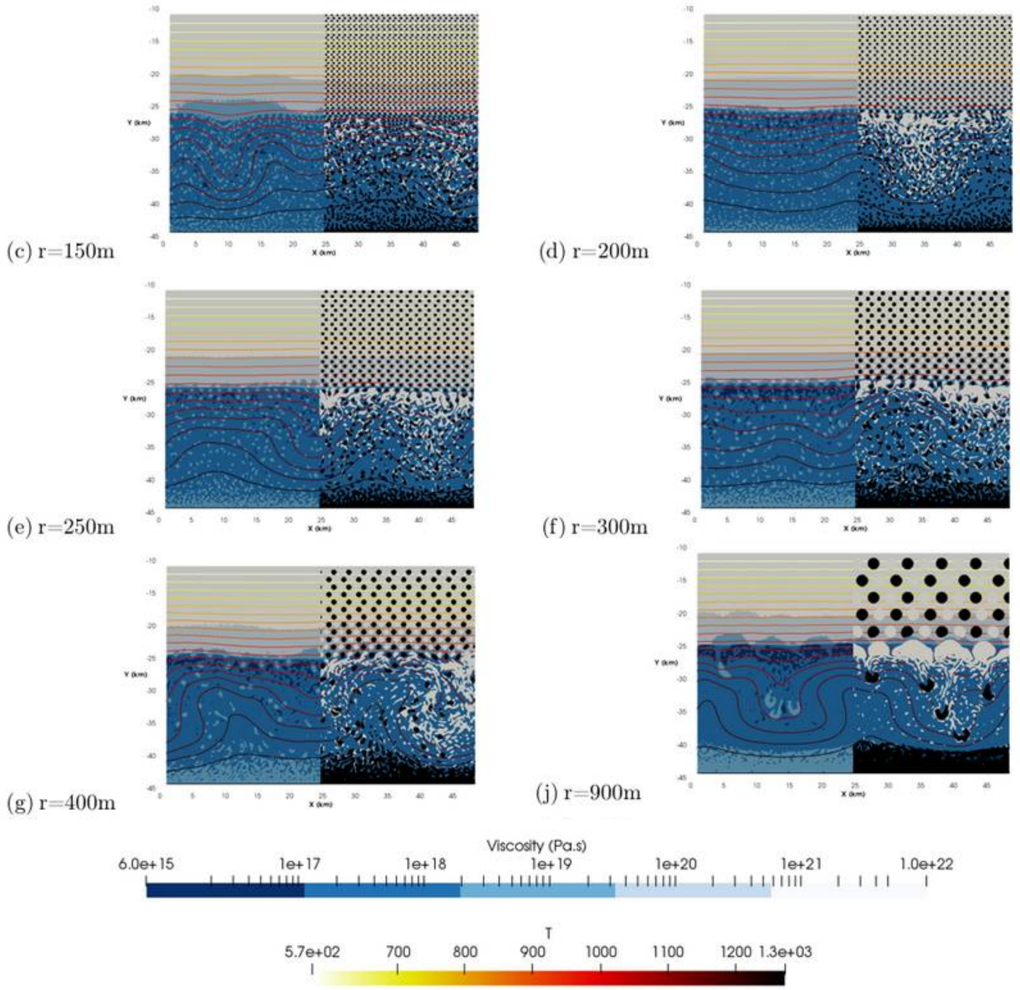


Louis Napoléon *in prep.*



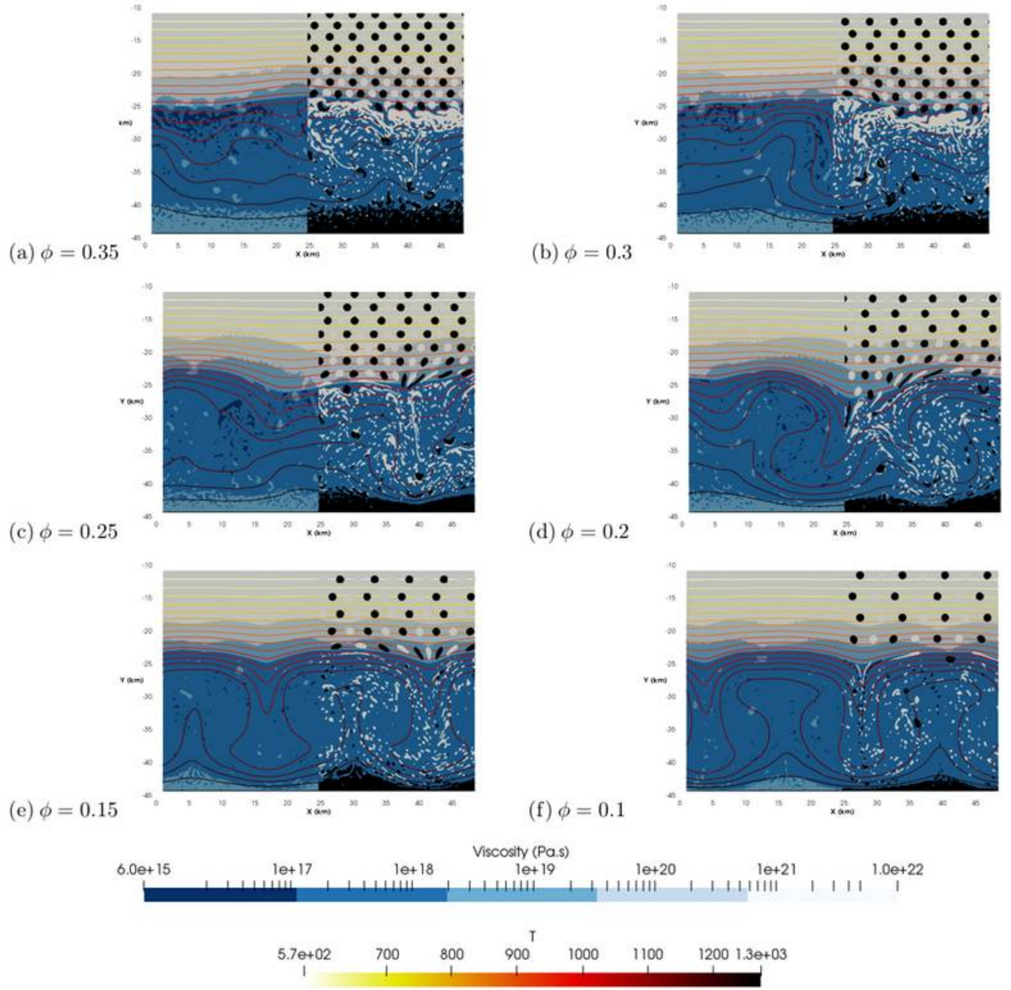
- ‘Domes’ of light inclusions develop and record 3 ‘cycles’ of ΔT~ 100°C every ca. 1 My, from ca. 4 My to 9 My.
- Large domes of ~12 km contain small ‘domes’ of ~4 km.

Influence of inclusions size



