Connecting the Grad-Shafranov reconstruction technique to Flux-Rope models

Bachelors' thesis. Engineering Physics and Mathematics.

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Motivation

Grad-Shafranov Reconstruction Technique (GSRT)

- It has been widely used in the past 20 years.
- It was first used to analyse structures in the magnetopause[1].
- It has also been used to reconstruct MFR from in-situ measurements of spacecrafts in the solar wind[2].
- It is based on the Grad-Shafranov equation (GS), which is a fundamental MHD equation.

CC & EC models

- Internal distribution of the magnetic field and current density.
- Also applied to in-situ measurements to reconstruct MFR.

[1] Hau & Sonnerup. Two-dimensional coherent structures in the magnetopause - Recovery of static equilibria from single-spacecraft data (Journal of Geophysical Research, 1999.)

[2] Hu & Sonnerup. Reconstruction of magnetic flux ropes in the solar wind (Geophysical Reseacrh Letters, 2001)

Motivation

- How can we connect them? Can we complement them?
- Can we extract field and geometrical information from the GSRT and use it as parameters in the CC and EC models?

Contents

The Grad-Shafranov equation

- It is the equilibrium equation in ideal magnetohydrodynamics.
- Derived from:

$$\begin{cases} \nabla \cdot \boldsymbol{B} = 0 & \text{(Gauss' law)} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} & \text{(Ampère's law)} \\ \nabla p = \boldsymbol{j} \times \boldsymbol{B} & \text{(Momentum eq. for plasma in equilibrium)} \end{cases} \tag{1}$$

- It can be expressed in different coordinate systems.

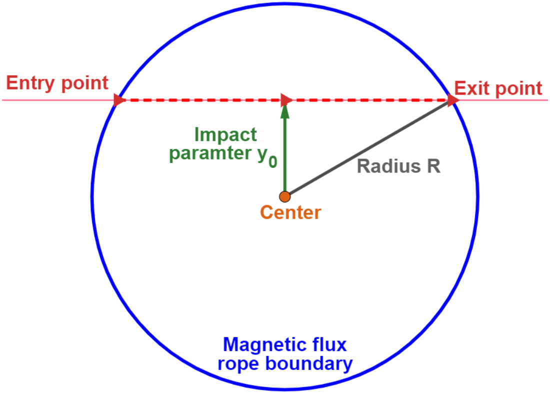
Axial symmetry $\frac{\partial}{\partial z} = 0$ (cartesian coordinates)

$$\nabla_\perp^2 A_z(x,y) = \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu_0 \frac{d}{dA_z}\left(p + \frac{B_z^2}{2\mu_0}\right) \tag{2}$$

Contents

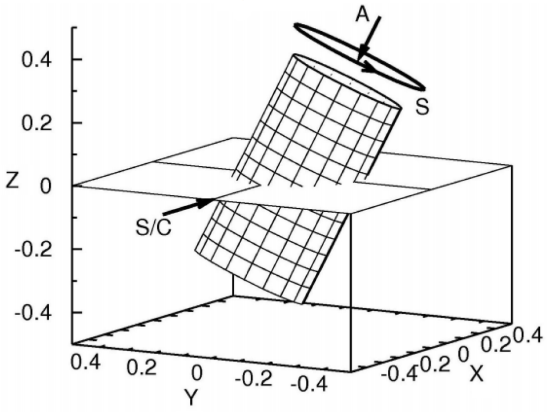
GS reconstruction technique

- **Input**: spacecraft measurements (in RTN) every T seconds.
  1. Magnetic field: B_r, B_t, B_n.
  2. Plasma velocity: v_r, v_t, v_n.
  3. Pressure: p



GS reconstruction technique

- **Output**:
  1. Hoffmann-Teller velocity: $\boldsymbol{v_{HT}}$
     Frame of reference velocity moving with the MFR.
  2. MFR axis orientation: $\phi \in [0, 360)$, $\theta \in [0, 180]$.
  3. Reconstructed $A_z(x, y)$ in the whole cross section.



Contents

Connection with the CC & EC models

- The 2016 circular-cylindrical (CC) model of **Nieves-Chinchilla** et. al.[3] assumes planar and axial symmetry (no dependence in φ nor z, only in r).
- It uses the following equations:

$$\begin{cases} \nabla \cdot \boldsymbol{B} = 0 & \text{(Gauss' law)} \\ \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} & \text{(Ampère's law)} \end{cases} \tag{3}$$

And assumes $J_r = 0$ and that J_φ and J_z are polynomial in r.

