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Accurate calculation of pressure forces on cells defined by steeply sloping coordinates

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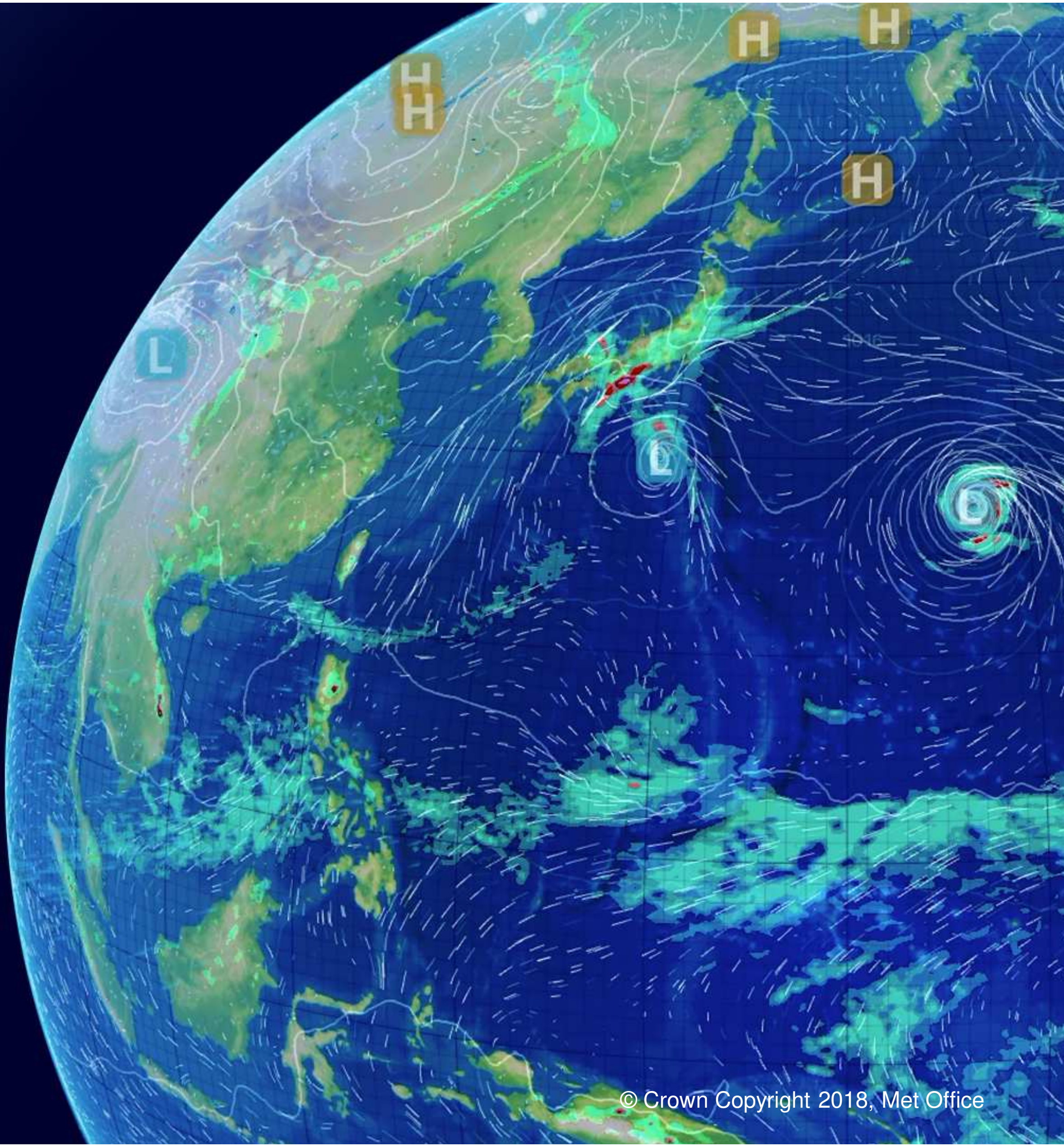

Outline

1. Motivation/context
2. HPG schemes considered
3. Results for isolated sea-mount test case
4. Initial results for Atlantic Margin Model
5. Work in progress
6. Treating tracers as grid cell mean values
7. Summary
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1. Motivation/context

Motivation is desire to move away from stepped bathymetry

- Poor representation of overflows
- Uneven (hence noisy) vertical velocities
- Unclear implications for vortex stretching
- step-like side-walls along the continental slope

Context

- Work on multi-envelope bathymetry (Diego Bruciaferri & James Harle)
- Work on Brinkman penalisation (Laurent Debreu & Gurvan Madec)
- Desire to move to more generalised vertical coordinates

2. HPG schemes considered

This section

- describes the types of schemes we are exploring
- explains their main ideas by illustrating the calculations they involve

2.1 Forces on faces (Lin 1997, Adcroft et al. 2008, Engwirda et al 2017)

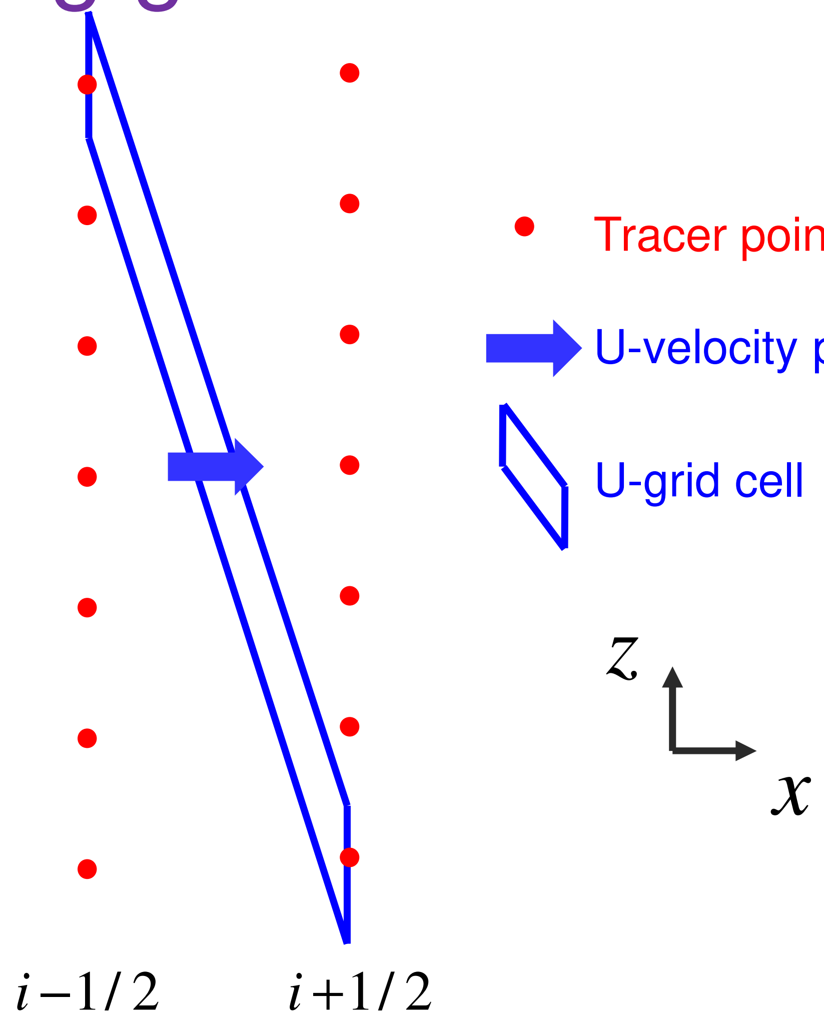
2.2 Density Jacobian with constrained cubic splines (djc) (Shchepetkin & McWilliams 2003; djc)

2.3 Interpolation to a common level using constrained cubic splines (Hedong Li, prj). This scheme is currently used by the Met Office's Atlantic Margin Models (AMM)