- Small intrusions (r<200 m) not 'seen' by the large scale flow
- Large inclusions reduce to ~350 m with advective flow

Influence of inclusions concentration



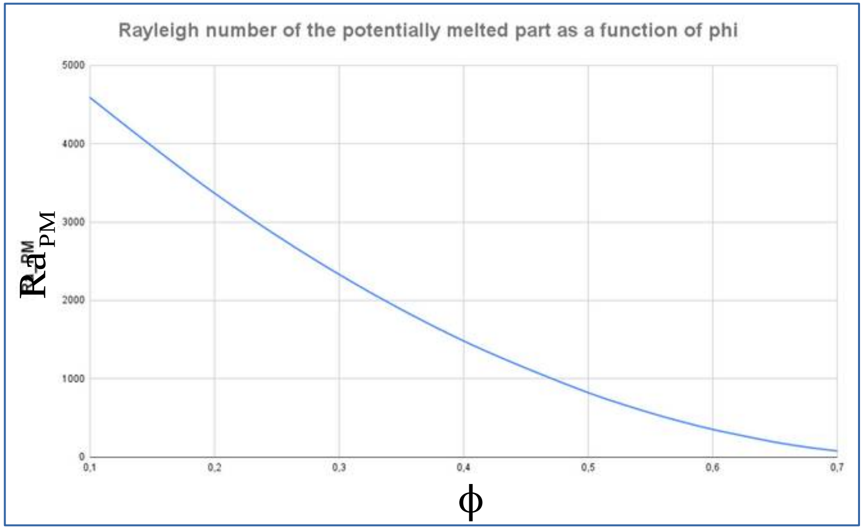
- Small concentration (ϕ<0.3) not 'seen' by large scale flow
- High concentration (ϕ>0.35) always lead to doming.