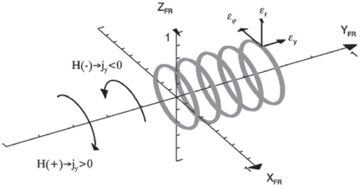


Figure: Current density scheme.

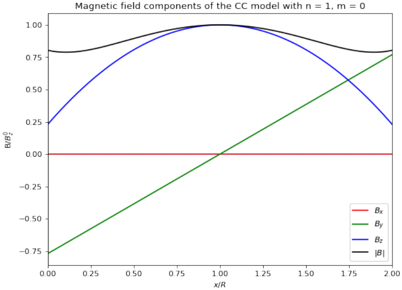
[3] A circular-cylindrical flux-rope analytical model for magnetic clouds (The Astrophysical Journal, 2016)

Connection with the CC & EC models

- The resulting magnetic fields are:

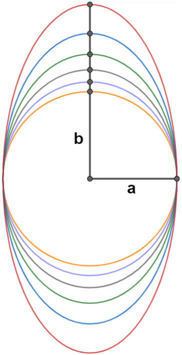
$$\begin{cases} B_r = 0 \\ B_\varphi = \mu_0 \beta_m \dfrac{r^{m+1}}{m+2} \\ B_z = B_z^0 - \mu_0 \alpha_n \dfrac{r^{n+1}}{n+1} \end{cases} \tag{4}$$

with $n \geq 1$, $m \geq 0$.



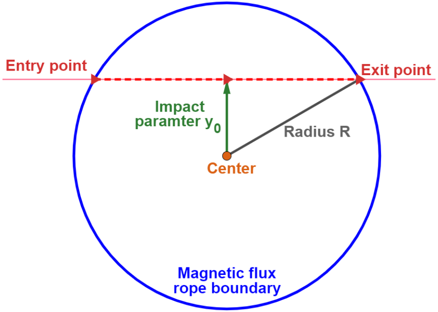
Connection with the CC & EC models

- The **2018 elliptical-cylindrical (EC) model** of **Nieves-Chinchilla et. al.**[4] is a generalization of the CC model. It uses similar equations in an elliptical geometry.
- One of the most important parameter is the **distortion** $\delta = \frac{a}{b} \in (0, 1]$, defined as the ratio of the small and the large axis of the ellipse.



[4] Elliptic-cylindrical Analytical Flux Rope Model for Magnetic Clouds (The Astrophysical Journal, 2018)

1. **Case 1: CC model**
   - We simulate the crossing of a spacecraft through a MFR whose magnetic field is given by the CC model.
   - We use the GSRT and determine the parameters.
   - Check whether the simulated and reconstructed parameters are in agreement.



Reconstruction steps

1. **Input parameters (CC model)**

Field parameters:

- $n = 1, \ m = 0$
- $\tau = 1.3, \ C_{10} = 1, \ B_z^0 = 10$ nT
- Helicity $s = -1$

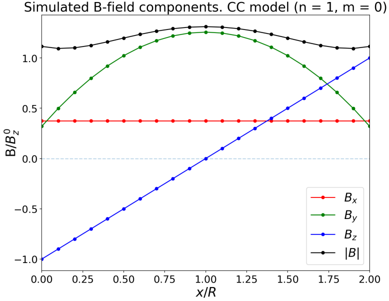
Geometrical parameters:

- Orientation $\phi = 90°, \ \theta = 00°$ (MFR laying on the y axis)
- Radius $R = 0.07$ AU
- Impact parameter $y_0 = 35\%$
- Relative speed $u_s = 450$ km/s

$$\begin{cases} B_r = 0 \\ B_\varphi = s \dfrac{B_z^0}{C_{10}} \left(\dfrac{r}{R} \right) \\ B_z = \dfrac{B_z^0}{\tau} \left[\tau - \left(\dfrac{r}{R} \right)^2 \right] \end{cases}$$

2. **Simulated data**

We simulate the trajectory across the MFR and its corresponding magnetic field.

