HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Theoretical explanation for the formation of zebra stripes in the absence of electric or magnetic field fluctuations

Adnane Osmane
University of Helsinki, FI

EGU Spring meeting
26.05.2022

@Adnane_Osmane

Zebra stripes: characteristics

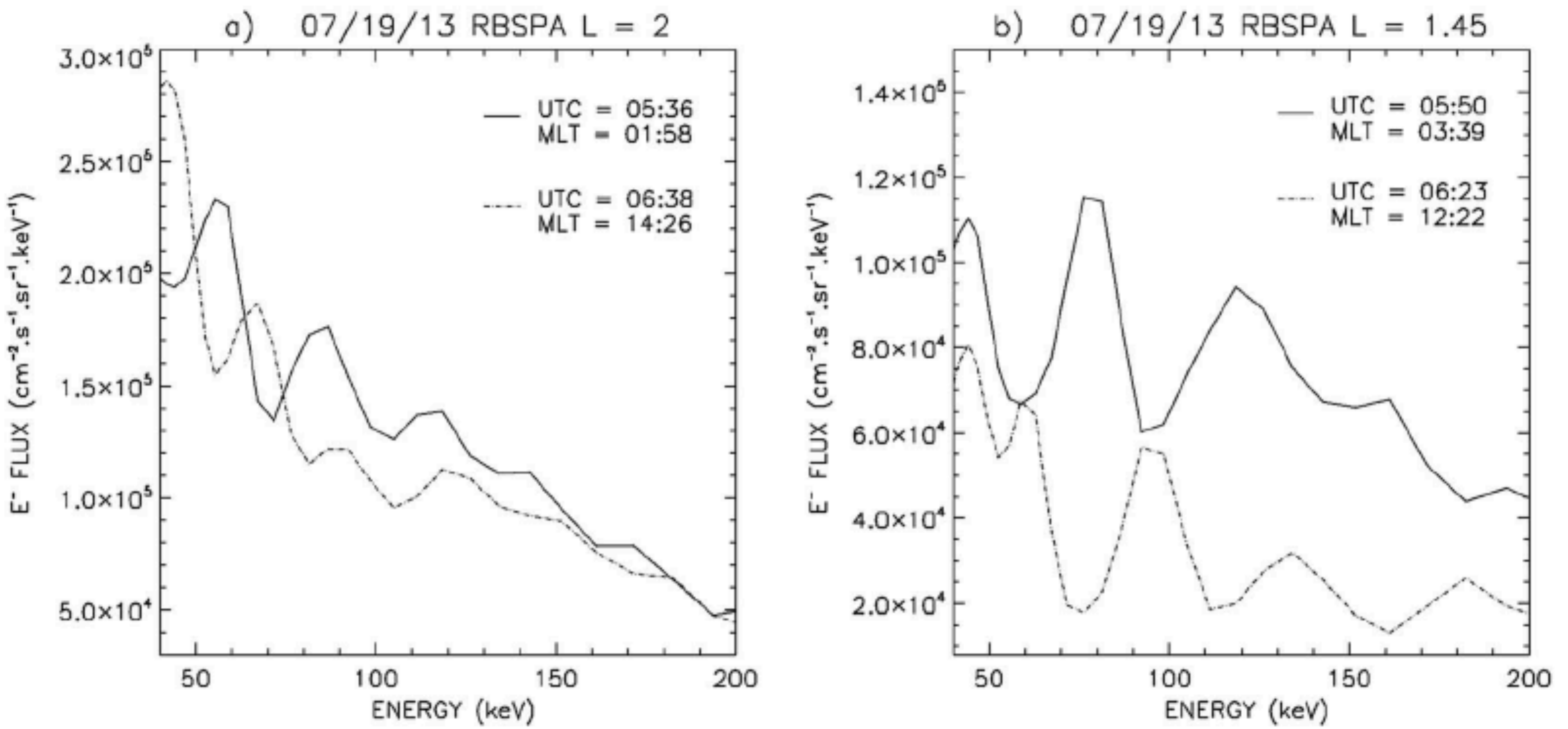


Figure 1. Peak-to-valley inversions observed when comparing differential electron fluxes measured during the successive inbound and outbound crossing of (a) the $L = 2$ shell and (b) the $L = 1.45$ shell.

Structured peaks and valleys observed on spectrograms of energetic electrons (10s-100s keV) trapped in the inner radiation belts.

