



A novel trans-dimensional inversion algorithm to model deformation sources with unconstrained shape in finite element domains

Erica De Paolo^{1,2}, Nicola Piana Agostinetti^{3,4}, Elisa Trasatti¹

¹ *Istituto Nazionale di Geofisica e Vulcanologia, Osservatorio Nazionale Terremoti, Rome, Italy*

² *Department of Physics and Astronomy, University of Bologna, Bologna, Italy*

³ *Department of Earth and Environmental Sciences, University of Milano-Bicocca, Milan, Italy*

⁴ *Geophysics Section, Dublin Institute for Advanced Studies, Dublin, Ireland*



Design of a composite source method using FEM

- Deformation source modeling approaches typically rely on **a-priori shape assumptions**
- Attempts in literature simulating cavities as **aggregates** do not always fulfill continuum mechanics principles and are limited to elastic half-space, or use expensive re-meshing
- The stress field of a **uniform distribution of couples of forces** is equivalent to that caused by uniform normal stresses applied to the internal surface of a pressurized cavity (**Fig.1**)
- This concept is implemented in a **FEM mesh**, applying a **stress tensor** to the faces of cubic solid elements, creating an **elementary forward model** suitable to be assembled in a source of potentially any shape (**Fig.2**)
- We obtain a geometry-free source model, in a full FE space, allowing for **pre-computed solutions** on surface to be uniformly scaled with 6 scaling factors.

Fig. 1

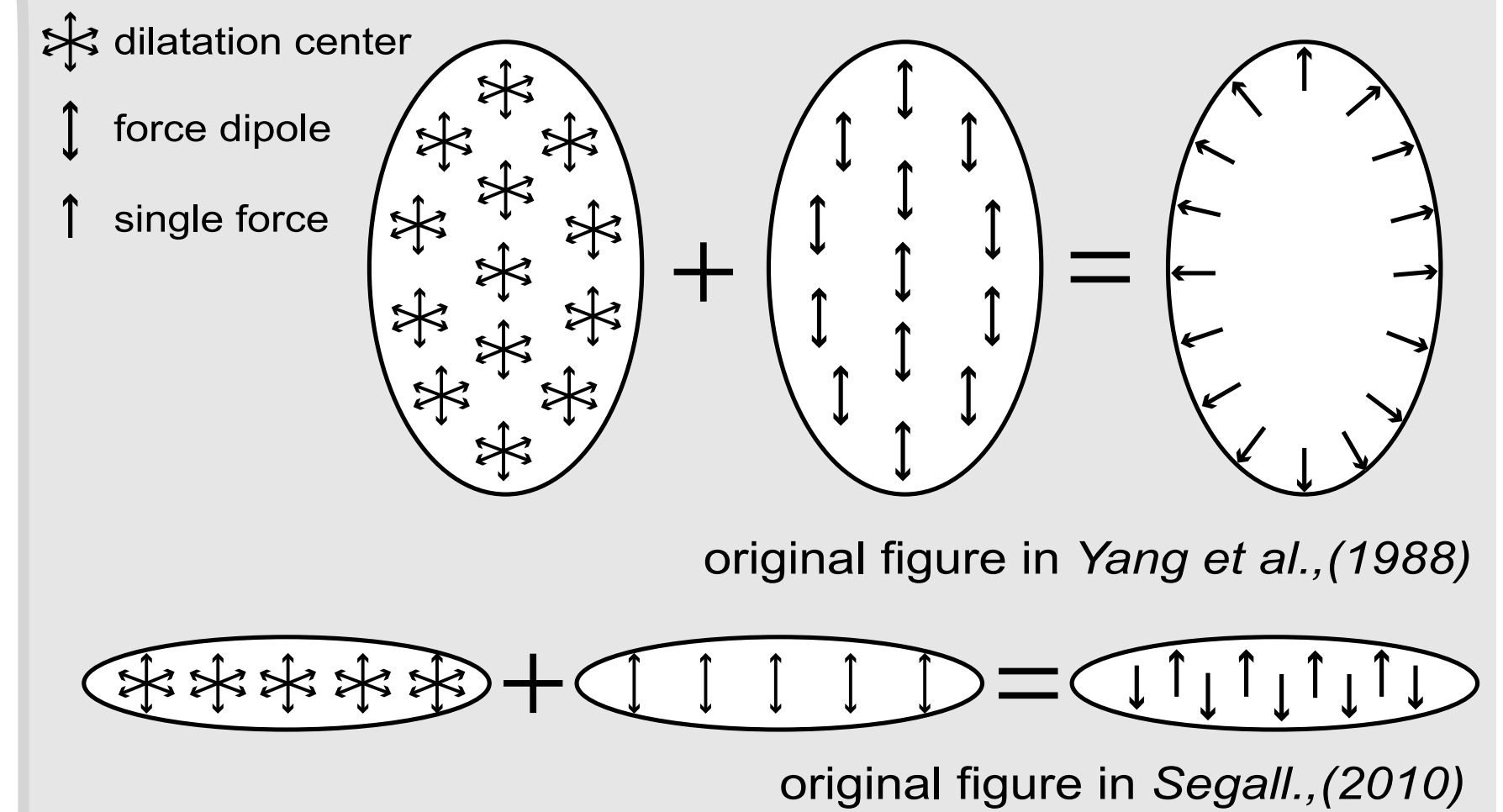
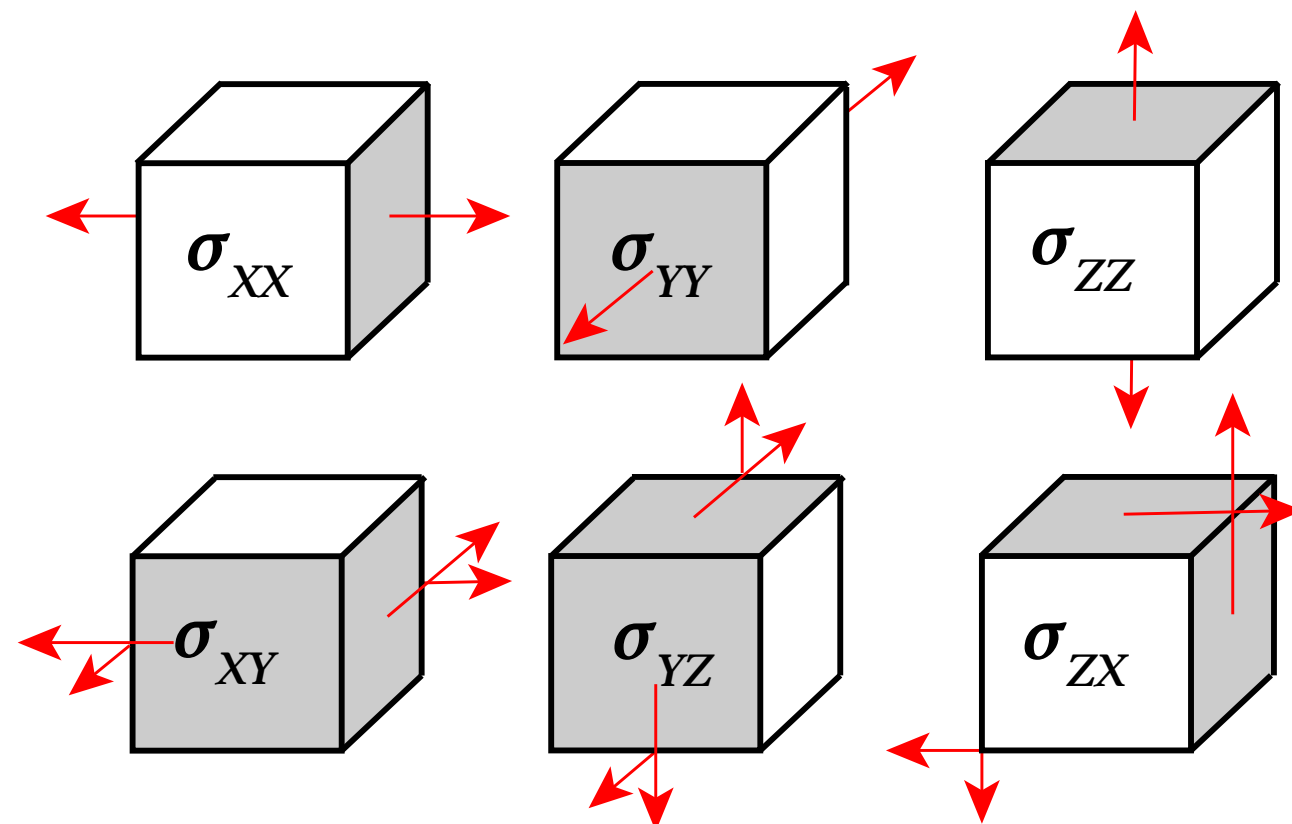


