A link between turbulent cascade and gyrotropic pressure instabilities in compressible and magnetized fluids.

Laboratoire de Physique des Plasmas

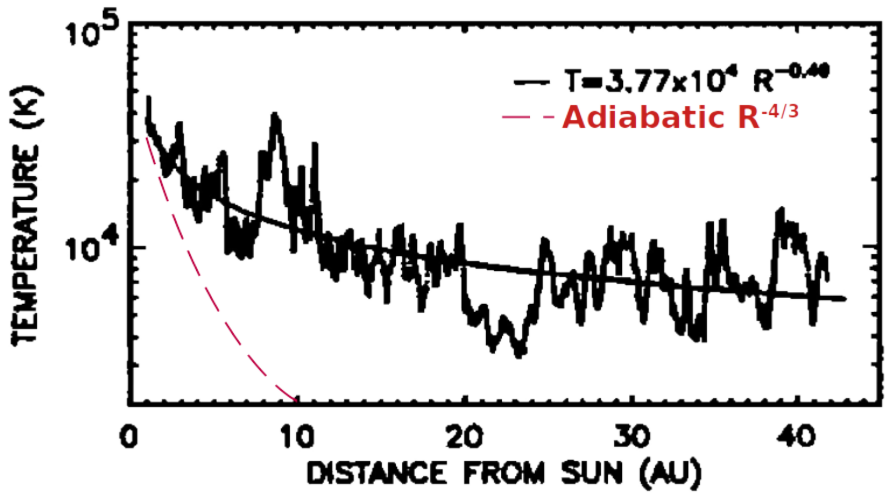
Pauline SIMON

PhD student at Laboratoire de Physique des Plasmas, École Polytechnique, France
from the 01/10/2020 to the 30/09/2023

Directors : Fouad SAHRAOUI, Sébastien GALTIER

École Doctorale d'Astronomie & Astrophysique d'Île-de-France

The heating issue of the Solar Wind



Properties of the solar wind

- Collisionless

Missions launched in the Solar Wind reported a **non-adiabatic profil of temperature.**

[Barnes 1992, Richardson 1995]

Solution: the turbulent cascade of total energy

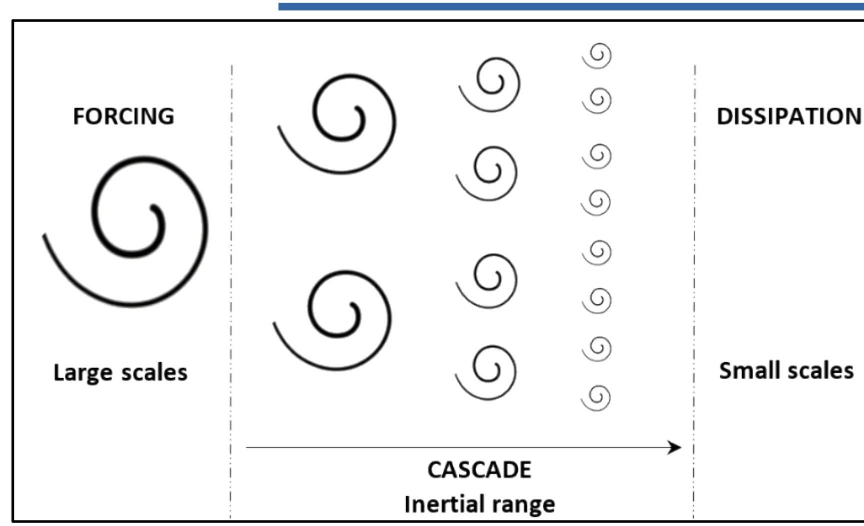
Kolmogorov's hypothesis :

- Large scale forcing and small scale dissipation
- Statistical stationarity and homogeneity
- Large Reynolds numbers

Statistical temporal derivation of a correlation function between 2 points...