2.4 Subtraction of a locally defined “reference” profile (a new scheme)

2. Steeply sloping grid cells

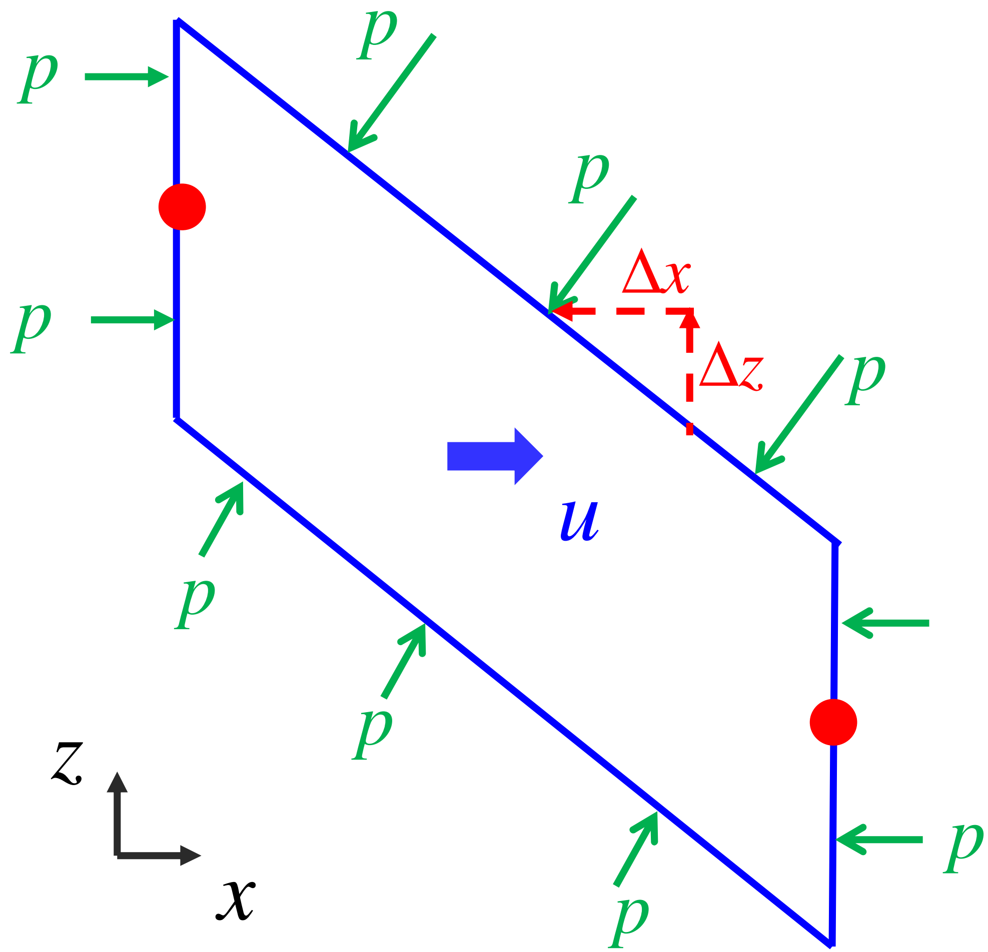
- Over steep bathymetry, terrain-following coordinates give steeply sloping grid-cells



2.1a Forces on faces

- The net horizontal pressure force on these cells can be calculated as the sum of the forces on the faces of the cell (Lin 1997).
- This is a good "conservative" framework.
- The horizontal force on the upper face segment Δx is - $p\ \Delta z$
- So the total force on the cell is

$$F_x = -\oint_C \left(p\frac{\partial z}{\partial s}\right) ds$$

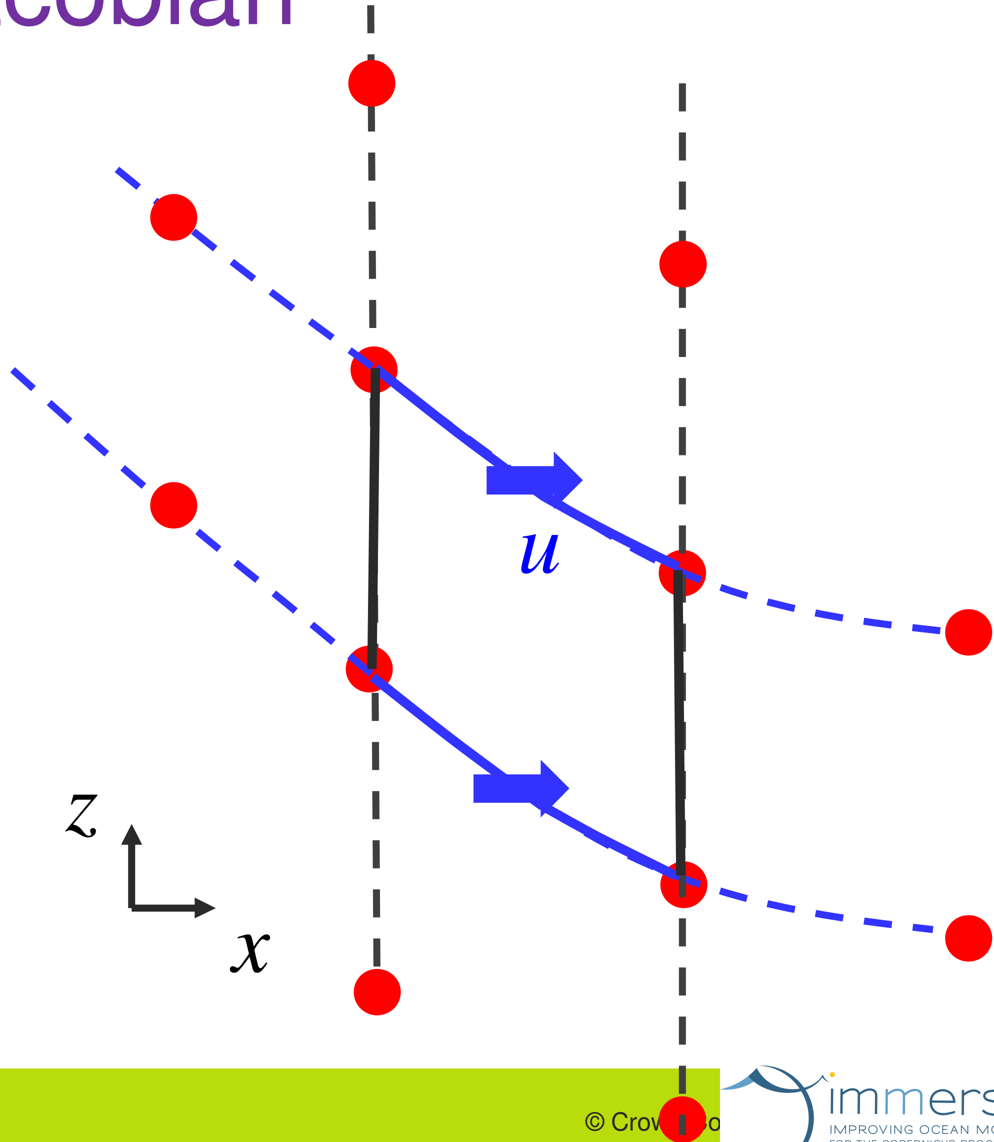


2.2 Density Jacobian

- Shchepetkin & McWilliams (2003) derives a pseudo-flux form of the density Jacobian

$$\mathfrak{I} = -g \oint_C \left(\rho \frac{\partial z}{\partial s}\right) ds \qquad F = \int_z^{z_{\mathrm{srf}}} \mathfrak{I} \, ds$$

- Constrained cubic splines (Kruger 2002) are used to construct the density along each of the faces of the "cell"
- These ensure there are no "overshoots"
- They "require" boundary conditions – problem for hybrid s-z coordinates?
- The reconstruction in the top-half cell is not as accurate as elsewhere