Fig. 2 The elementary unit mechanism



Original figure in Trasatti et al. (2008)

$\sigma_{XX}, \sigma_{YY}, \sigma_{ZZ}, \sigma_{XY}, \sigma_{YZ}, \sigma_{ZX}$ Unitary forces of 1 MPa are applied to the faces of each element

$u_{XX}, u_{YY}, u_{ZZ}, u_{XY}, u_{YZ}, u_{ZX}$ Computation of corresponding displacements on surface

$U_{XX}, U_{YY}, U_{ZZ}, U_{XY}, U_{YZ}, U_{ZX}$ Sum of contributions from each element in the assembly

$s_{XX}, s_{YY}, s_{ZZ}, s_{XY}, s_{YZ}, s_{ZX}$ Estimation of scale parameters to be uniformly applied to U_{ij}

$$U_{tot}(x,y,z) = s_{XX}U_{XX} + s_{YY}U_{YY} + s_{ZZ}U_{ZZ} + s_{XY}U_{XY} + s_{YZ}U_{YZ} + s_{ZX}U_{ZX}$$

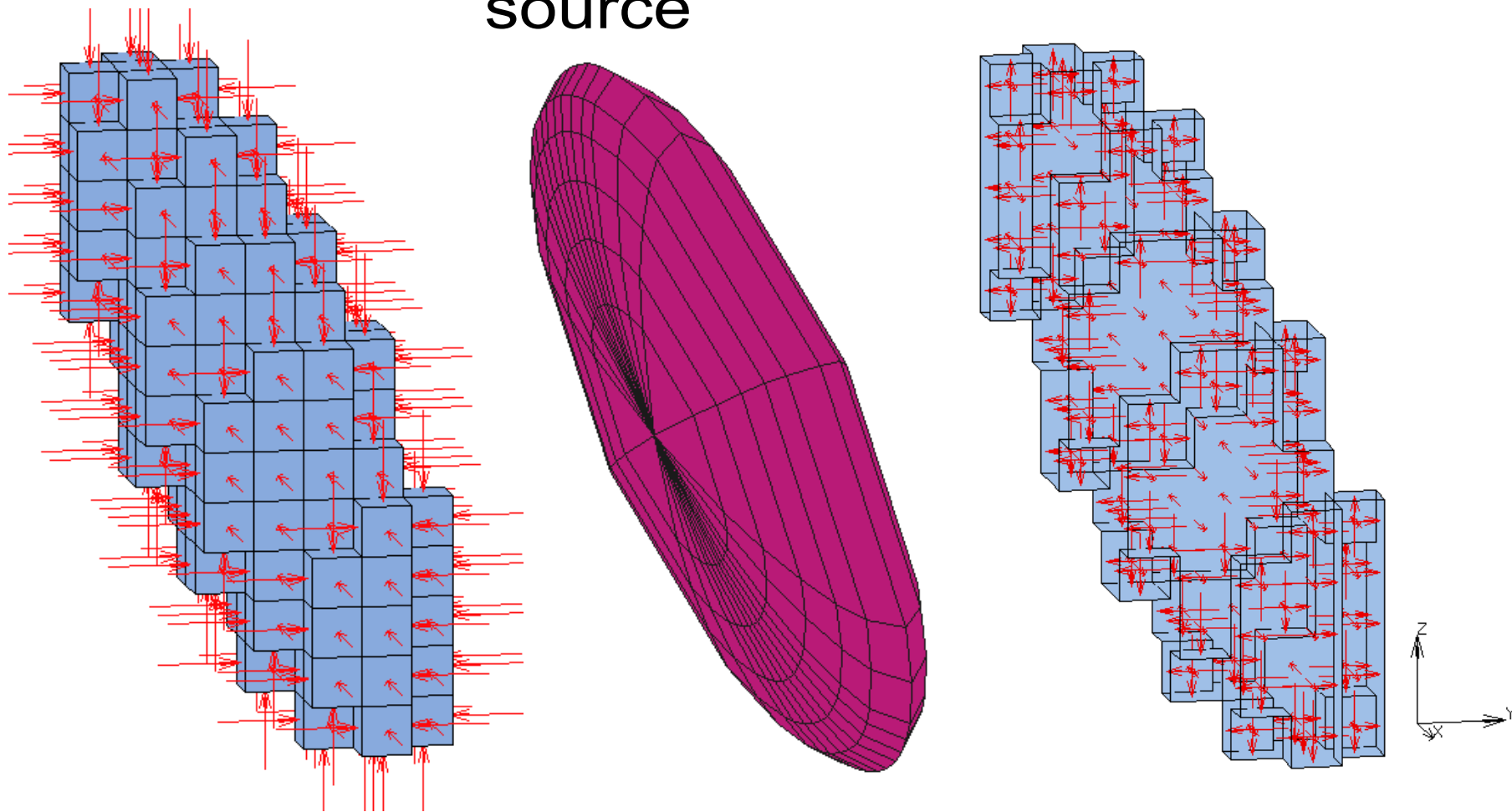
