Quantification of the cross helicity cascade in compressible MHD simulations

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Introduction

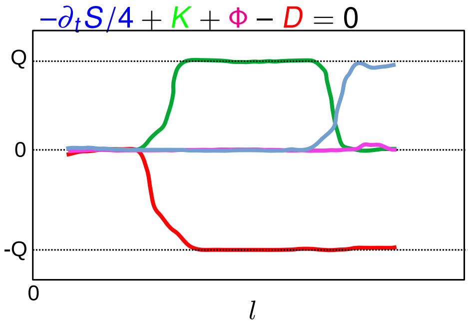
- Karman-Howarth-Monin (KHM) equations provide a way to estimate the energy cascade rate (and the dissipation rate) of turbulent flows without resorting to phenomenological models.
- Fluctuations in the interplanetary medium are generally subsonic, but can still reach non-negligible levels of compressibility, specially in the slow winds, the magnetosheaths or close to the Sun. This motivates the development of KHM equations that account for compressible effects.
- High cross helicity (correlation of magnetic and velocity fluctuations) inhibits non-linear energy transfer in plasma turbulence. Like energy, cross helicity is also transfered towards smaller scales by non-linear interactions.
- We derived the KHM equation of cross helicity for the compressible MHD equations. We analyzed 3D compressible MHD simulations and used the KHM equations to analyze the cross helicity cascade.

Karman-Howarth-Monin equations

The KHM equations provide the evolution of second order structure functions $S(\boldsymbol{\ell}) =< \delta \boldsymbol{a} \cdot \delta \boldsymbol{b} >$, where $\boldsymbol{\ell}$ is the separation scales, $\boldsymbol{a}$ and $\boldsymbol{b}$ are two generic fields, $\delta \boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}+\boldsymbol{\ell}) - \boldsymbol{a}(\boldsymbol{x})$, and $< \cdots >$ is the volum average. From the compressible MHD equations, one can derive an equation valid in three-dimensional separation space

$$-\partial_t S/4 + K + \Phi - D = 0$$

where the first term in the left-hand side corresponds to the temporal decay, the second to non-linear transfer, the third to pressure-dilatation effects and the fourth to turbulence dissipation minus the turbulent heating rate Q.
The information is reduced to a 1D plot through isotropization of each term (see diagram).



Tipically, the term K is written as $K =< \nabla \cdot \boldsymbol{Y} >$ with $\boldsymbol{Y}$ being a third order structure function. In order to substantially reduce the computational cost of calculating the KHM terms, we have left them all in the form $< \delta \boldsymbol{a} \cdot \delta \boldsymbol{b} >$.

KHM equations for cross-helicity and energy

KHM equations for cross helicity

Temporal decay term:

$$\partial_t S_H/4 = \partial_t \langle \delta \boldsymbol{u} \cdot \delta \boldsymbol{B} \rangle /4$$

Non-linear transfer terms:

$$K_H =< \delta \boldsymbol{B} \cdot \delta((\boldsymbol{u} \cdot \nabla)\boldsymbol{u}) - \delta \boldsymbol{B} \cdot \delta(\rho^{-1}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}) - \delta \boldsymbol{u} \cdot \delta(\nabla \times (\boldsymbol{u} \times \boldsymbol{B})) >$$

Terms associated to dissipation:

$$D_H = \left\langle -\delta \boldsymbol{B} \cdot \delta(\rho^{-1}\nabla \cdot \boldsymbol{\tau}) - \eta \delta \boldsymbol{u} \cdot \delta(\Delta \boldsymbol{B}) \right\rangle$$

Pressure-dilatation terms:

$$\Phi_H = -\left\langle \delta \boldsymbol{B} \cdot \delta(\rho^{-1}\nabla P) \right\rangle$$

KHM equations for energy

Temporal decay term:

$$\partial_t S_E/4 = \partial_t \left\langle \delta(\sqrt{\rho}\boldsymbol{u}) \cdot \delta \boldsymbol{B} \right\rangle /4$$

Non-linear transfer terms:

$$K_E =< \delta(\sqrt{\rho}\boldsymbol{u}) \cdot \delta((\boldsymbol{u} \cdot \nabla)\boldsymbol{u}) - \delta(\sqrt{\rho}\boldsymbol{u}) \cdot \delta(\sqrt{\rho}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}) - \delta \boldsymbol{B} \cdot \delta(\nabla \times (\boldsymbol{u} \times \boldsymbol{B})) >$$

Terms associated to dissipation:

$$D_E = \left\langle -\delta(\sqrt{\rho}\boldsymbol{u}) \cdot \delta(\rho^{-1/2}\nabla \cdot \boldsymbol{\tau}) - \eta \delta \boldsymbol{B} \cdot \delta(\Delta \boldsymbol{B}) \right\rangle$$

Pressure-dilatation terms:

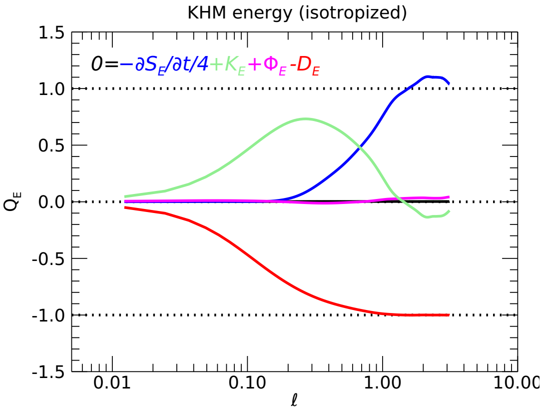
$$\Phi_E = -\left\langle \delta(\sqrt{\rho}\boldsymbol{u}) \cdot \delta(\rho^{-1/2}\nabla P) \right\rangle$$

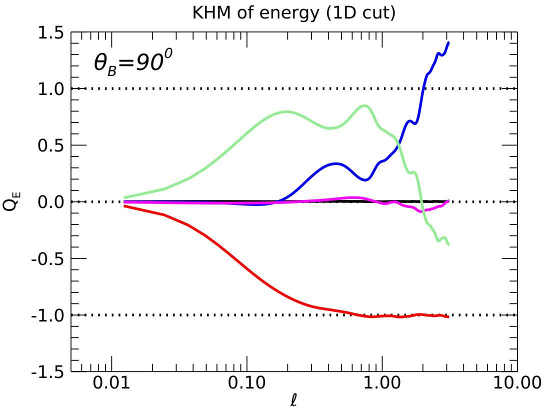
Numerical set-up

- We present here the analisys of a 3D compressible MHD simulation with periodic boundary conditions, 512^3 grid-points resolution and a strong mean magnetic field $b/B_0 = 0.25$.
- Turbulence is free-decaying, shear velocity and magnetic fluctuations are initially injected within a sphere in Fourier space of radius $k_{cut-off} = 4$. Velocity and magnetic fluctuations are also at equipartition initially.
- The peak of turbulent activity is reached at $t = 6t_{NL}^0$, where $t_{NL}^0 = u_{rms}/L_0$ is the initial turn over time, L_0 the size of the numerical domain and u_{rms} the root mean square of the initial velocity fluctuations. The KHM terms have been computed at $t = 8t_{NL}^0$ for all cases presented here.
- Normalized cross helicity is initially set at $\sigma_c = 0.8$ and turbulent Mach number at $M = 0.8$, in order to enhance the possible effects of cross helicity and plasma compressibility on the KHM terms.

Energy cascade

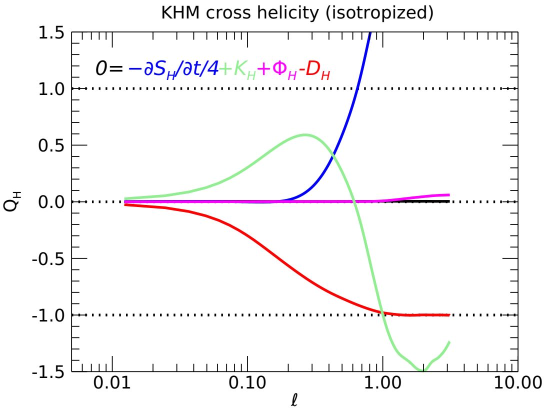
Negligible pressure-dilatation effects. Direct cascade of energy at intermediate scales and small inverse cascade at large scales. Isotropization masks longer inertial range and larger values of the non- linear transfer term K_E.

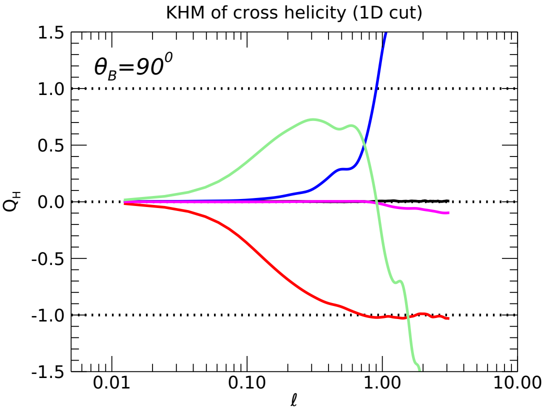




Cross helicity cascade

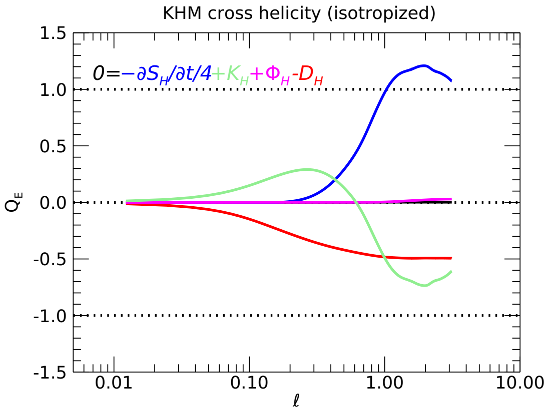
Similar to the energy cascade. Same anisotropy as the energy cascade. Larger negative non-linear transfer K_H. Shorter inertial range and shifted towards large separation scales.