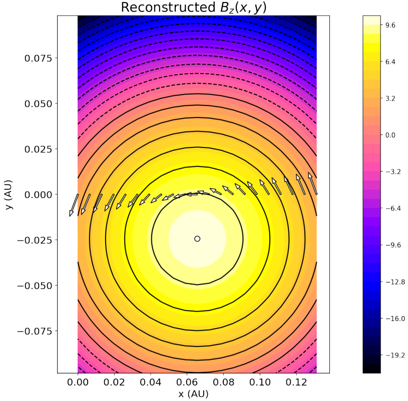


Reconstruction steps

3. **Reconstructed axis**

   $\phi = 90^\circ$, $\theta = 90^\circ$ ✓

4. **Reconstructed cross-section**



- $B_z^0 = 9.9999$ nT ✓
- Impact parameter $y_0 = 0.02393$ AU, corresponding to 34.191%. ✓
- The axial current $J_z(x, y)$ is constant and equal to 1.169 pA for the given parameters. ✓

Reconstruction of B_z

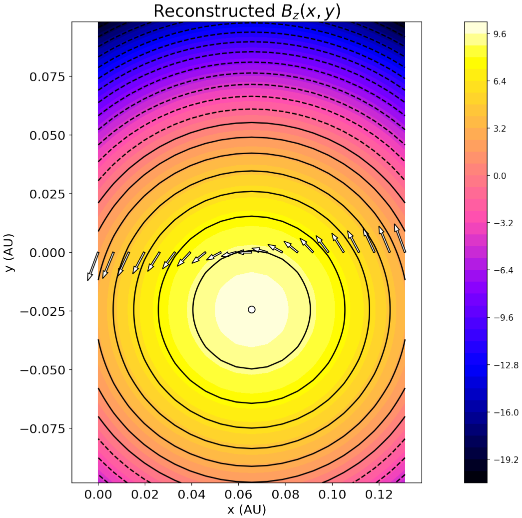


Figure: Reconstructed cross-section of $B_z(x, y)$.

Reconstruction of B_x and B_y

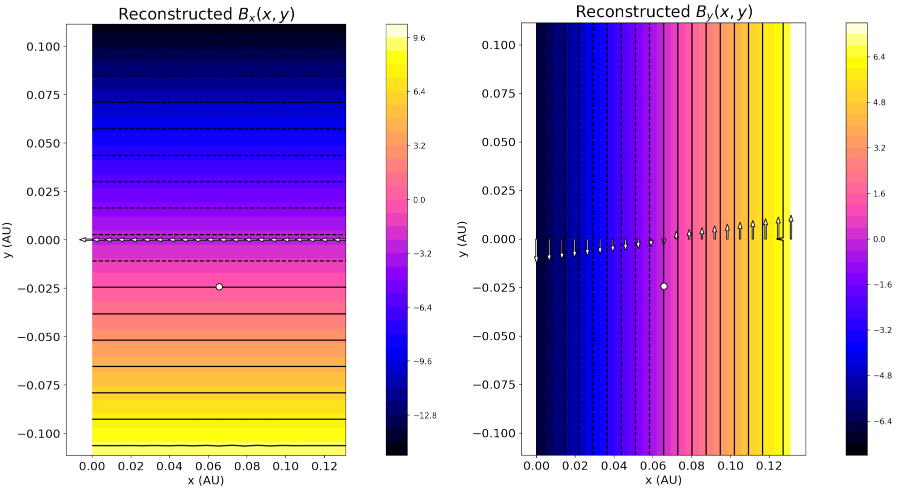
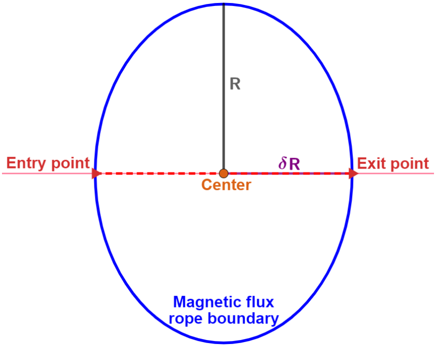


Figure: Reconstructed cross-section of $B_x(x, y)$ and $B_y(x, y)$.

1. **Case 2: EC model**
    - We simulate the crossing of a spacecraft through a MFR whose magnetic field is given by the EC model.
    - We use the GSRT and determine the parameters.
    - Check whether the simulated and reconstructed parameters are in agreement.