Synthetic testing and validation

Fig. 3

FEM assembly

analytical
source

FEM cavity



- Synthetic tests prove the **equivalence with the deformation fields** produced by corresponding **cavities**, with uniform pressure applied to the boundaries **Fig.3**
- Sources with oblique symmetry axes (dip $\neq 90^\circ$, strike $\neq 0^\circ$) require also the shear components i.e. all 6 scaling factors (**see example below**)

FEM assembly vs analytical source

$a = 2951 \text{ m}$ (volume eq) $b/a = 0.333$
dip = 60° Strike = 45°

$S_{xx} = 2.654$ $S_{yy} = 2.654$ $S_{zz} = 2.155$

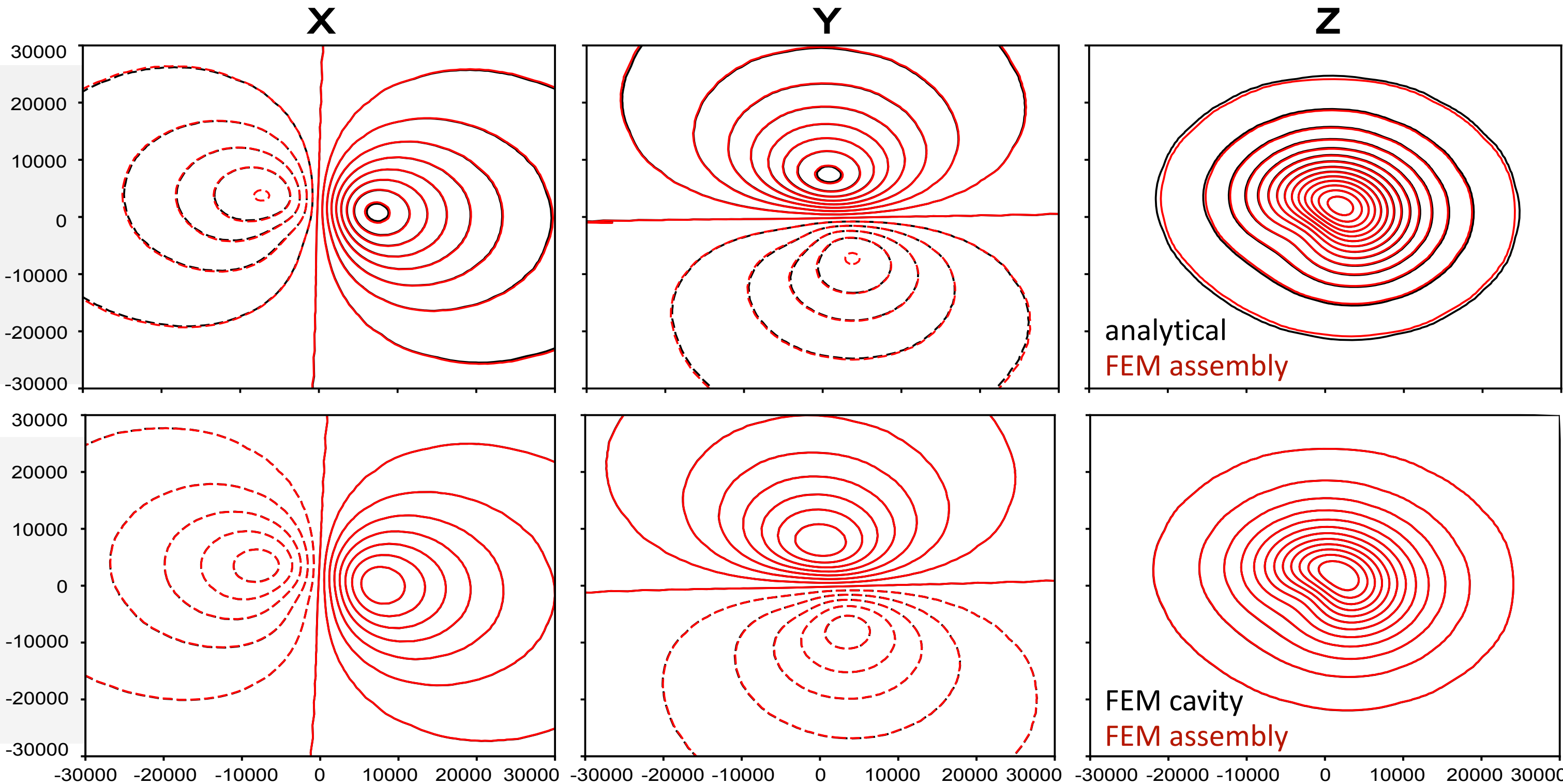
$S_{xy} = -0.095$ $S_{yz} = 0.236$ $S_{zx} = 0.237$