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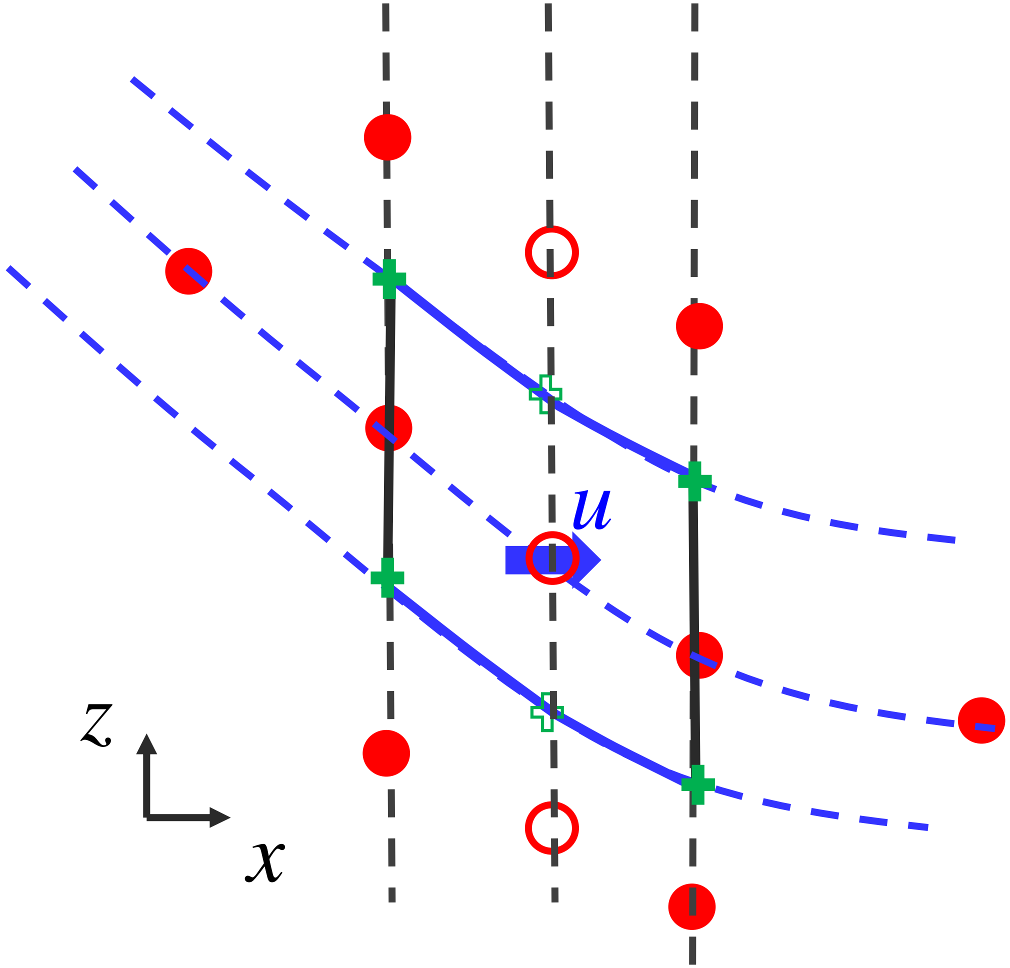
2.1b Forces on faces (higher order)

$$F = -\oint_C \left(p\frac{\partial z}{\partial s}\right) ds$$

- Use cubic polynomials to interpolate density & height (z) along s=const (blue) lines
- Use quadratic polynomials to interpolate density & z in the vertical (black lines)
- Integrate to determine the pressures then the forces on the faces
- Use Simpson's rule on upper & lower faces
- Use off-centred interpolation in vertical near the boundaries (rather than bcs)



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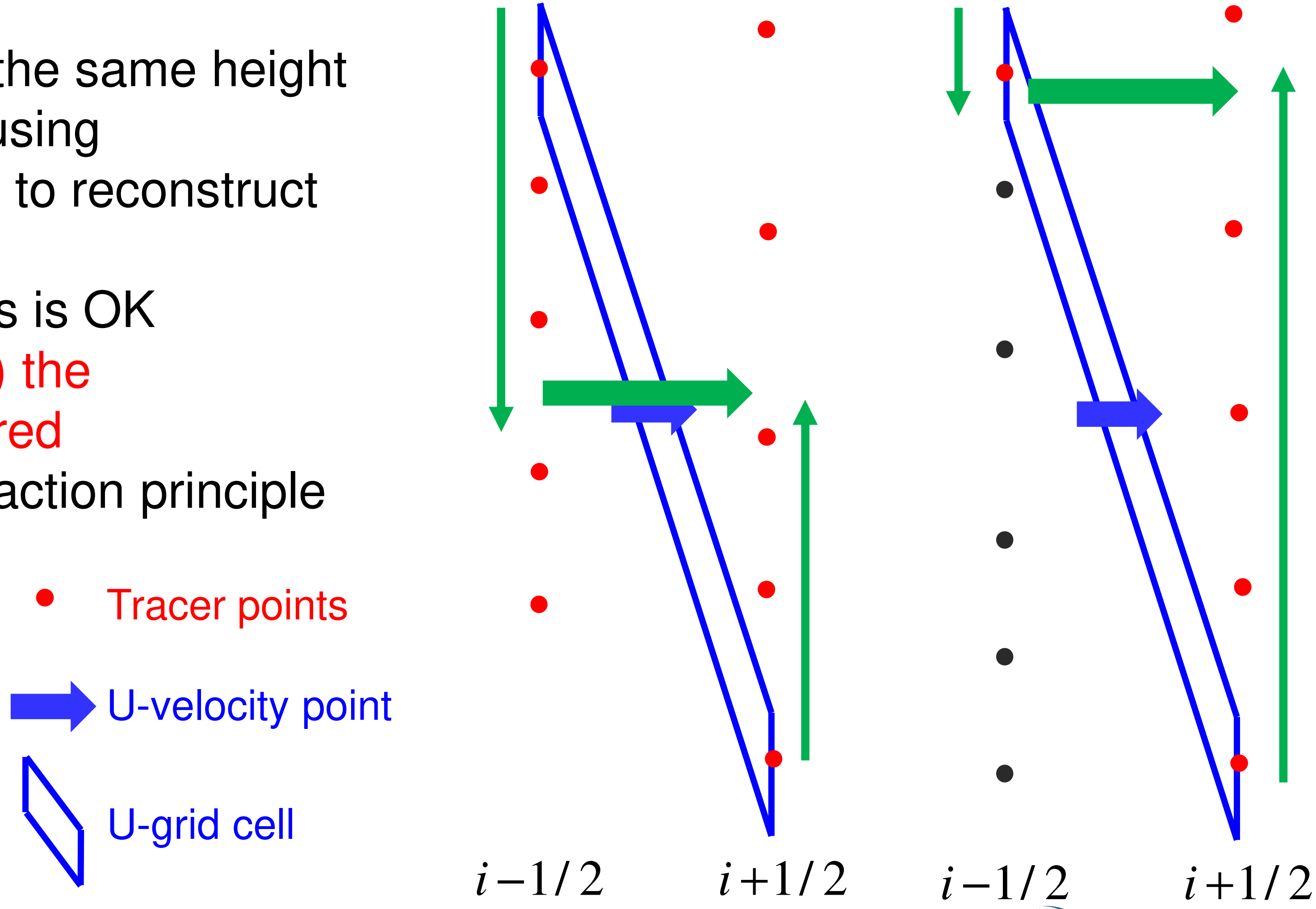
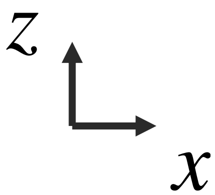
2.3 Pressure “Jacobian”

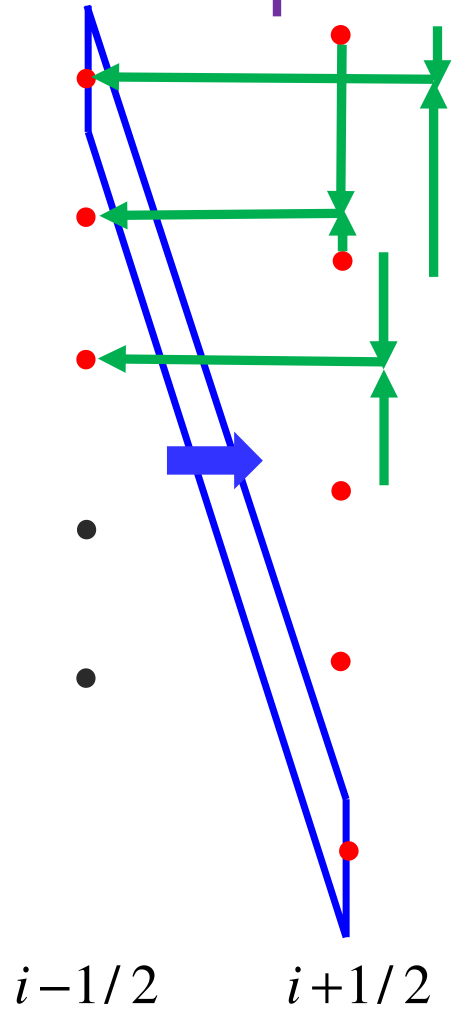
- Calculate pressure at the same height on both sides of the cell using constrained cubic splines to reconstruct the density field
- At mid-depths (left) this is OK
- Near the bottom (right) the interpolation is uncentred
- There is no action=re-action principle