Reconstruction steps

1. **Input parameters (EC model)**

Field parameters:

- $n = 1,\ m = 0$
- $\tau = 1.3,\ C_{10} = 1,\ B_z^0 = 10$ nT
- Helicity $s = -1$

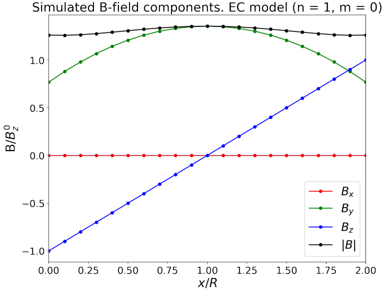
Geometrical parameters:

- Orientation $\phi = 90^\circ$, $\theta = 00^\circ$ (MFR laying on the y axis)
- Distortion $\delta = 0.75$, ellipse rotation $\xi = 0$
- Radius $R = 0.07$ AU
- Impact parameter $y_0 = 0$
- Relative speed $u_s = 450$ km/s

$$\begin{cases} B_r = 0 \\ B_\varphi = s\dfrac{B_z^0}{C_{10}}\left(\dfrac{r}{R}\right) \\ B_z = \dfrac{B_z^0}{\tau}\left[\tau - \left(\dfrac{r}{R}\right)^2\right] \end{cases}$$

2. **Simulated data**

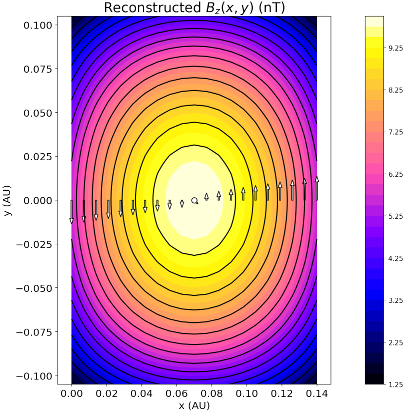
We simulate the trajectory across the MFR and its corresponding magnetic field.



Reconstruction steps

3. **Reconstructed axis**
   $\phi = 90°$, $\theta = 90°$ ✓

4. **Reconstructed cross-section**



- $B_z^0 = 10.000nT$ ✓
- Impact parameter $y_0 = 0$ AU. ✓

Reconstruction of B_z and J_z

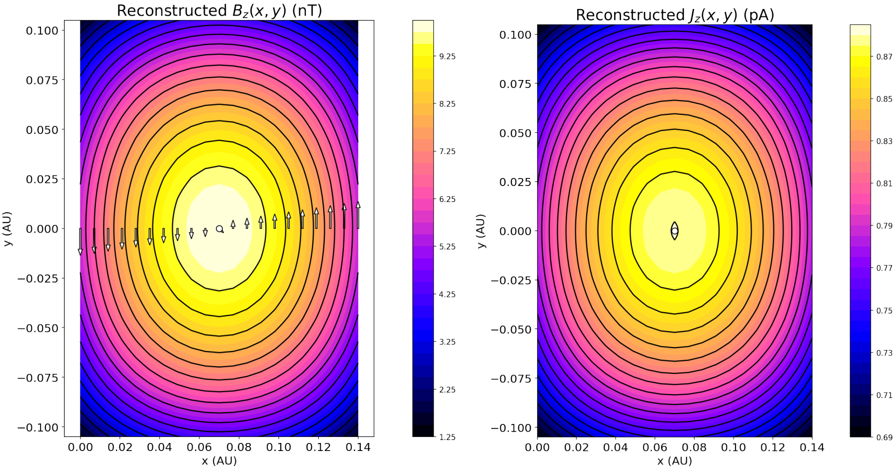


Figure: Reconstructed cross-section of $B_z(x, y)$ and $J_z(x, y)$.