... Exact Law:

$$-4\frac{\varepsilon}{\rho_0} = \nabla_\ell \cdot < |\delta \mathbf{v}|^2 \delta \mathbf{v} >$$



Exact laws for magnetized and compressible plasmas

Previous exact laws:

- Incompressible (PP98) [Politano & Pouquet 1998]
  $\rho \propto 1$ and $D_t u = 0$
- Isothermal (derived directly with an explicite internal energy expression)
  [Banerjee & Galtier 2013][Andrés & Sahraoui 2017]
  $p \propto \rho$ and $u \propto \ln(\rho/\rho_0)$

Polytropic law $p \propto \rho^\gamma$ and $u \propto \dfrac{p}{\rho(\gamma-1)}$

General exact law for an isotropic pressure
[Simon & Sahraoui 2021 (ApJ)]

$$D_t u = -\frac{p}{\rho} \nabla \cdot \mathbf{v}$$

General exact law for a tensorial pressure
[Simon & Sahraoui 2022 (PRE)]

$$D_t u = -\frac{\overline{\overline{P}}}{\rho} : \nabla \mathbf{v}$$

Gyrotropic laws:

$$\overline{\overline{P}} = p_\perp \overline{\overline{I}} + (p_\| - p_\perp)\mathbf{bb} \text{ and } \rho u = \frac{1}{2} \overline{\overline{P}} : \overline{\overline{I}}$$

- **Compressible CGL (bi-adiabatic)**
- **Incompressible (Correction of PP98 depending on the pressure anisotropies)**

LPP

Gyrotropic laws: Exact law for CGL model (bi-adiabatique)

$$-4\varepsilon^{\mathrm{GYR}} = \nabla_\ell \cdot \mathcal{F}^{\mathrm{GYR}} + \mathcal{S}^{\mathrm{GYR}} + \mathcal{S}'^{\mathrm{GYR}}$$

$$\begin{cases}
\mathcal{F}^{\mathrm{GYR}} = \langle \delta(\rho\mathbf{v})\cdot\delta\mathbf{v}\delta\mathbf{v} + \delta(\rho\mathbf{v_A})\cdot\delta\mathbf{v_A}\delta\mathbf{v} - \delta(\rho\mathbf{v_A})\cdot\delta\mathbf{v}\delta\mathbf{v_A} - \delta(\rho\mathbf{v})\cdot\delta\mathbf{v_A}\delta\mathbf{v_A}\rangle \\
\quad + \left\langle \delta\rho\delta\left(\frac{\mathbf{v_A}^2}{2}\left(\beta_\parallel[1+a_p]-1\right)\right)\delta\mathbf{v} - \delta\rho\delta\left(\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\right)\cdot\delta\mathbf{v}\right\rangle , \\
\mathcal{S}^{\mathrm{GYR}} = \left\langle \left(\rho\mathbf{v}\cdot\delta\mathbf{v} + \frac{1}{2}\rho\mathbf{v_A}\cdot\delta\mathbf{v_A} - \frac{1}{2}\mathbf{v_A}\cdot\delta(\rho\mathbf{v_A}) + \rho\delta\left(\frac{\mathbf{v_A}^2\beta_\parallel}{2}\right)\right)\nabla'\cdot\mathbf{v}'\right\rangle \\
\quad - \langle \rho\delta\left(\beta_\parallel[1-a_p]\mathbf{v_A}\mathbf{v_A}\right) : \nabla'\mathbf{v}'\rangle \\
\quad + \langle(-2\rho\mathbf{v}\cdot\delta\mathbf{v_A} - \rho\mathbf{v_A}\cdot\delta\mathbf{v} + \delta(\rho\mathbf{v})\cdot\mathbf{v_A})\,\nabla'\cdot\mathbf{v}'_\mathbf{A}\rangle \\
\quad + \left\langle\left((\delta\rho)\frac{\mathbf{v_A}^2}{2}[a_p\beta_\parallel+1]\mathbf{v} - \rho\delta\left(\frac{\mathbf{v_A}^2}{2}[a_p\beta_\parallel+1]\right)\mathbf{v}\right)\cdot\frac{\nabla'\rho'}{\rho'}\right\rangle \\
\quad + \left\langle\left((\delta\rho)\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\cdot\mathbf{v} - \rho\delta\left(\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\right)\cdot\mathbf{v}\right)\cdot\frac{\nabla'\rho'}{\rho'}\right\rangle , \\
\mathcal{S}'^{\mathrm{GYR}} = \text{conjugate}\left(\mathcal{S}^{\mathrm{GYR}}\right) .
\end{cases}$$

With: $\beta_\parallel = \frac{p_\parallel}{p_M}$, $a_p = \frac{p_\perp}{p_\parallel} = \frac{T_\perp}{T_\parallel}$

[Simon & Sahraoui 2022 (PRE)]

LPP

Gyrotropic laws : a link between turbulence and instabilities ?