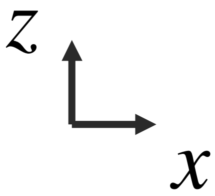


2.4 Subtraction of a reference profile

- Choose the deepest point in the stencil
- Interpolate the density profile at this point vertically to all points of the stencil using preferred form of cubic interpolation
- Subtract off this density profile at all points in the stencil
- Use your preferred form of hpg scheme
- With low-order hpg schemes this avoids difficult interpolations along s-coordinates and at boundaries
- If the original scheme has an action=re-action principle, we can construct one for the new scheme too



- Tracer points
- U-velocity point
- U-grid cell



3. Results for sea-mount test case

This section describes results for a “standard” sea-mount test case

The results are summarised in section 3.4

3.1 Seamount Test case

Idealised Configuration Testing

Classic HPG test case of Beckmann and Haidvogel (1993) (will be available as NEMO test case in vn 4.2)

- Isolated seamount in an E-W periodic channel
- Ocean initialised at rest with exponentially decaying density profile *defined as point-values*
- s-coordinate domain (s coordinates equally spaced)
- 380km x 280km x 4500m domain with
  - Shallow gradient case:1000m seamount
  - Steep gradient case: 4050m seamount
- Full non-linear momentum equations, 2nd order centred tracer advection
- One can calculate the pressures and integrals along each face exactly – this helps with de-bugging! The symmetries help with debugging too

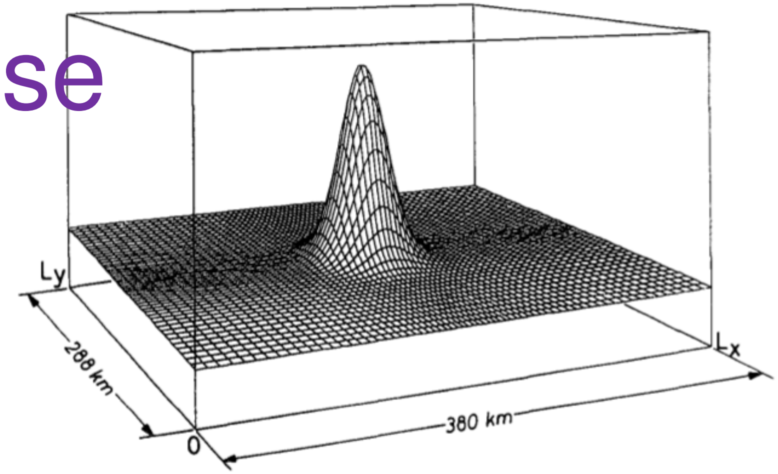


Figure A: Idealised Seamount config. Beckmann and Haidvogel (1993)

Parameter	Value
Δx	$4000m$
n_{levels}	10
$\mathrm{depth}_{\mathrm{max}}$	$4500m$
$\mathrm{H}_{\mathrm{seamount}}$	$4050m$
S=(NH)/(fL)	2
A_M	$2000m^2/s$
$r_{max}=\delta H/(2\overline{H})$	0.21

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3.2 Experiment naming

- sco – standard 2nd order scheme for s-coordinates
- djc – density Jacobian constrained cubic spline (ccs) (SMcW 2003)
- djr – djc with "reference" subtracted using ccs (djr_ccs) or pure cubic (djr_cub)

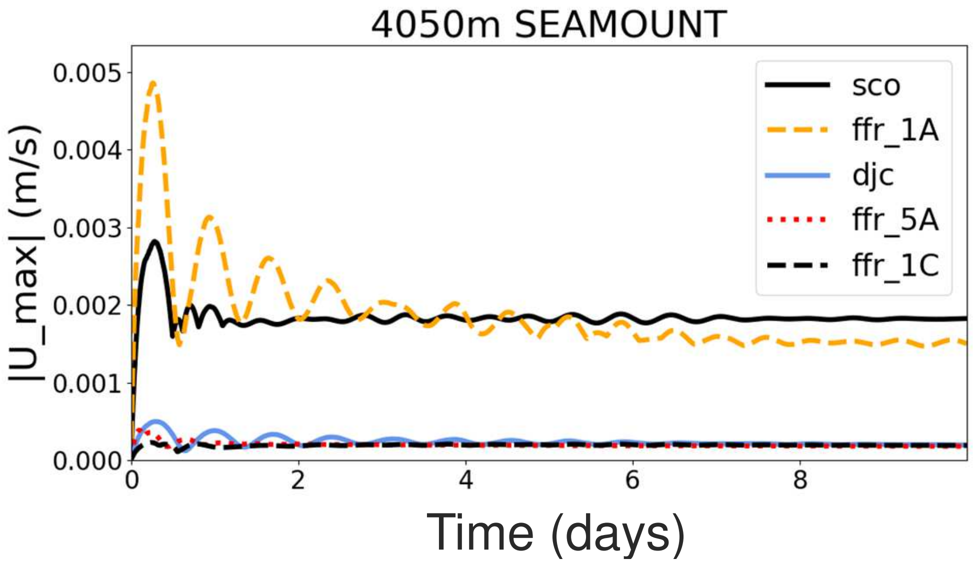
ffr#L – forces on faces

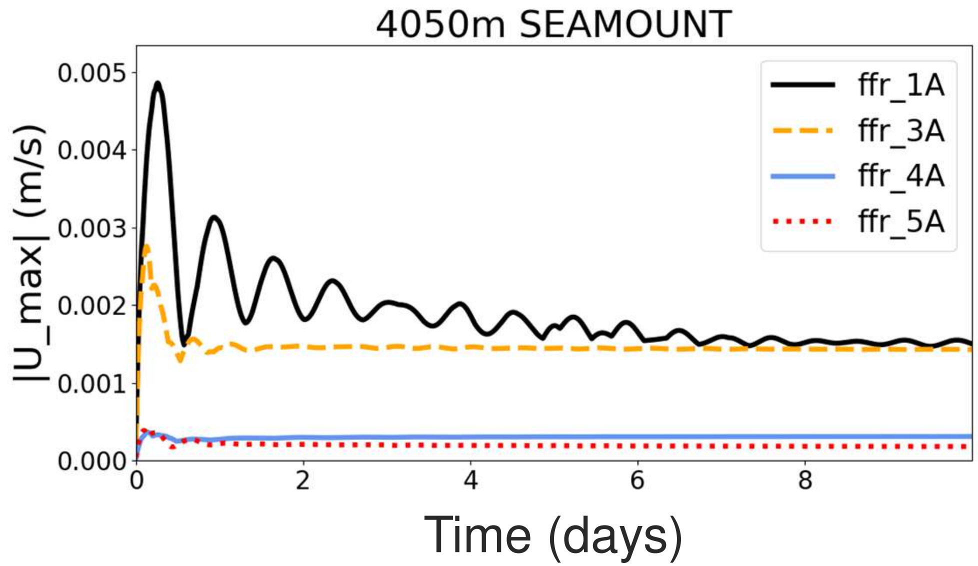
#	Description
1	2nd order (original Lin scheme)
3-5	Density cubic in vertical;
3	2nd order on const s
4	ccs on const s
5	cub on const s

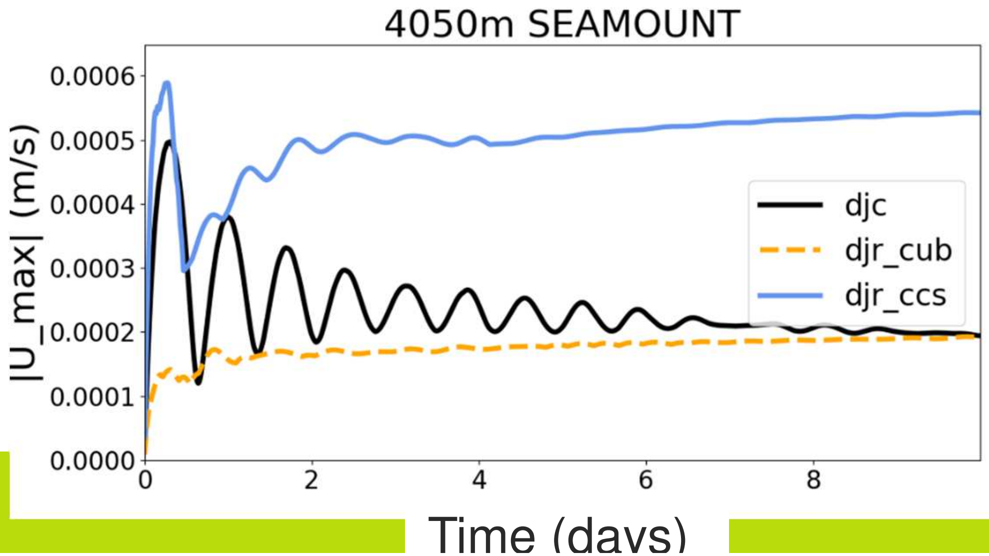
L	Reference subtracted
A	none
B	ccs
C	cub