FEM assembly vs FEM cavity*

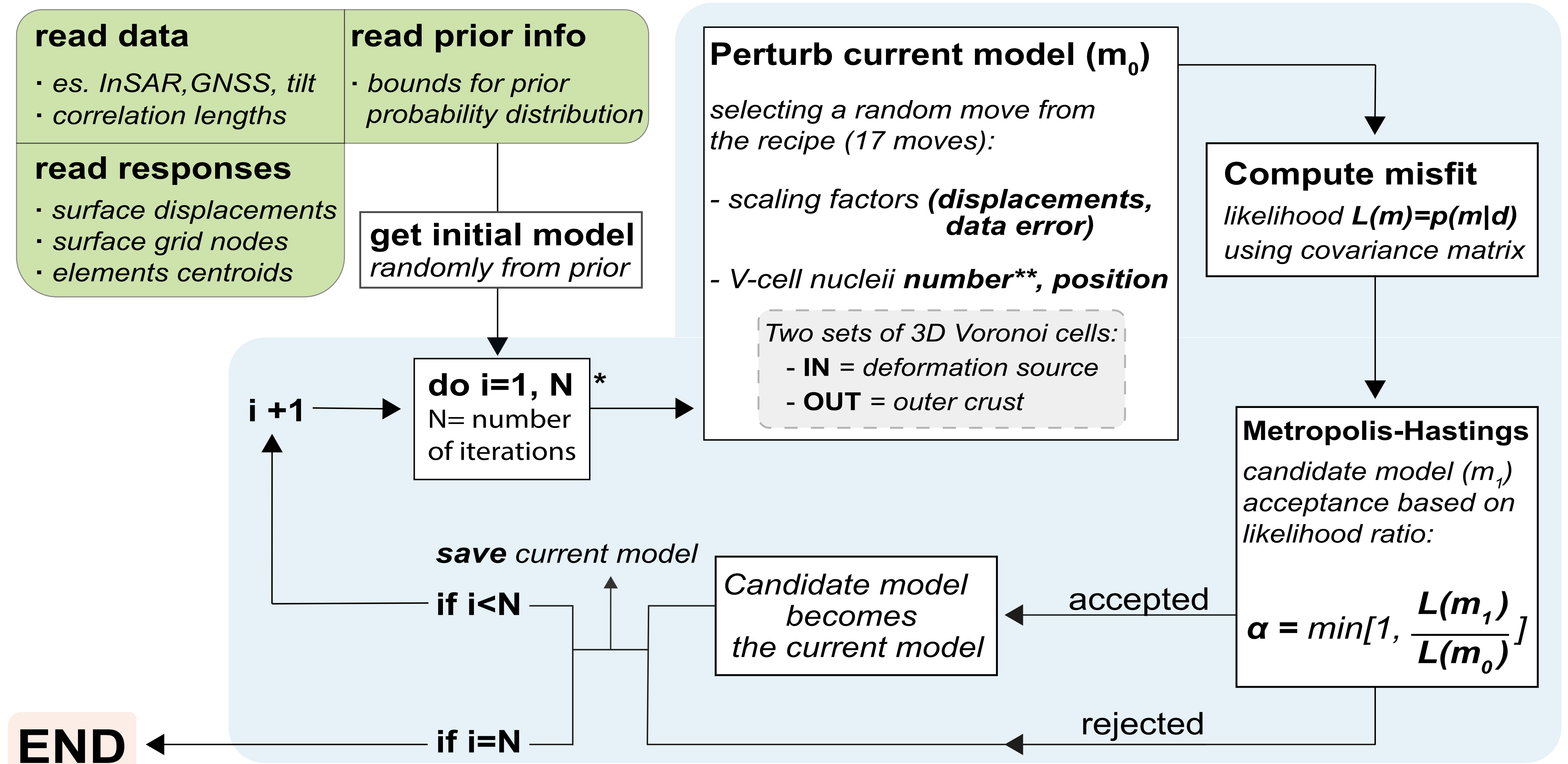
* pressure applied to cavity boundaries

$S_{xx} = 2.755$ $S_{yy} = 2.755$ $S_{zz} = 2.172$

$S_{xy} = -0.153$ $S_{yz} = -0.179$ $S_{zx} = -0.179$



Inversion algorithm flowchart



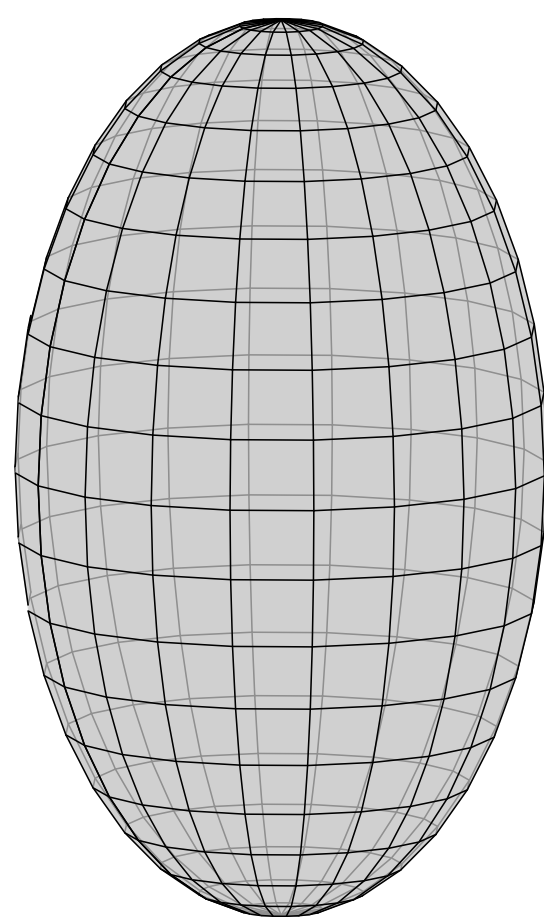
posterior probability density (PPD) of model parameters from saved models