We modify K and Ra to account for the influence of concentration

- Krieger-Dougherty (1959) : exponential law of the effective viscosity of a particles mixture in suspension in a viscous fluid, as a function of the particles volume fraction :

$$K_{eff}^{PM} = \mu_1^0 \left(1 - \frac{\phi}{\phi_{max}}\right)^{-\frac{5}{2}\phi_{max}} exp\left(2.5 + \left(\frac{1 - M_S}{M_S}\right)^{0.48} (1 - M_S)\right)$$

$$Ra_{PM} = \frac{\rho \alpha g \Delta T_{PM} \left(H_T/2\right)^2}{\kappa K_{eff}^{PM}}$$



Louis Napoléon in prep.

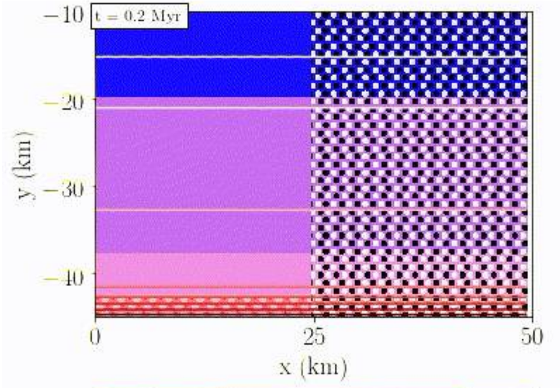
Conclusions

1) The imbricated migmatite domes of Naxos explained by combined convective and diapiric instabilities of felsic, partially melting orogenic crust during **~ 10 My**.

2) Three regimes of **convection, diapirism and layering**. Convection tends to **destroy** domes, but they are **preserved** if basal heating ceases soon enough.

3) Specific range of sizes (r>300m) and concentration (ϕ>0.3) of heterogeneities allow for « dome » formation = consistent with theoretical sub-scale porous media fluid flow.

4) OpenFoam VOF method is appropriate to model such segregation processes, in 3D.



Louis Napoléon et al., J.G.Int 2020, 2021, + (3D in prep.)
https://doi.org/10.1093/gji/ggab510
https://doi.org/10.1093/gji/ggaa141

(A)Dimensional parameters

Preliminary validation of our numerical method :

Geophysical Journal International

Geophys. J. Int. (2020) 222, 474–506
doi: 10.1093/gji/ggaa141
Advance Access publication 2020 March 20
GJI Geodynamics and Tectonics

3-D numerical modelling of crustal polydiapirs with volume-of-fluid methods

Aurélie Louis-Napoléon,[1,2] Muriel Gerbault,[2] Thomas Bonometti,[1] Cédric Thieulot,[3] Roland Martin[2] and Olivier Vanderhaeghe[2]

Adimensional parameters,
accounts for the power law consistency (K_{eff})*:*

$$\frac{K_{eff}^{field}}{K_{eff}^{sim}} = \left(\frac{Ar^{field}}{Ar^{sim}}\right)^{-\frac{1}{2n}} \left(\frac{H^{field}}{H^{sim}}\right)^{\frac{2n+1}{2n}} . \quad (24)$$

Using $n = 1$ in (24), we can write $\mu_0^{field}/\mu_0^{sim} = (Ar^{field}/Ar^{sim})^{-1/2}(H^{field}/H^{sim})^{3/2}$, which

$$\mathcal{T} = q^{-1} = \left(\frac{2K_{eff}}{\Delta\rho g H}\right)^n, \quad \mathcal{L} = H, \quad \mathcal{U} = qH^2, \quad \mathcal{P} = \rho(qH)^2, \quad \mathfrak{T} = \Delta T$$