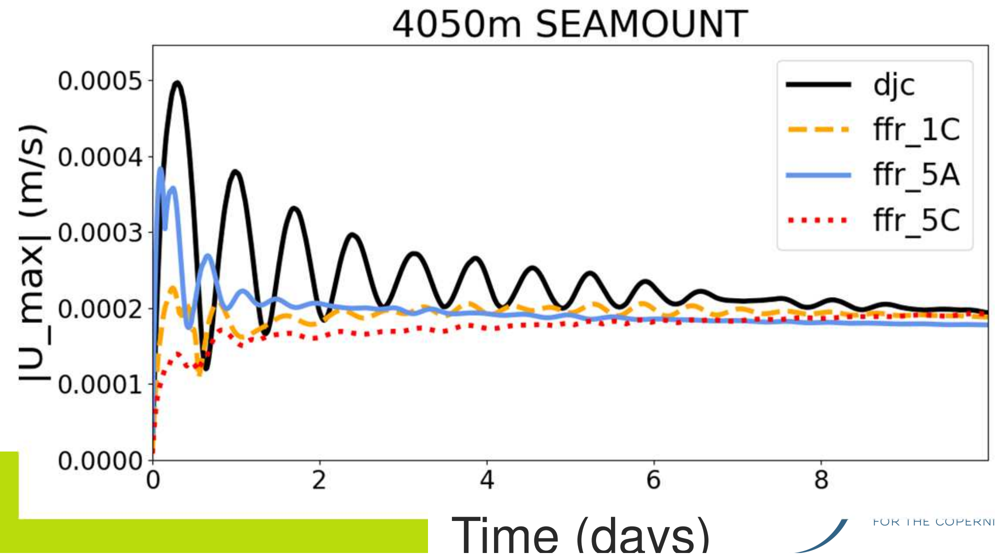
3.3 Results 10-day integrations

Maximum velocity in domain (m/s)









3.4 Summary of results after 10 days

- Pressure Jacobian (prj) scheme does well for shallow slopes but has v>0.15 m/s for cases shown on previous slide
- For other schemes, initial transients die down after @10 days
- sco and original Lin scheme (ffr_1A) give similar long-term 2.10^{-3} m/s error
- Density Jacobian (djc) and higher order Lin (ffr_4A & ffr_5A) give long-term 2.10^{-4} m/s error
- Subtracting reference field using pure cubic (ffr_1C) gives similar long-term error to djc and higher order ffr schemes (2.10^{-4} m/s error)
- Subtracting reference field using constrained cubic spline is less successful

3.5 Exploration of pressure forces (pictures in additional slides)

- The spurious forces are greatly reduced when a reference field is subtracted
- This is true initially and after 10 days of integration (see additional slides)
- But the velocities for djc and djr schemes after 10 days are similar in magnitude (their spatial patterns are very different)
- So the dynamical balance is different (radial hpg error in geostrophic balance with zonal velocities)
- Does this have implications for performance near the equator?
- Most of the error in djc originates from the top level (in these test cases)

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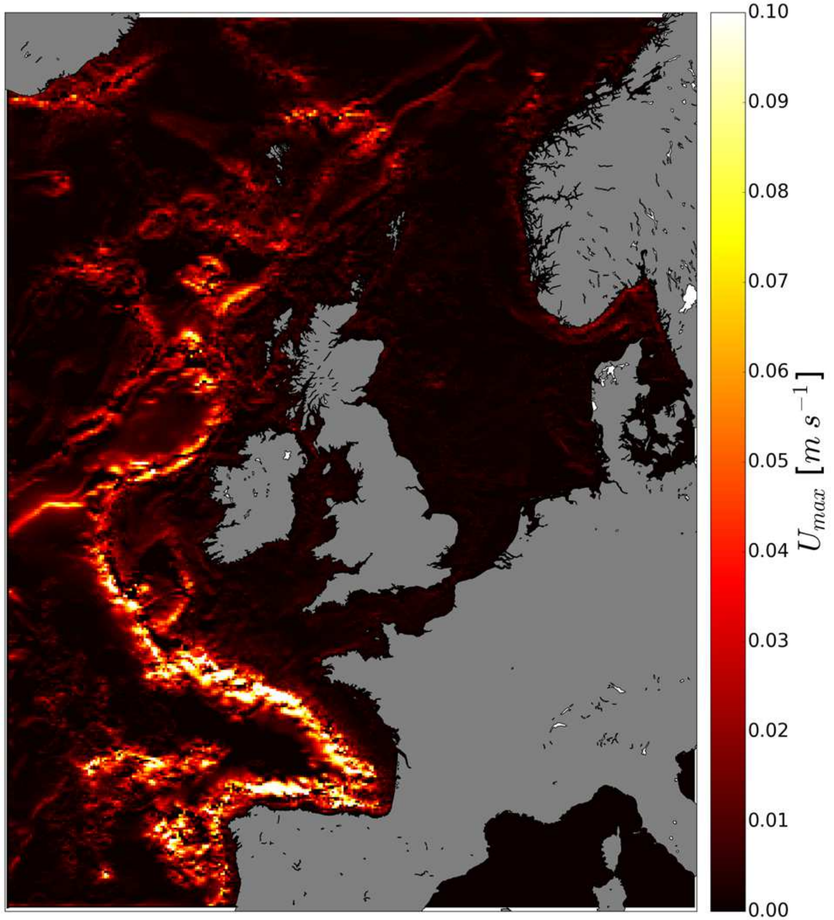
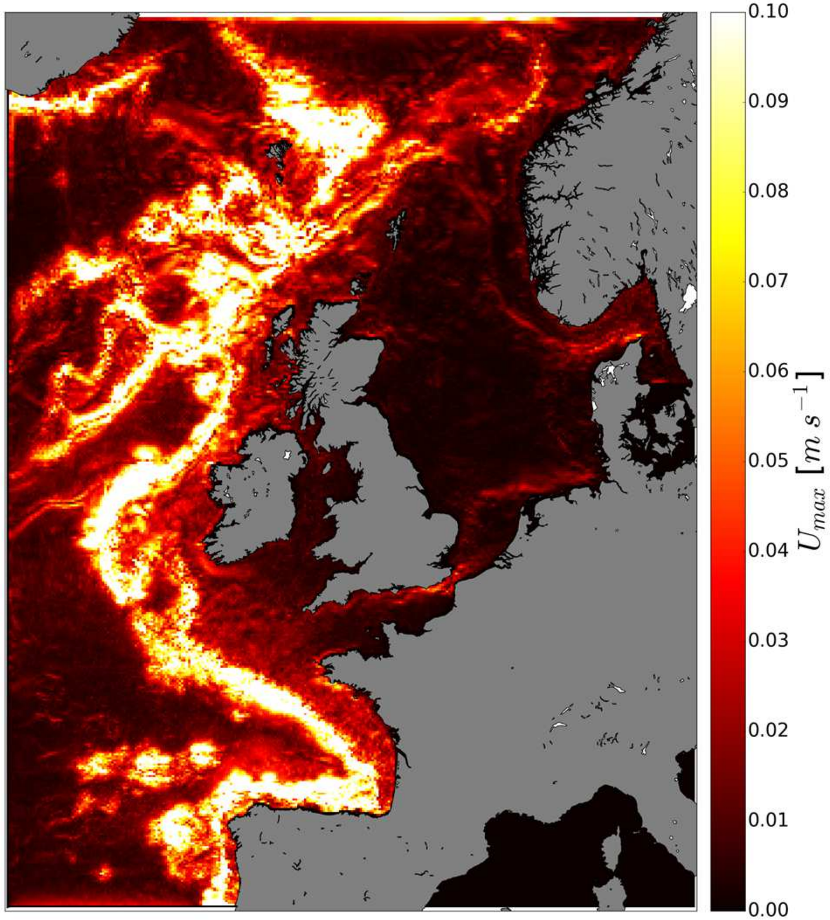
3. Initial results for Atlantic Margin Model