Reconstruction of B_x and B_y

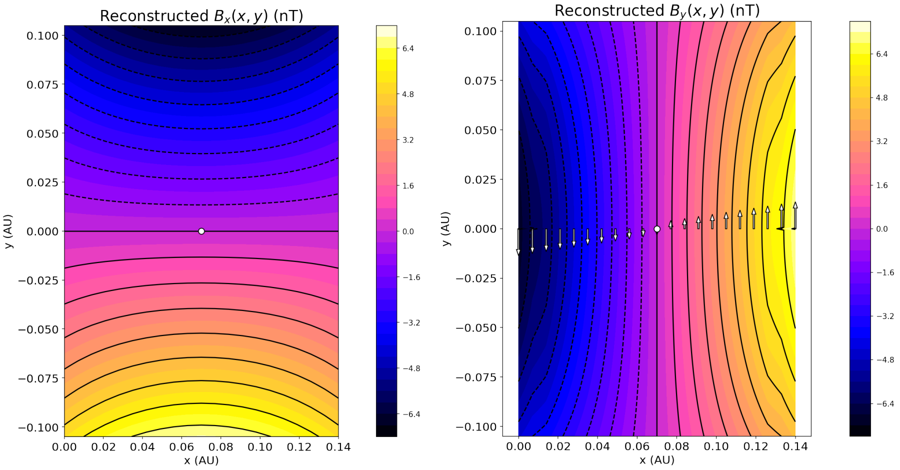
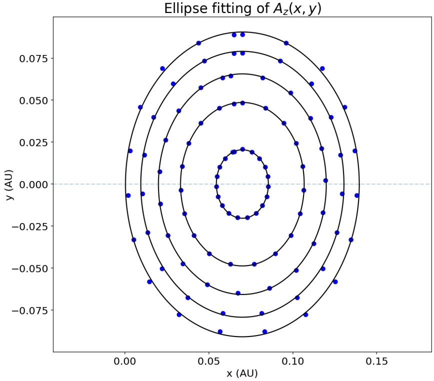


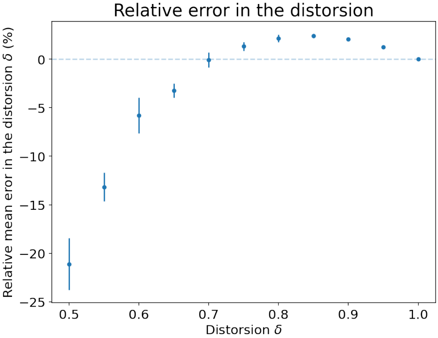
Figure: Reconstructed cross-section of $B_x(x, y)$ and $B_y(x, y)$.

3. **Case 3: Distortion determination with the EC model**
   - We have repeated case 2 for values of the distortion δ ranging from 0.5 (elongated) to 1 (circle).
   - We have fitted an ellipse for every level curve of the $A_z(x, y)$ cross-section plot (see below for $\delta = 0.75$).



Connection with the EC model

- The estimation is very good for $\delta = 1$, with a standard deviation of 0.1%.
- As we increment the distortion (decrease δ), the accuracy in the determination of δ decreases.
- Still for $\delta = 0.6$, the relative error is $(5.8 \pm 1.8)\%$



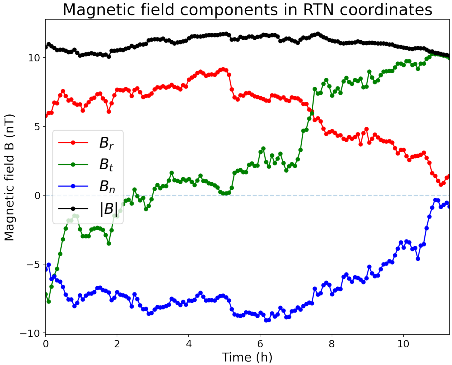
Contents

Real event analysis

- We have used the STEREO-A Event on 23 May 2007 event as a benchmark event.
- We compare our results to the ones by C. Möstl et. al.[5].

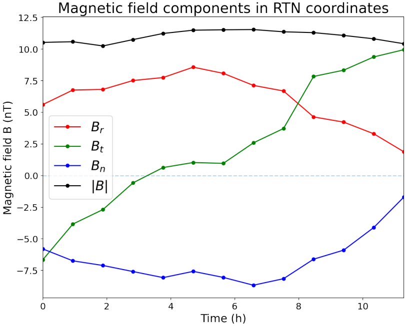
[5] Optimized Grad – Shafranov Reconstruction of a Magnetic Cloud Using STEREO-Wind Observations (Solar Phys, 2009).