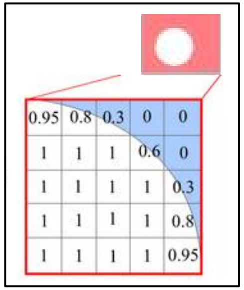
$$t^* = \frac{t}{\mathcal{T}}, \quad x_i^* = \frac{x_i}{H}, \quad U^* = \frac{U}{qH^2}, \quad P^* = \frac{P}{\rho(qH)^2}, \quad g^* = \frac{g}{q^2 H},$$

Key adimensional parameters values :

	Time scale	**Archimedes**	**Rayleigh**	**Rayleigh-Roberts**	**Prandtl**
	q^{-1} (years)	$Ar = \frac{2\rho q^{2-\frac{1}{n}} H^2}{K_{eff}}$	$Ra = \frac{2q}{\kappa/H^2}$	$Ra_H = \frac{2qH^4 H_r}{\kappa^2 C_p \Delta T}$	$Pr = \frac{Ra}{Ar}$
Natural field	642	$[10^{-22}; 10^{-15}]$	$[147; 10^5]$	$[10^{12}; 10^{15}]$	$[10^{20}; 10^{23}]$
Simulations	–	$[10^{-10}; 10^{-1}]$	$[147; 10^5]$	$[10^6; 10^9]$	$[10^5; 10^{10}]$

Open∇Foam and the VOF Method, equations

Phase transport C_i : $\quad \frac{\partial C_i}{\partial t^*} + \mathbf{U}^* \cdot \nabla^* C_i = -\nabla^* \cdot \boxed{(\mathbf{U}_r^* C_r)}$



Mass conservation : $\quad \nabla^* \cdot \mathbf{U}^* = 0$

Conservation of momentum : $\quad \frac{\partial \mathbf{U}^*}{\partial t^*} + \mathbf{U}^* \cdot \nabla^* \mathbf{U}^* = -\nabla^* P^* + \mathbf{g}^* + \nabla^* \cdot [\frac{2}{Ar}(\nabla^* \mathbf{U}^* + (\nabla^* \mathbf{U}^*)^T)]$

