# Weakly Constrained LETKF for Convective-Scale Data Assimilation

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# Our study

- Data assimilation (DA) on convective scales should update hydrometeors
- Assimilation of observations such as radar reflectivity or cloud products are important for prediction on convective scales
- Janjic et al. 2014 show that is important to preserve both positivity and mass with DA algorithm when estimating variables that should be non-negative
- Operationally used DA algorithms when updating hydrometeors clip negative values to zeros, modifying mass
- Here, we propose a fast, easy to implement modification of LETKF that is able to weakly preserve both properties of mass conservation for each hydrometeor variable and non-negativity.

# Weakly Constrained LETKF

$$\begin{aligned}\min_{x} \quad J(\mathbf{x}) &= J_b(\mathbf{x}) + J_o(\mathbf{x}) + J_m(\mathbf{x}) \\ &= \frac{1}{2}(\overline{\mathbf{x}}_k^b - \mathbf{x})\mathbf{P}_k^{b^{-1}}(\overline{\mathbf{x}}_k^b - \mathbf{x})^T + \frac{1}{2}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]\mathbf{R}_k^{-1}\left[\mathbf{y}_k^o - \mathbf{H}(\mathbf{x})\right]^T \\ &+ \frac{1}{2}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}(\mathbf{x})\right]^T\end{aligned}$$

- **m** is a vector quantity, whose elements are the domainwise (global) integral of hydrometeors.
- **S** is operator which calculates the domainwise (global) integral for each of the microphysical spieces,

Constraint on mass is up to accuracy $\mathbf{M}_k$

$$\mathbf{M}_k = \frac{1}{N_{ens} - 1}\sum_{i=1}^{N_{ens}}\left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]\left[\mathbf{m}_k^* - \mathbf{S}\left(\mathbf{x}_k^{b(i)}\right)\right]^T$$

# Weakly Constrained LETKF

For mass:

$$\overline{\mathbf{x}}_k^{a,M} = \quad = \overline{\mathbf{x}}_k^b + \mathbf{P}_k^{a,M}\mathbf{H}^T\mathbf{R}_k^{-1}\left(\mathbf{y}_k^o - \overline{\mathbf{y}}_k^b\right) + \frac{1}{N_{ens}-1}\mathbf{X}_k^{a,M}\left(\mathbf{S}\mathbf{X}_k^{a,M}\right)^T\mathbf{M}_k^{-1}\left[\mathbf{m}_k - \mathbf{S}\left(\overline{\mathbf{x}}_k^b\right)\right]^T.$$

$$\mathbf{W}_k^{a,M} = \left[(N_{ens}-1)\mathbf{I} + \left(\mathbf{H}\mathbf{X}_k^b\right)^T\mathbf{R}_k^{-1}\mathbf{H}\mathbf{X}_k^b + \left(\mathbf{S}\mathbf{X}_k^b\right)^T\mathbf{M}_k^{-1}\mathbf{S}\mathbf{X}_k^b\right]^{-1},$$

# Weakly Constrained LETKF

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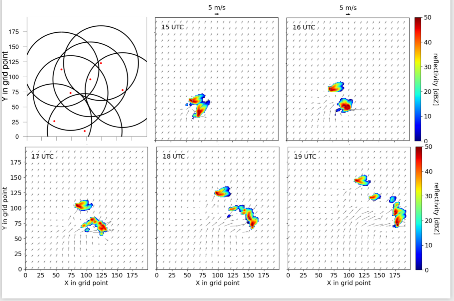
For positivity:

- To avoid spurious convection (Aksoy et al., 2009) and overestimated analysis increments (Zeng et al., 2021), a non-negative threshold value is set for very small reflectivities. We call those data clear-air reflectivity data
- Radar reflectivity data depend nonlinearly on hydrometeors. Further, reflectivity data (including clear-air reflectivity data) are available at radar observation locations, therefore not in every grid point of the model
- By assimilating additional clear-air reflectivity data, we are asking in the approximate weak sense that non-negativity is preserved in the analysis of hydrometeors

# Experimental setup

Idealized setup for radar DA



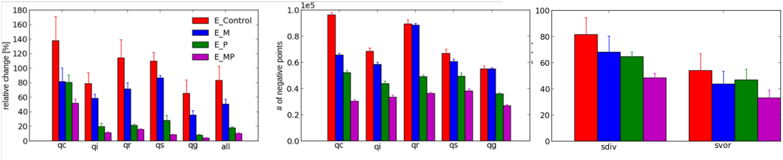
Zeng et al. 2021: Assimilating radar radial wind and reflectivity data in an idealized setup of the COSMO-KENDA system, Atmospheric Research, **249**, 105282, https://doi.org/10.1016/j.atmosres.2020.105282.

- COSMO model with a 2-km horizontal resolution
- Efficient Modular VOlume scanning RADar Operator (EMVORADO, Zeng et al., 2014, 2016)
- Both radial wind and reflectivity data are assimilated
- Ensemble size is 80
- Observations perturbed with Gaussian noise with a standard deviation of 5.0 dBZ and 1.0 m/s

# Results: Impact of the different constraints

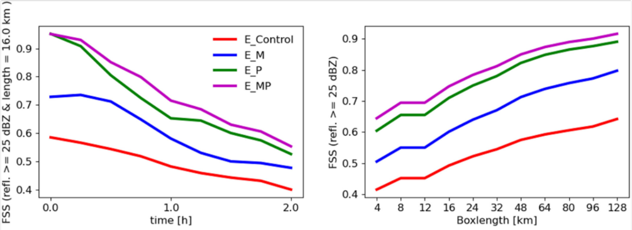
1. $E_{Control}$ Radar reflectivity and wind assimilated
2. $E_M$ In addition mass constraint
3. $E_P$ In addition positivity constraint (clear-air reflectivity data )
4. $E_{MP}$ Both constraint

If clear-air reflectivity data are assimilated, a threshold value of 5 dBZ is set, that is, all reflectivity values smaller than 5 dBZ are set to 5 dBZ. If clear-air reflectivity data are not assimilated, all reflectivity values smaller than 5 dBZ are set to missing values.



Janjic, T. and Y. Zeng, 2021, Weakly constrained LETKF for estimation of hydrometeor variables in convective-scale data assimilation, Geophysical Research Letters, **48**, e2021GL094962, https://doi.org/10.1029/2021GL094962.

# Conclusion



- Assimilation of clear-air reflectivity reduces the number of grid points with negative values of hydrometeor variables in pre-analyses and mitigates the biased increase and uncertainties in the total mass.
- Weak constraints on mass conservation is effective (especially in reducing mass bias) but not as effective as assimilating clear-air reflectivity.
- The best results are obtained by a combination of both constraints.
- Method proposed here is well suited for the assimilation of radar data in convection permitting models.
- It is appropriate for estimation of high dimensional state and requires only minor changes to the already existing implementation of LETKF.

## Main References.

**Janjic and Zeng 2021** " Weakly Constrained LETKF for Estimation of Hydrometeor Variables in Convective-Scale Data Assimilation", *Geophysical Research Letters*, **48**, e2021GL094962, https://doi.org/10.1029/2021GL094962

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**Janjic, et al. 2014**. "Conservation of mass and preservation of positivity with ensemble-type Kalman filter algorithms." *Monthly Weather Review* 142.2: 755-773.