In-situ measurements of the spacecraft



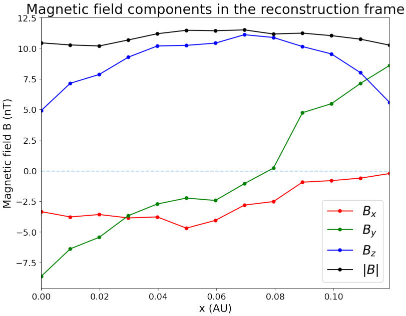
Reconstruction steps

1. **Data filtering**: we resample the measurements to $m = 13$ data points.



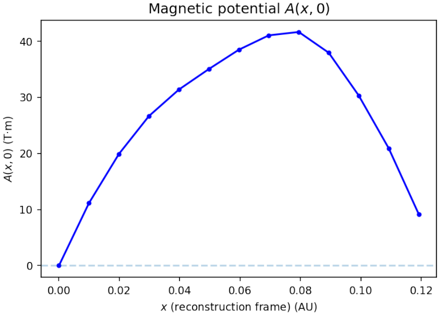
2. **Hoffmann-Teller velocity**:
   $\mathbf{v}_{HT} = (343.4, -2.3, -12.3)$ km/s

3. **Axis orientation**: $\phi = 40°$, $\theta = 148°$

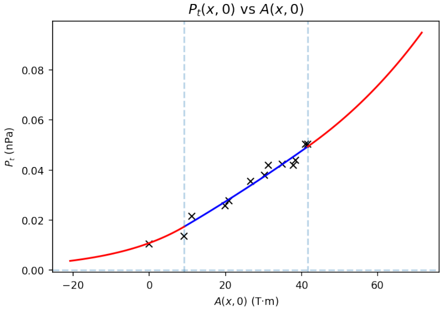


Reconstruction magnitudes

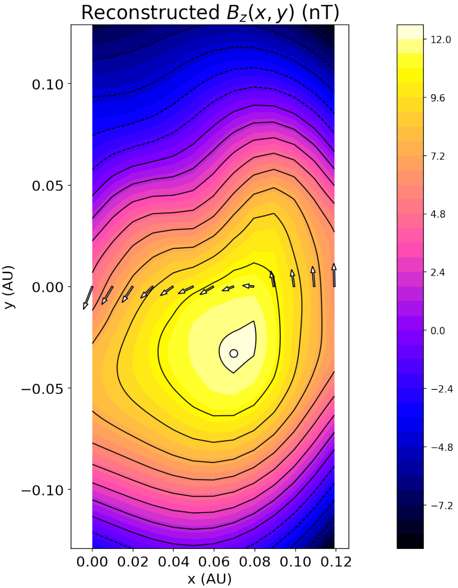
- **Magnetic potential $A_z(x, 0)$**

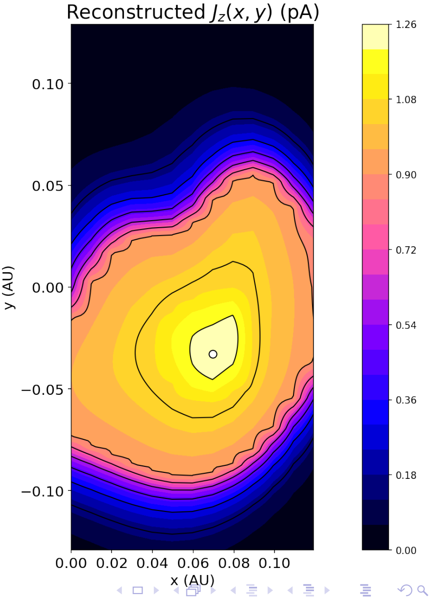


- **Transverse pressure vs $A(x, 0)$**

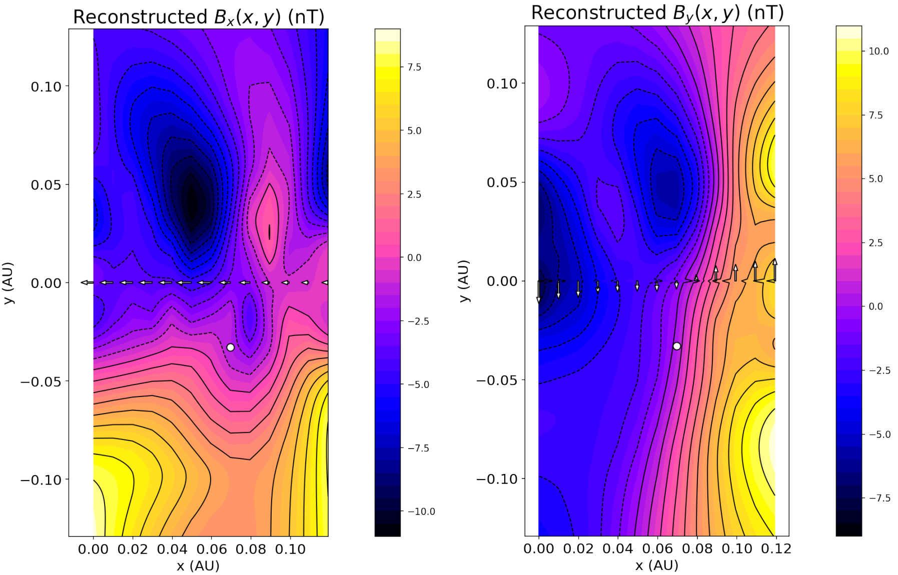


Reconstruction of B_z and J_z

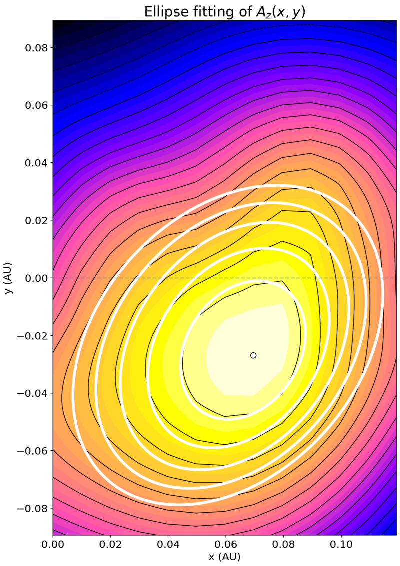




Reconstruction of B_x and B_y



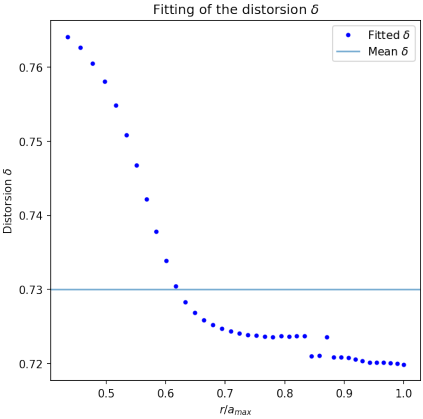
Distortion



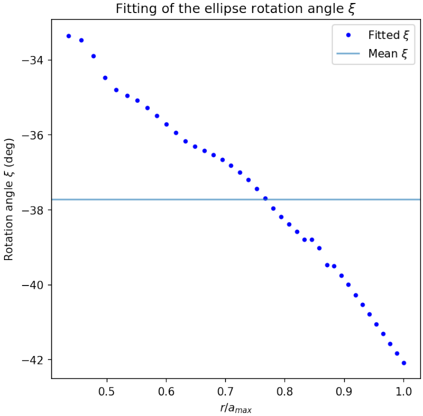
Ellipse fitting of $A_z(x, y)$
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
y (AU)
0.00
0.02
0.04
0.06
0.08
0.10
x (AU)

Distortion and rotation angle

- **Distortion**
  $\delta = 0.730 \pm 0.136$



- **Rotation angle**
  $\xi = (-37.71 \pm 2.45)^\circ$



Contents

Conclusions and final remarks

- We have successfully connected the GSRT to the CC and EC models.
- GSRT can determine with accuracy the distortion, up to $\delta = 0.6$.
- There is still work to do: try other axis orientations.
- Could be generalized to allow for multi-spacecraft measurements.
- The combination of GSRT and the CC and EC models makes both of them more robust.

That's all folks!

Thank you!

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I want to show my gratitude to NASA GSFC for enabling me to do this research and sponsoring my AGU and EGU participation. I also want to thank CFIS-UPC for granting me the opportunity to carry out my Bachelor's thesis at NASA GSFC and financially supporting it. I wish to extend my special thanks to Dr. Teresa Nieves Chinchilla and all the team for the support, attention and advice given throughout all my thesis.