- Model domain shown in next slide; 7 km horizontal grid spacing (AMM7)
- 51 vertical levels; envelope bathymetry; rmax = 0.24
- Initial conditions; horizontally uniform T(z) representative of winter conditions; S=35
- Results shown after 30 days of integration
- No tracer diffusivity; biharmonic Smagorinsky lateral viscosity
- FCT advection (4th order in horizontal, 2nd order in vertical)
- Open lateral boundaries using initial conditions
- HPG schemes used: prj (operational); djc (Shchepetkin & McWilliams)

3. Results for Atlantic Margin Model after 30 days Spurious current (m/s) (max in vertical)





djc (Shcheptkin & McWilliams)

prj (operational at the moment)

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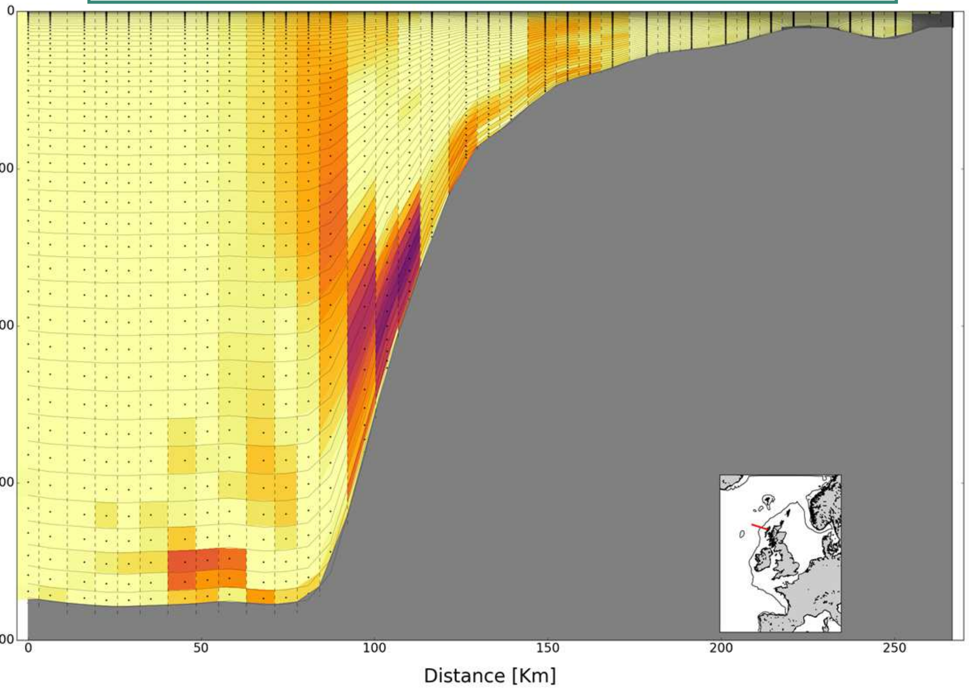
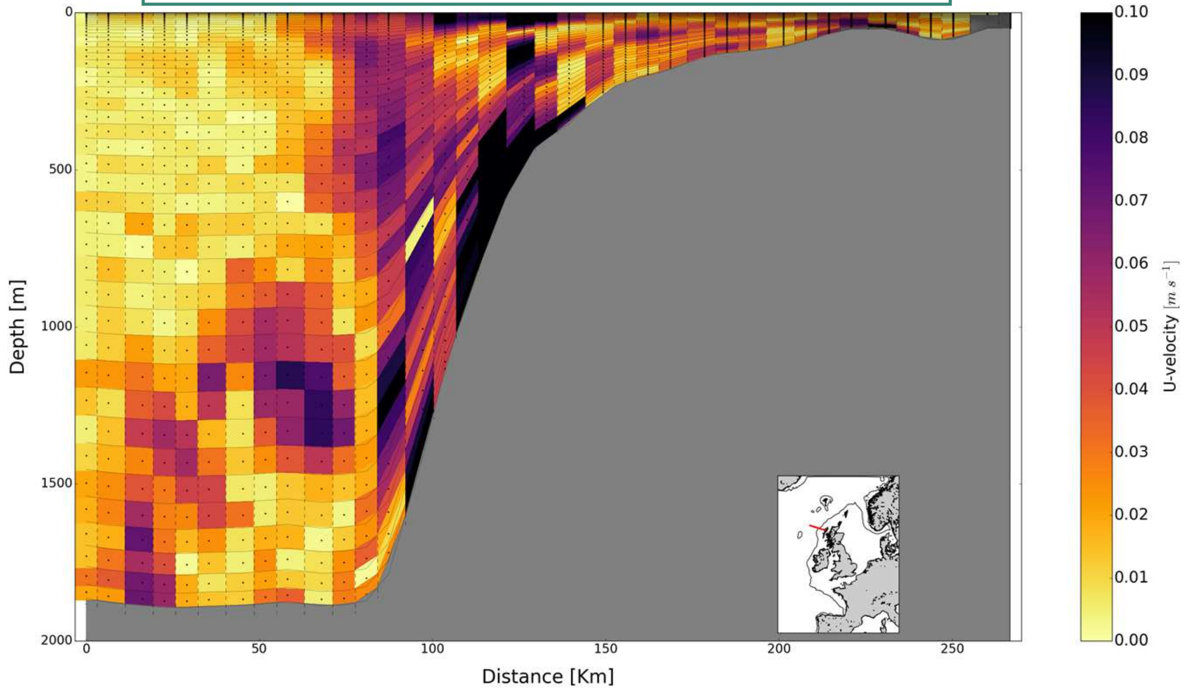
3. Results for AMM7 after 30 days Cross-sections of horizontal current (m/s)

djc (Shcheptkin & McWilliams)



prj (operational at the moment)



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5. Work in progress

- Constraining the quadratic reconstruction of vertical density profile in the forces on faces scheme
- Extending “test” cases
  - Repeating sea-mount test case with stretched grids; envelope bathymetries; smaller values of f
  - Extending AMM7 tests to new schemes
  - Initialise fields using grid cell mean-values rather than point values (as a sensitivity test)
- Intend to submit a paper documenting the schemes and results

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6. Treating tracers as grid cell mean values

- The forces on faces scheme is inherently a finite volume scheme
- So tracers (and density) should be treated as grid cell mean values but …
  - This will make the pointwise density field less smooth (so the results won't look as good)
  - Subtraction of a reference profile is more difficult / less attractive
- Selecting appropriate reconstruction limiters is harder
  - Constrained cubic splines are difficult to use (we've tried)
  - Adcroft et al. (2008) extend PPM mapping to cubics and quartics
  - Engwirda & Kelley (2016) show how to re-construct fields using WENO functions
  - Shchepetkin & McWilliams 2001 (unpublished) discuss several techniques for doing it
  - Can be complicated and needs to be done for each of ji, jj and jk directions
- Grid cells ought to be treated as 3D cells not 2D slices
  - but this would greatly increase cost (and complexity). So not worth it?

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7. Summary

- We've implemented several hpg schemes within NEMO
  - Density Jacobian using constrained cubic splines (Shchepetkin & McWilliams 2003; djc)
  - A set of forces on faces schemes using quadratic & cubic reconstructions of density
  - Implemented subtraction of a locally defined "reference" profile for both above schemes
- Results for the isolated sea-mount test case for the new schemes are competitive (and in some respects better than the djc scheme)
- Next steps
  - Further testing with sea-mount and AMM7
  - Constrain vertical re-construction of density (with forces on faces scheme)
  - Publish results
  - Calculations treating values as grid-cell-means (longer term)

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8. References

Adcroft, A., Hallberg, R., Harrison, M., 2008. A finite volume discretization of the pressure gradient force using analytic integration. Ocean Modelling 22, 106-113.

Beckmann, A and Haidvogel, D. 1993. Numerical Simulation fo Flow around a Tall Isolated Seamount. Part 1: Problem Formulation and Model Accuracy. *Journal of Physical Oceanography*. pp1736-53.

Engwirda, D. and M. Kelley 2016 A WENO-type slope-limiter for a family of piecewise polynomial methods. arXiv:1606.08188v1

Kruger, C.J.C. "Constrained Cubic Spline Interpolation for Chemical Engineering Applications," 2002

Lin, S.J., 1997. A finite-volume integration method for computing pressure gradient force in general vertical coordinates. *Quart. J. Roy. Meteor. Soc*. 123, 1749–1762.