* parallel independent chains computation allowed

** trans-dimensional moves changing the model dimension

Comparison of source configurations

A analytical source (Yang et al. 1988)



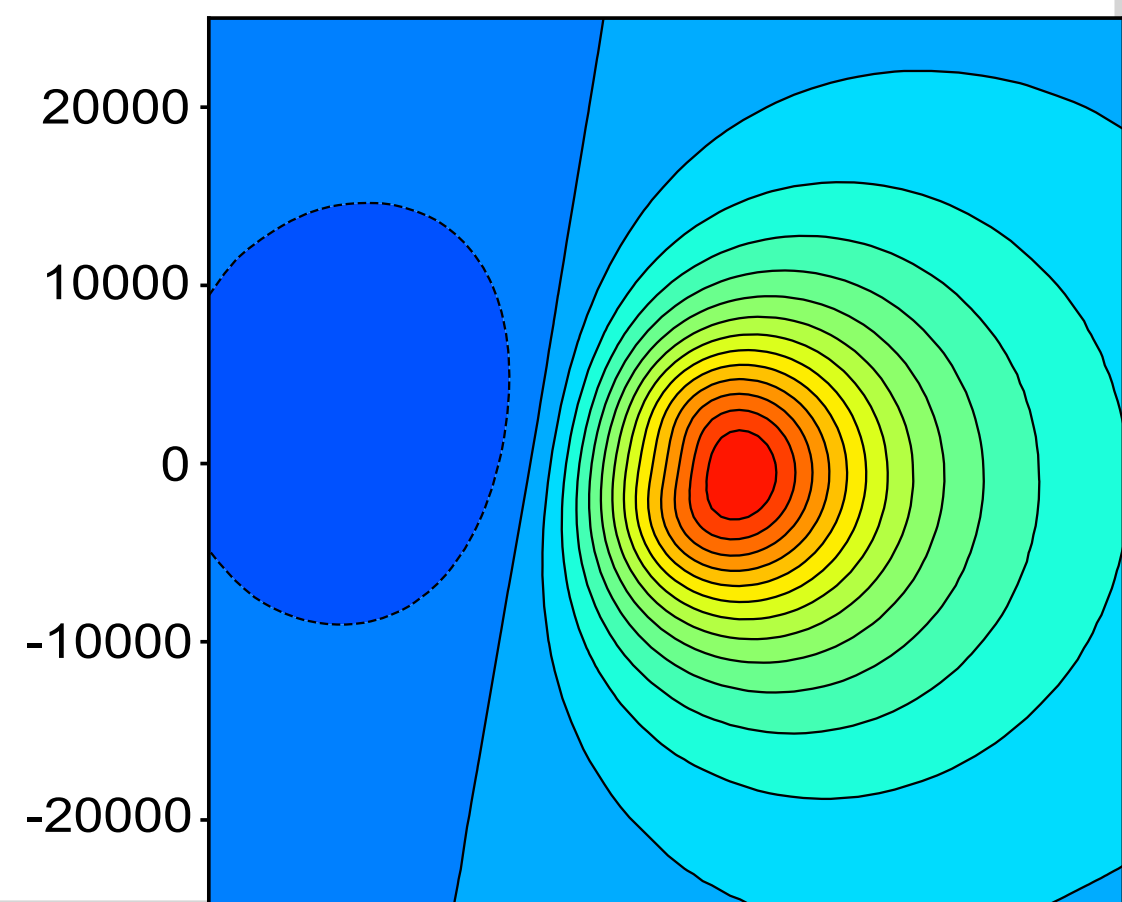
$a = 1500 \text{ m}$

$b/a = 0.5$

dip = 90°

strike = 0°

$\Delta P = 1 \cdot 10^6 \text{ Pa}$



$N^\circ \text{elements} = 30$ (Vol = $3.7 \cdot 10^6 \text{ m}^3$)