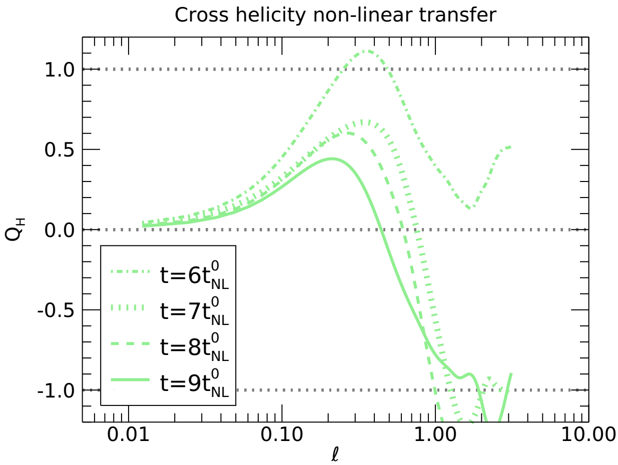
Back transfer of cross helicity cascade?

Cross helicity is not positively defined. Thus, negative K_H at large scales does not necessarely correspond to an inverse cascade. A direct cascade at all scales that changes the sign of cross helicity for a certain range of scales also has negative K_H. However, in such case, all KHM terms change the sign when normalized by Q_E. In this case, the the other KHM terms of cross helicity keep the same sign across all scales.



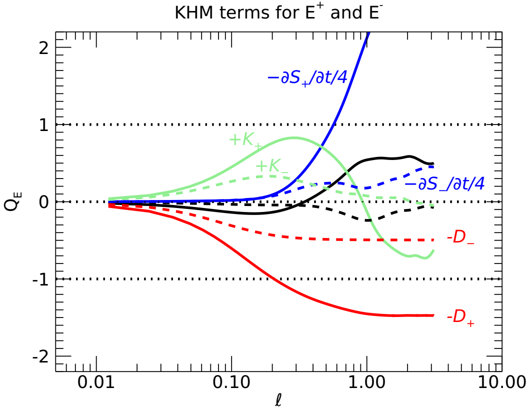
Temporal evolution of cross helicity cascade

The inverse non-linear transfer at large scales of the cross helicity cascade remains for several turnover times.



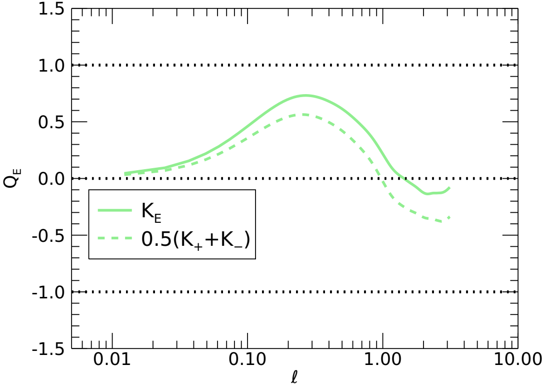
Cascade of pseudo-energies

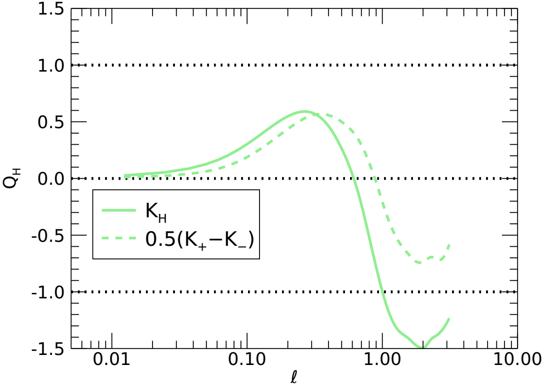
Assuming incompressibility, it is possible to compute the KHM equations for the pseudo-energies $E^{\pm} = |z^{\pm}|^2$ where $\boldsymbol{z}^{\pm} = \boldsymbol{u} \pm \boldsymbol{B} / \sqrt{4\pi < \rho >}$ are the Elsässer fields.
Due to non-negligible compressibility in the simulation, the KHM equations for $E^{\pm}$ are not satisfied (errors are represented as black lines). Even if the errors are taken into account, one can state that there is a direct non-linear transfer of E^{-} at all scales. Conversely, there is an inverse cascade of E^{+} at large scales and a direct at intermediate scales.



Incompressible and compressible non-linear transfers

Incompressible non-linear transfers for energy ($(K_+ + K_-)/2$) and cross helicity ($(K_+ - K_-)/2$) underestimate the real K_E and K_H about 10% of their repective heating rates. Conversely, the back transfer for cross helicity is largely underestimated and the energy one is overestimated.





Summary

- The KHM equations for cross helicity have allowed to show the presence of an inverse cascade of this quantity at large scales and a direct cascade at intermediate scales. Such configuration remains for several turnover times.

- The incompressible formulation of the KHM equation for cross helicity largely underestimates the back transfer at large scales.

- Energy and cross helicity cascades (direct and inverse) develop preferentially in the directions transverse to $\boldsymbol{B}_0$.