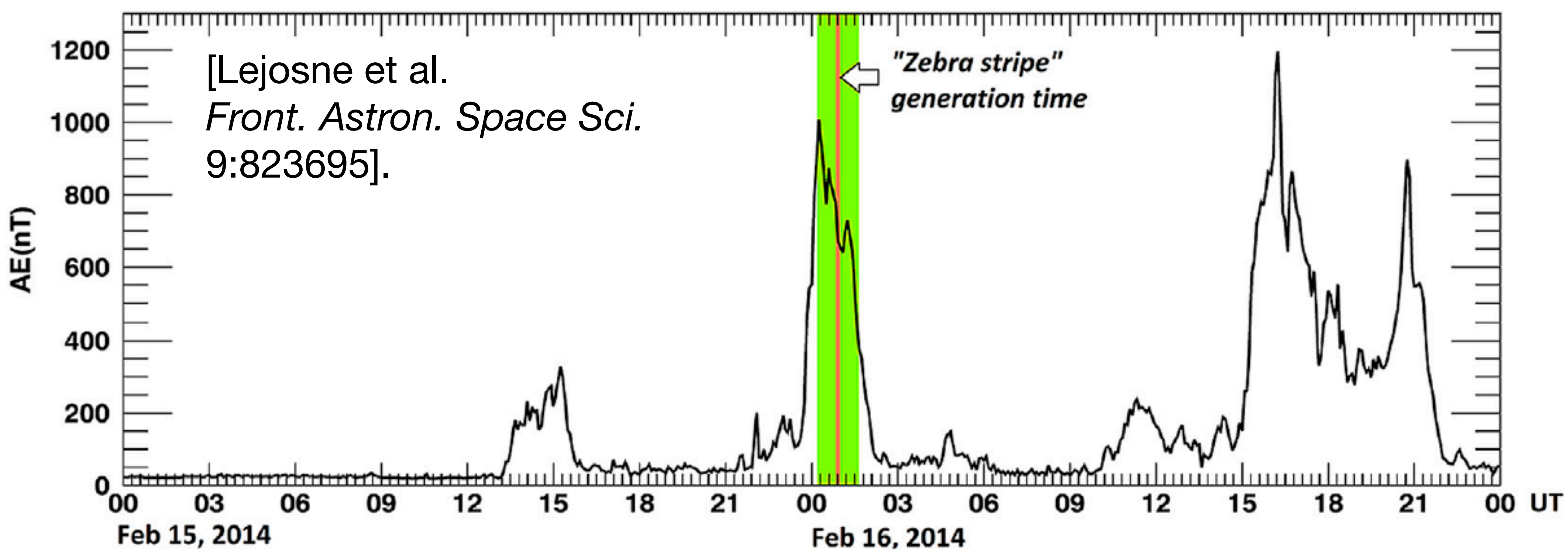
[Lejosne and Roederer, JGR, 121, (2016)]

Zebra stripes: characteristics

- Observed since 1965 by Imhof and Smith [PRL, 14] and characterised with (1) central energy peaks varying as 1/L and (2) peaks of constant drift period.

- Stripes are correlated with Kp & Dst indices when observed by DEMETER, associated with substorm onsets, and are commonly observed i.e., 80% of the time during a 100 days of measurements [Datlowe, 1985].

- Stripes are transients, forming on the order of the drift period, [e.g. Imhof et al. 1981a;Sauvaud et al. 2013], and Ukhorskiy et al. 2014 argues that number of stripes increases with time, so they eventually get too thin to be resolved.



Why is it worth our time and effort?

- Modelling of the Earth's radiation belts is now a mature field (>60 years), and zebra stripes can be used to test the theoretical apparatus we have built.
- Transient coherent structures that are commonly observed in the inner belts and contrasts with dynamics in the outer belts.
- If sustained by an electric field, it could be used as a diagnostic of radiation belts.
  [See Lejosne et al. *Front. Astron. Space Sci.* 9:823695 and references therein].

Mechanisms for creation of stripes

- Drift-resonant mechanism to accelerate (& decelerate) particles in some energy interval.

  *However, it has several and notable inconsistencies, e.g.
  —Zebra stripes can form during quiet geomagnetic times when ULF waves are too weak to be efficient.
  —Energy of stripes (peaks or valleys) scale as 1/L, whereas drift-resonance would preserve the first adiabatic invariant and would lead to $1/L^3$ dependence.
  —Difficult for drift resonance to operate for particles with a broad range of drift frequencies (e.g., electrons spanning a wide range of energies).

- Still some wave-particle drift interactions (not necessarily resonant), but due to electric fluctuations induced from diurnal (daily) variations of the outer magnetosphere (kinematic model of Lejosne and Roederer, JGR, 121, 2016).

Mechanisms assume temporally or spatially varying electric fields that have characteristic timescales comparable to the drift period of a given population.