$S_{xx} = 2.577$

$S_{yy} = 2.577$

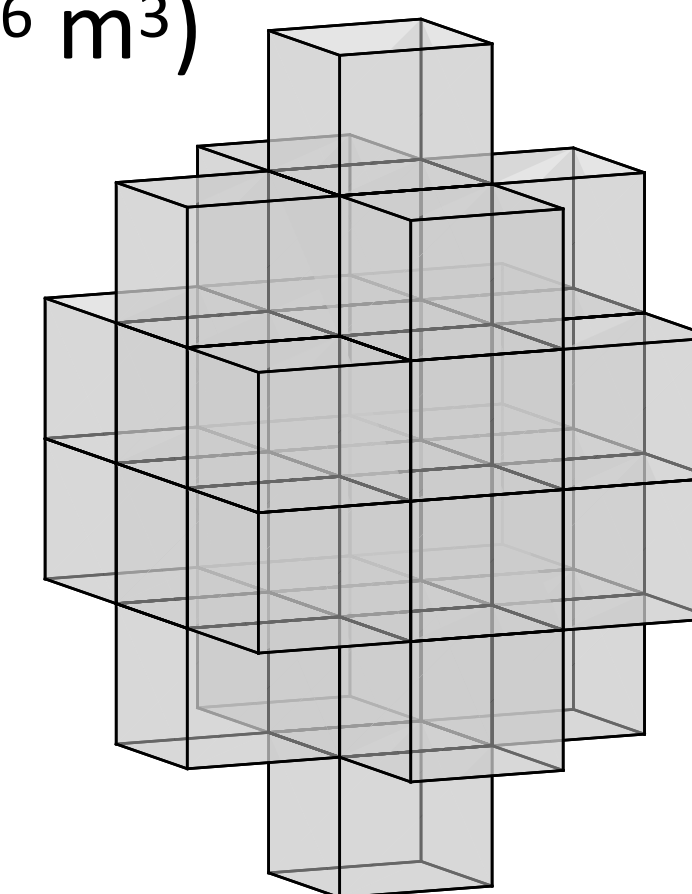
$S_{zz} = 1.994$

$S_{xy} = 0.000$

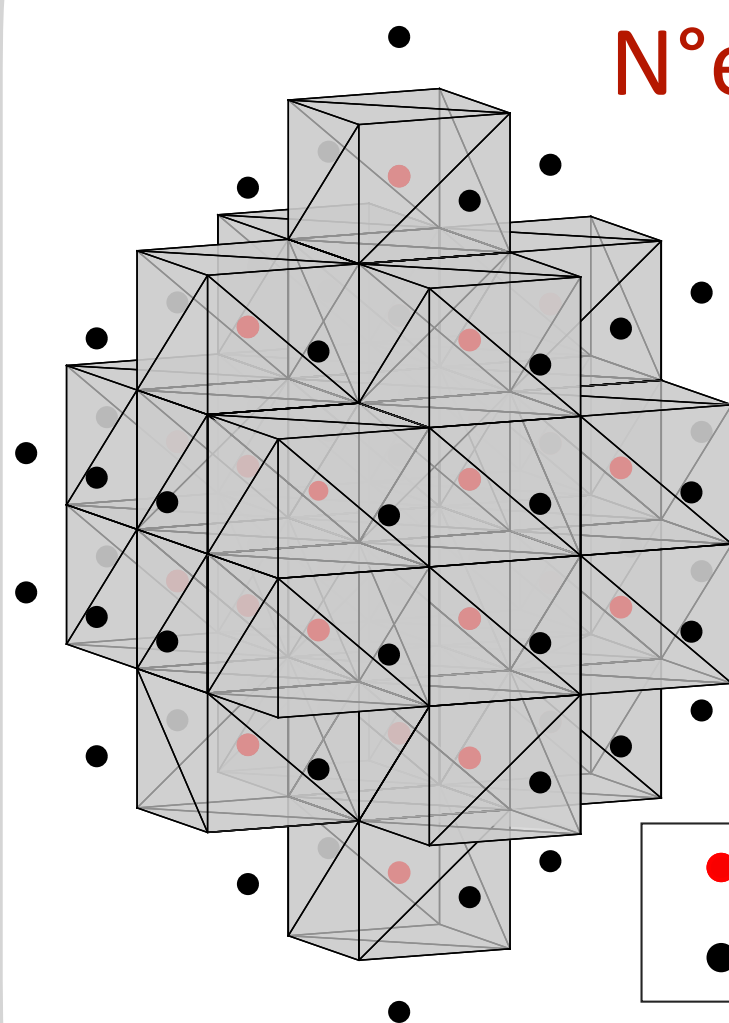
$S_{yz} = 0.000$