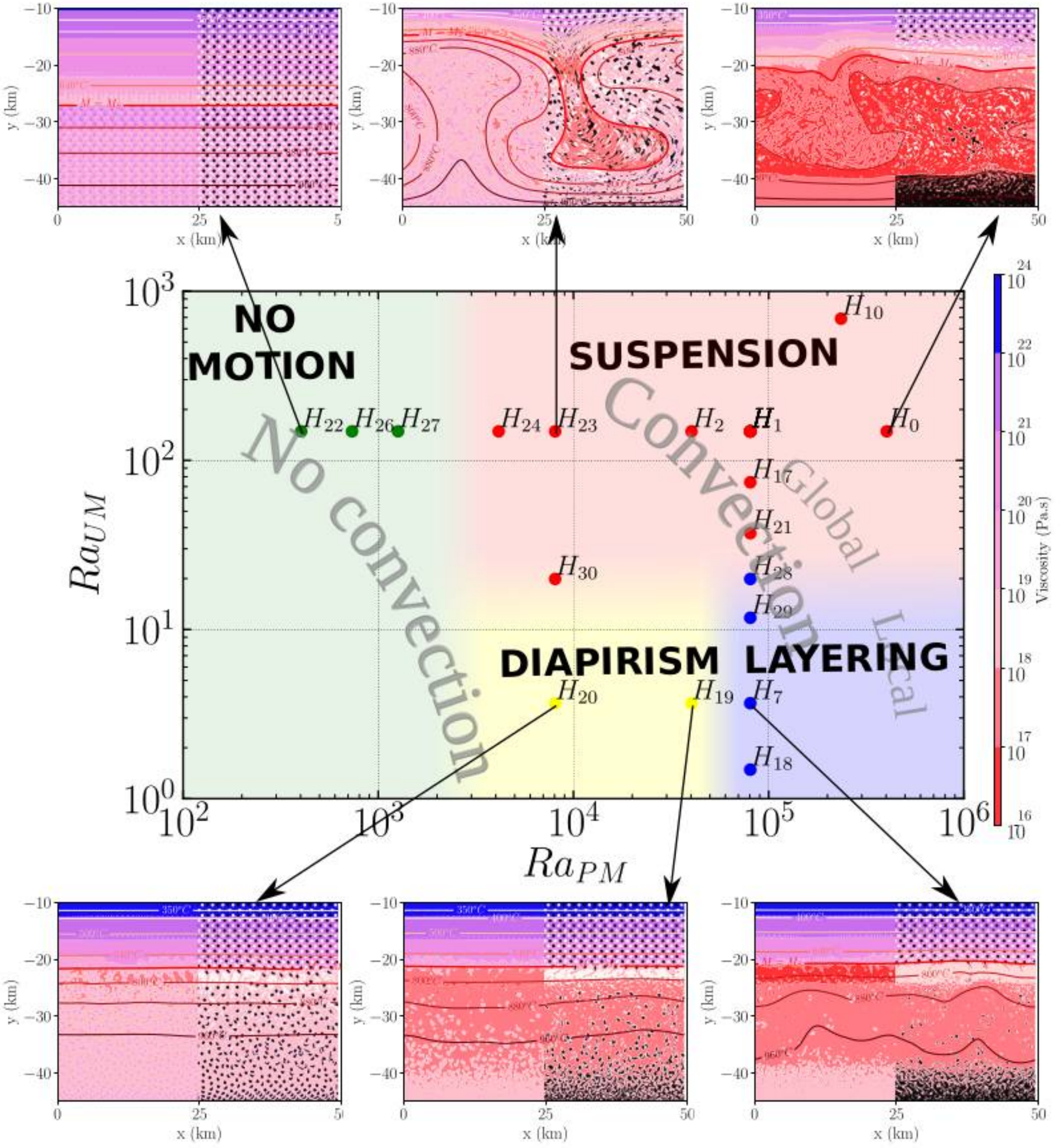
Heat Equation : $\quad \frac{\partial T^*}{\partial t^*} + \mathbf{U}^* \cdot \nabla^* T^* = \nabla^* \cdot (\frac{2}{Ra} \nabla^* T^*) + \frac{2Ra_H}{Ra^2}$

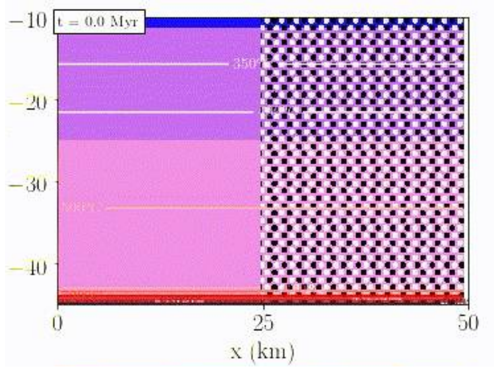
$Ar = \frac{2\rho q^{2-\frac{1}{n}} H^2}{K_{eff}}$ (generalized Archimedes number),

$Ra = \frac{2q}{\kappa / H^2}$ (generalized Rayleigh number),

$Pr = \frac{Ra}{Ar} = \frac{K_{eff}}{\rho \kappa q^{1-\frac{1}{n}}}$ (generalized Prandtl number).

$Ra_H = \frac{2qH^4 H_r}{\kappa^2 C_p \Delta T}$ (generalized Rayleigh-Roberts number).





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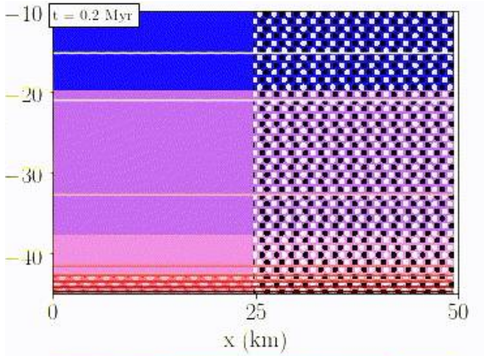

Melting and Convection