Drift-kinetics [Hazeltine, PoP, 15, 1973]

- In a magnetized plasma, charge particle's motion can be split into a fast gyration around the local magnetic field and the motion of its guiding centre.

- For a strong background magnetic field, the small parameter ε can be defined as:

$$\varepsilon = \frac{\rho}{l} = \frac{mv}{qBl} \ll 1, \quad \frac{\omega}{\Omega} = \frac{m\omega}{qB} \sim \varepsilon \ll 1.$$

- In this limit, for electric and magnetic fields that have long spatial and slow temporal variations, one can build a kinetic theory of guiding-centres.

$$\frac{\partial}{\partial t}(B\langle f\rangle) + \nabla\cdot(B\dot{\mathbf{r}}\langle f\rangle) + \frac{\partial}{\partial v_{\|}}(B\dot{v}_{\|}\langle f\rangle) + \frac{\partial}{\partial \mu}(B\dot{\mu}\langle f\rangle) = 0,$$

$$\dot{\mathbf{r}} = \left(v_{\|} + \frac{\mu}{q_s}\mathbf{b}\cdot\nabla\times\mathbf{b}\right)\mathbf{b} - \frac{\mathbf{E}\times\mathbf{b}}{B} + \frac{v_{\|}^2}{\Omega_s}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{q_sB}\mathbf{b}\times\nabla B,$$

Many compelling reasons to address the problem kinetically, e.g., be assured of conservation of phase-density (Liouville's theorem).

[Osmane et al., in prep. for ApJ Suppl.]

Drift-kinetics theory [Hazeltine, PoP, 15, 1973]

- We can split the distribution in terms of a slowly evolving background and a perturbation that has fast temporal variations: $\Omega_d = 3\mu/q\gamma r^2$

$$f(r,\varphi,t) = f_0(r) + \sum_m \delta f_m(r,0) e^{-im\Omega_d t} e^{im\varphi}.$$

- We can then solve the kinetic problem for equatorial particles trapped in a magnetic dipole in the absence of electric and magnetic fluctuating fields:

$$\frac{\partial}{\partial t}(B\langle f\rangle) + \nabla\cdot(B\dot{\mathbf{r}}\langle f\rangle) + \frac{\partial}{\partial v_\parallel}(B\dot{v}_\parallel\langle f\rangle) + \frac{\partial}{\partial \mu}(B\dot{\mu}\langle f\rangle) = 0,$$

$$\dot{\mathbf{r}} = \left(v_\parallel + \frac{\mu}{q_s}\mathbf{b}\cdot\nabla\times\mathbf{b}\right)\mathbf{b} - \frac{\mathbf{E}\times\mathbf{b}}{B} + \frac{v_\parallel^2}{\Omega_s}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{q_s B}\mathbf{b}\times\nabla B,$$

Resulting equation can be solved exactly in absence of electric field fluctuations !

Drift-kinetics theory [Hazeltine, PoP, 15, 1973]

- We can split the distribution in terms of a slowly evolving background and a perturbation that has fast temporal variations: $\Omega_d = 3\mu/q\gamma r^2$

$$f(r,\varphi,t) = f_0(r) + \sum_m \delta f_m(r,0) e^{-im\Omega_d t} e^{im\varphi}.$$