Mellor, G. L., L.-Y. Oey and T. Ezer 1998 Sigma Coordinate Pressure Gradient Errors and the Seamount Problem. *J. Atmos. Ocean. Tech*., 15, 1122-1131

NEMO Manual version 4.0.1. “NEMO ocean engine”, Scientific Notes of Climate Modelling Center, 27 — ISSN 1288-1619, Institut Pierre-Simon Laplace (IPSL), doi:10.5281/zenodo.1464816.

Shchepetkin, A.F., McWilliams, J.C., 2003. A method for computing horizontal pressure-gradient force in an oceanic model with a nonaligned vertical coordinate. *J. Geophys. Res*. 108, C3, 30390, doi:10.1029/2001JC001047.

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Additional Slides

Results for first time-step

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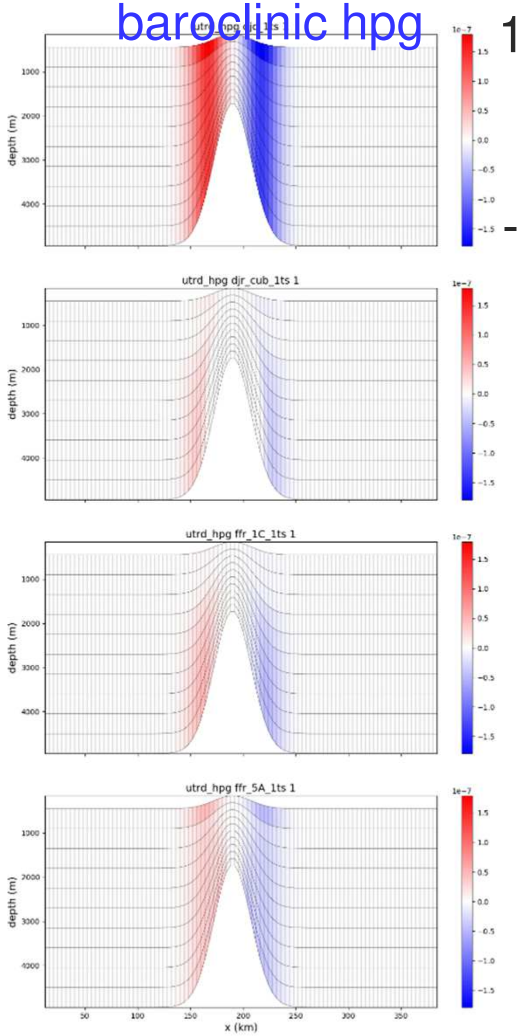
$\partial u/\partial t$ (ms^{-2}) from baroclinic hpg



1.510^{-7}

djc

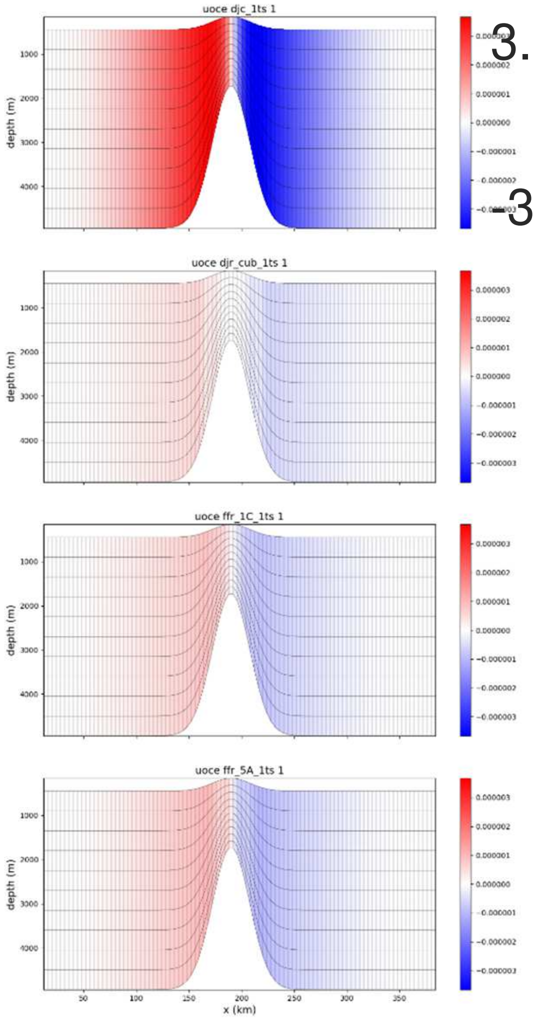
-1.510^{-7}

djr cub

ffr 1C

ffr 5A

u (m/s) at end of time-step



3.10^{-6}

-3.10^{-6}

- Initial acceleration in djc scheme dominated by errors in top cell
- Velocity errors in all schemes are dominated by the external mode

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Results for day 10

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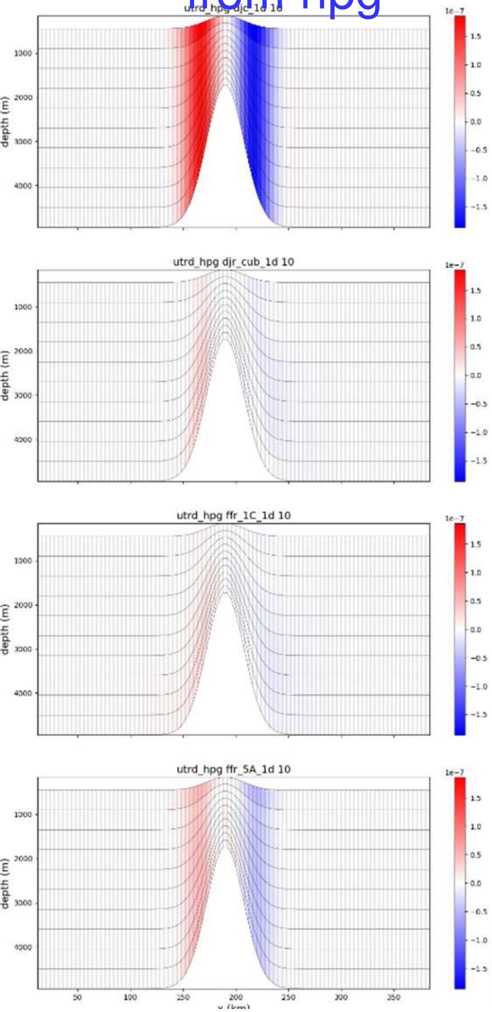
Baroclinic $\partial u/\partial t$ (ms^{-2}) from hpg



1.510^{-7}

djc

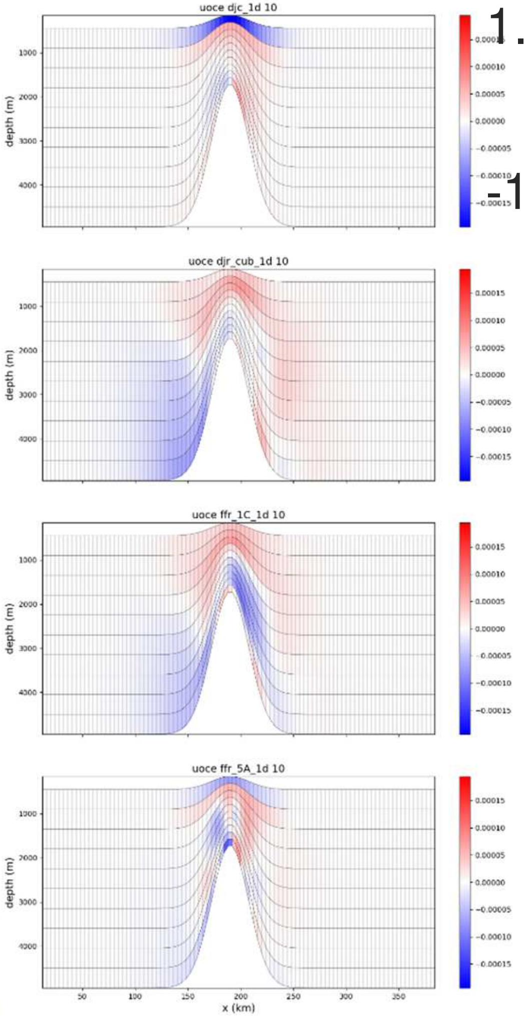
-1.510^{-7}

djr_cub

ffr_1C

ffr 5A

Last day mean u (m/s)



1.510^{-4}

-1.510^{-4}

- Accelerations in djc and ffr_5A largely unchanged
- Accelerations in djr cub and ffr_1C have reduced
- Velocity errors patterns are quite different from the acceleration pattern
- Velocity error patterns differ from one scheme to another
- Surface pressure gradients compensate the external mode
- I think the external mode is still oscillating