$S_{zx} = 0.000$

B FEM assembly



C Voronoi cells synthetic

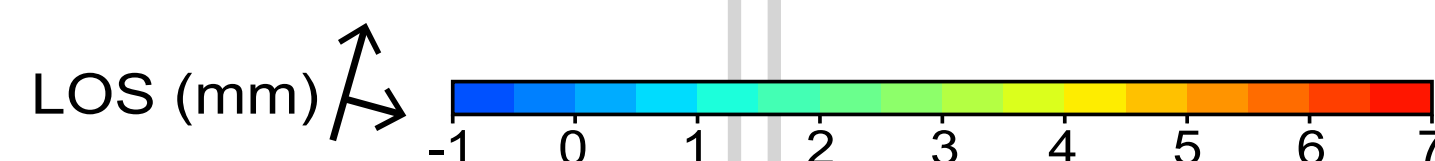
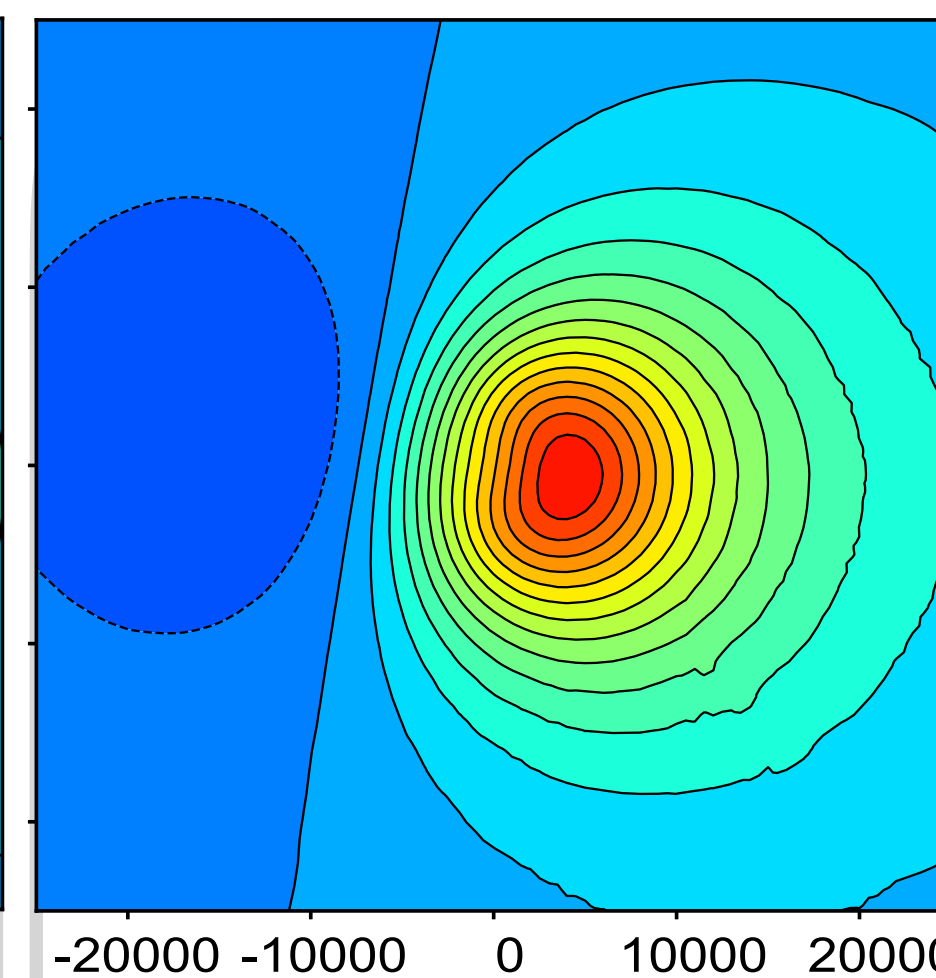
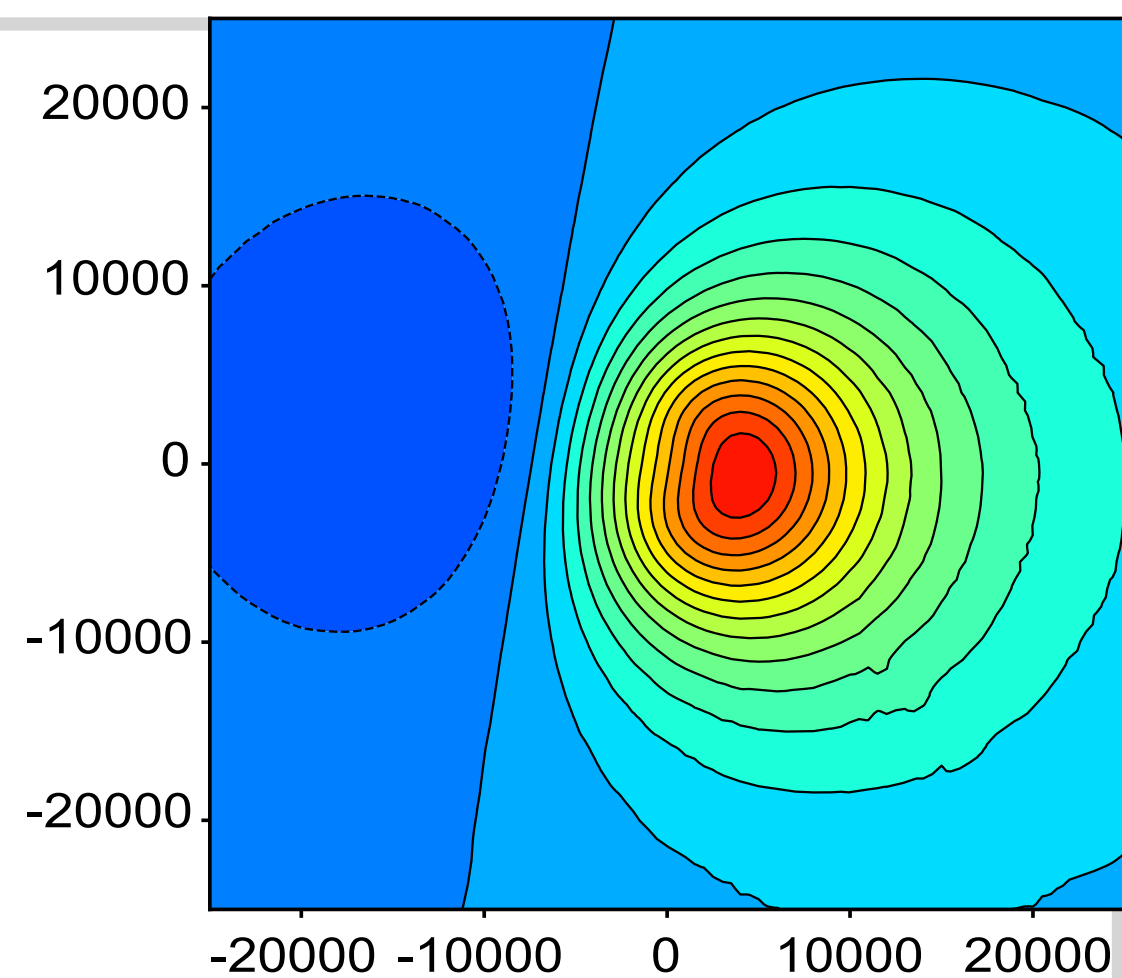