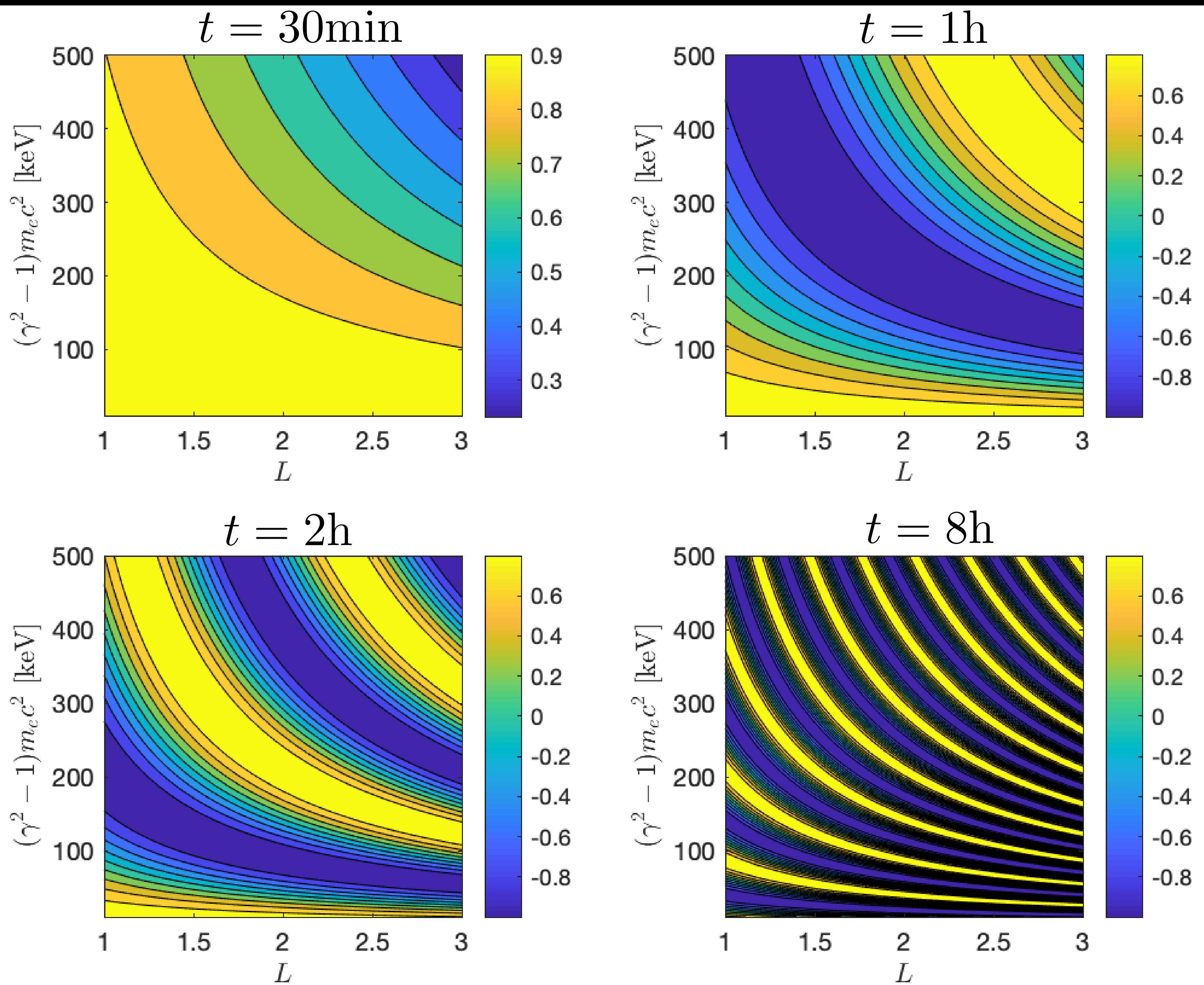
- We can then solve the kinetic problem for equatorial particles trapped in a magnetic dipole in the absence of electric and magnetic fluctuating fields:

$$\frac{\partial}{\partial t}(B\langle f\rangle) + \nabla\cdot(B\dot{\mathbf{r}}\langle f\rangle) + \frac{\partial}{\partial v_\parallel}(B\dot{v}_\parallel\langle f\rangle) + \frac{\partial}{\partial \mu}(B\dot{\mu}\langle f\rangle) = 0,$$

$$\dot{\mathbf{r}} = \left(v_\parallel + \frac{\mu}{q_s}\mathbf{b}\cdot\nabla\times\mathbf{b}\right)\mathbf{b} - \frac{\mathbf{E}\times\mathbf{b}}{B} + \frac{v_\parallel^2}{\Omega_s}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{q_s B}\mathbf{b}\times\nabla B,$$

Resulting equation can be solved exactly in absence of electric field fluctuations !

Analytical solution for phase-mixed distributions (without electric or magnetic fluctuations)



Our solutions shows many feature consistent with observations:
===> Stripes formation with energy peaks having a 1/L dependence.
===> Rapid formation on timescales comparable to drift period & progressive thinning of the stripes << 20h.

Conclusion

- Zebra stripes can arise due to linear phase-mixing when injected particles are brought at lower drift shells in the absence of electric field fluctuations.
  ===>They are equivalent to the Case-Van Kampen modes in electrostatic plasmas.

- Our solutions shows many feature consistent with observation, e.g.,
  ===> Stripes with energy peaks having 1/L dependence
  ===> Progressive thinning of the stripes with time and fast generation.

- The inclusion of electric or/and magnetic fluctuations does not prevent the formation of zebra stripes
  ===> models that incorporate electric field fluctuations will show zebra stripes.

$$\delta f_m(r,t) = \delta f_m(r,0)e^{-im\Omega_d t} - e^{-im\Omega_d t}\frac{\partial f_0(r)}{\partial r}\int_{-\infty}^{t} dt'\ e^{+im\Omega_d t'}\left(\text{Function of electric and/ or magnetic field fluctuations}\right)$$

[Osmane et al., in prep. for ApJ Suppl.]