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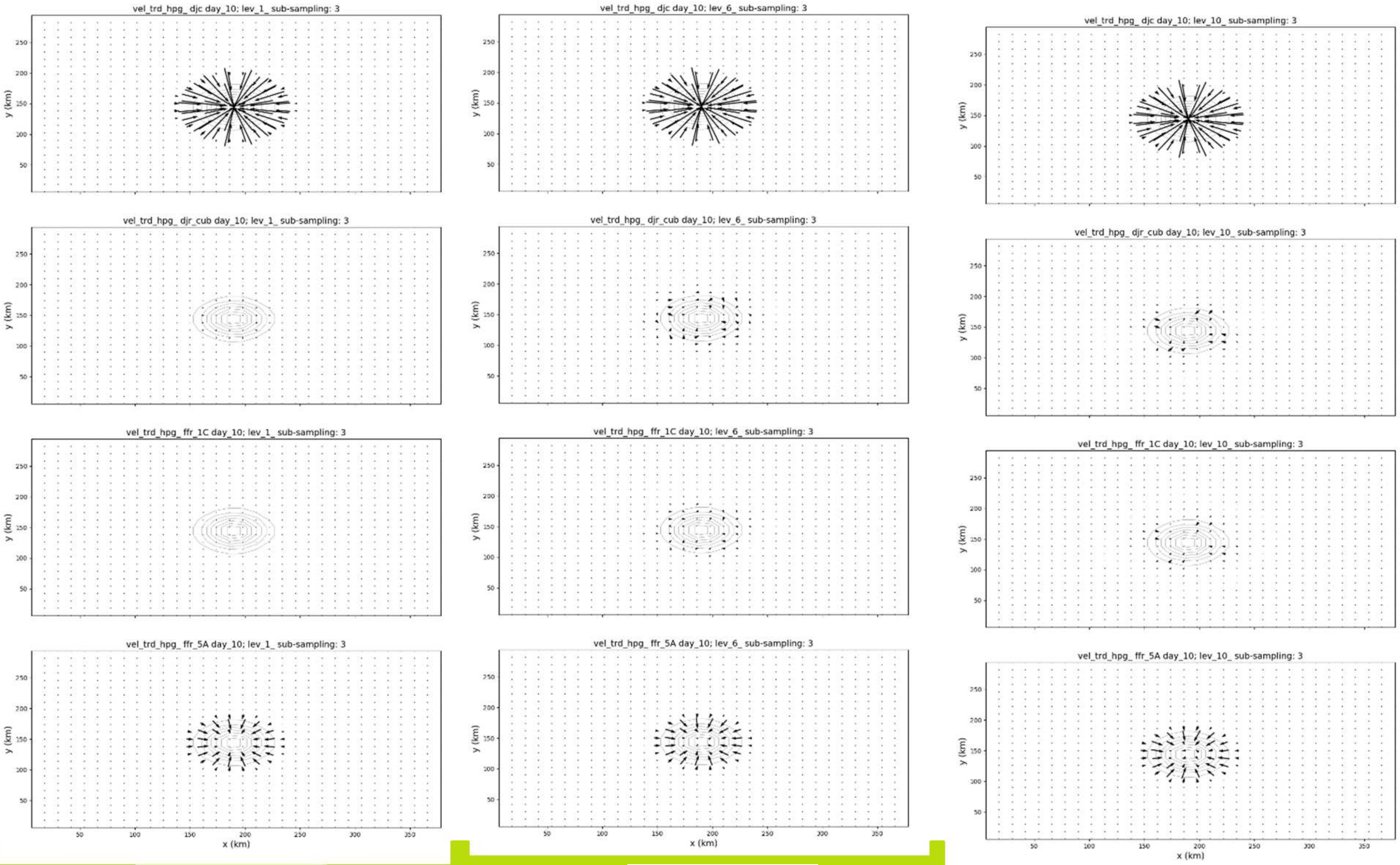
Results for day 10

Day 10 mean $\partial u/\partial t$ (ms^{-2}) from baroclinic hpg



djc – mainly radial errors

djr_cub – small "random errors

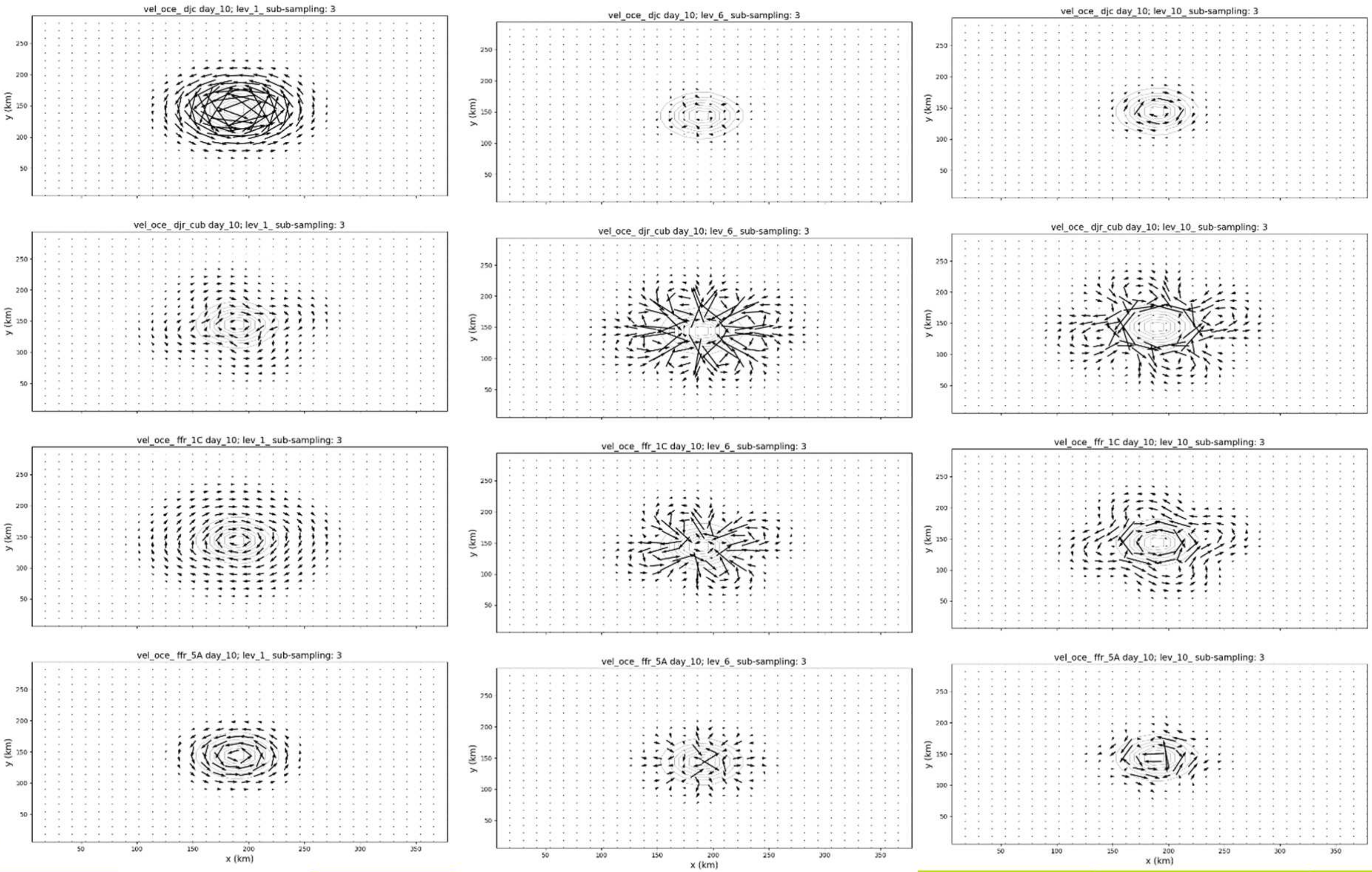
ffr 1C – small "random errors

ffr_5A – mainly radial errors

Level 1 | Level 6 | Level 10

Results for day 10

Day 10 mean u (ms^{-1}) from hpg (values still oscillating)



Level 1 | Level 6 | Level 10

djc – mainly zonal errors

djr_cub four lobes

ffr_1C four lobes

ffr 5A – mainly zonal errors