PP98 + Correction due to pressure anisotropy: [Simon & Sahraoui 2022 (PRE)]

$$-4\frac{\varepsilon}{\rho_0} = \underbrace{\nabla_\ell \cdot < (|\delta \mathbf{v}|^2 + |\delta \mathbf{v_A}|^2)\delta \mathbf{v} - 2\delta \mathbf{v} \cdot \delta \mathbf{v_A} \delta \mathbf{v_A} >}_{\text{PP98}} + < \delta(\beta_\parallel (1 - a_p)\mathbf{v_A v_A}) : \delta(\nabla \mathbf{v}) >$$

Depending on the sign of this term, the pressure anisotropies can diminish or reinforce the cascade.

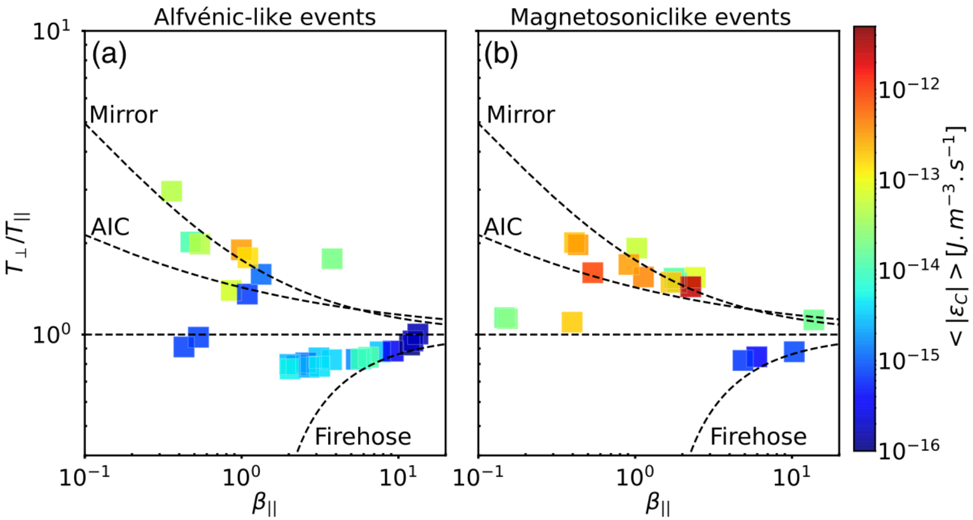
With: $\beta_\parallel = \frac{p_\parallel}{p_M}, a_p = \frac{p_\perp}{p_\parallel} = \frac{T_\perp}{T_\parallel}$

Conditions for firehose instabilities:

$a_p < 1 =>$: Firehose instabilities

In the compressible case, the analysis is the same but, this time, it is also possible to see mirror instabilities if $a_p > 1$.

[Hadid et al 2018]



Conclusion

Results:

- An extension of the theory of exact law that gives the mean cascade rate for compressible plasmas with tensorial pressure.
- A correction of the reknown Politano and Pouquet's incompressible law due to the anisotropy of pressure.
- A potential link between linear instabilities due to pressure anisotropy and turbulence, a non-linear process.

Simon & Sahraoui 2021
ApJ: vol 916, p49
arXiv:2105.08011

Simon & Sahraoui 2022
PRE: in press
arXiv:2112.03601

What's next ?

- Computing the new laws in turbulence simulation data to refine our understanding of the theory.
- Look at spacecraft data to confront theory and reality.

This presentation participates in OSPP

Outstanding Student & PhD candidate Presentation contest