$N^\circ \text{elements} = 30$

IN = 49

OUT = 78

- V-cell nuclei IN
- V-cell nuclei OUT



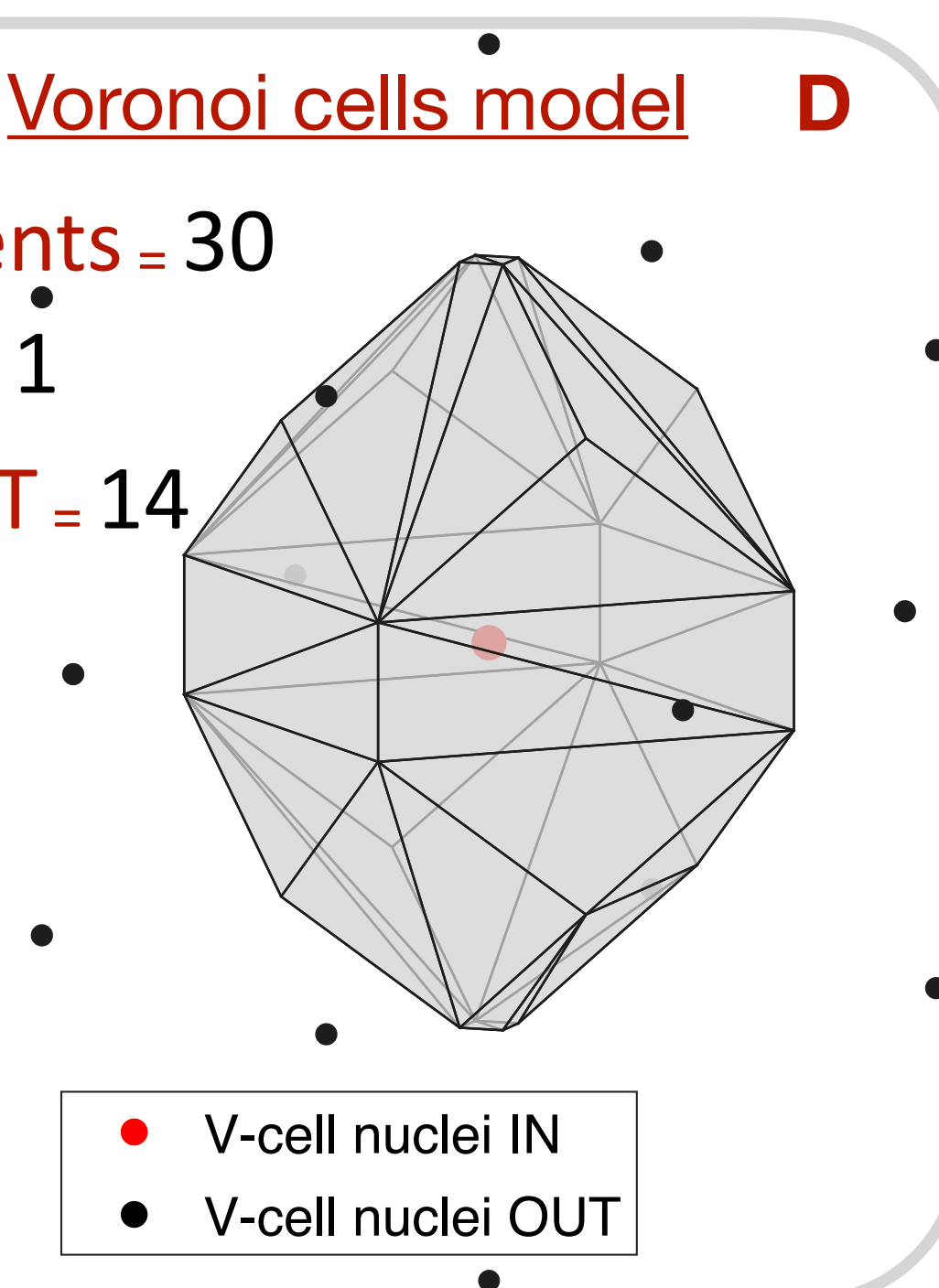
the algorithm tends to a parsimonious solution

D Voronoi cells model

$N^\circ \text{elements} = 30$

Cells IN = 1

Cells OUT = 14



- V-cell nuclei IN
- V-cell nuclei OUT

Conclusions and final remarks

- Modeling **free of a-priori geometry assumptions**, source shape defined by the data
- Advantages of **FEM domain potential complexities** (topography, 3D fully heterogeneous)
- Based on **pre-computed** set of solutions to be scaled, improving **cost-efficiency**
- Supported by a sophisticated bayesian trans-dimensional inversion algorithm
- **Rigorous** in terms of continuum mechanics
- **Work in progress:** Application to Long Valley Caldera, CA
- Promising approach in the view of more realistic source representations (irregular shapes)

Essential References:

Segall, P. (2010). Earthquake and volcano deformation. In Earthquake and Volcano Deformation. Princeton University Press.

Trasatti, E., Giunchi, C., & Agostinetti, N. P. (2008). Numerical inversion of deformation caused by pressure sources: application to Mount Etna (Italy). Geophysical Journal International, 172(2), 873-884.

Yang, X. M., Davis, P. M., & Dieterich, J. H. (1988). Deformation from inflation of a dipping finite prolate spheroid in an elastic half-space as a model for volcanic stressing. Journal of Geophysical Research: Solid Earth, 93(B5), 4249-4257.