The Regional Fractional Energy Balance Equation: Projections to 2100

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Agenda

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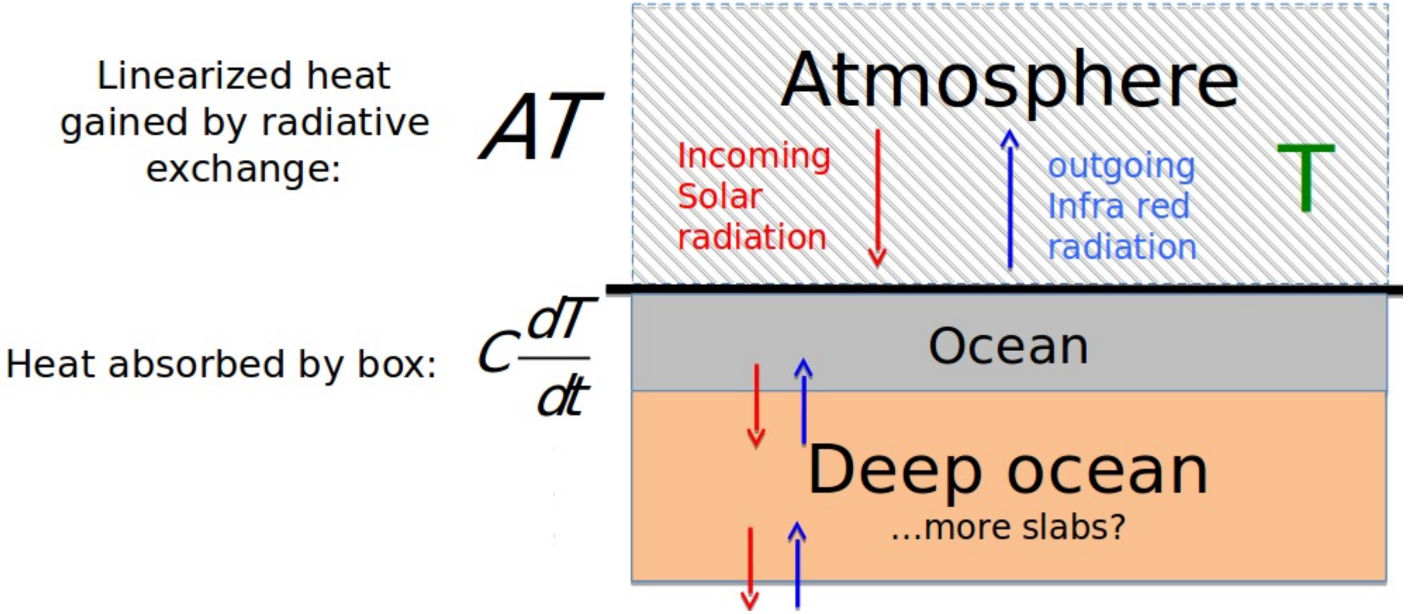
Energy Balance Models

- The classical energy balance equation (EBE) assumes the Earth is a uniform box or a set of N-boxes, we will consider N=1.

$$C\frac{dT}{dt} + AT = F(t)$$

- Solving with Green's functions we have:

$$T(t) = s\int_{-\infty}^{t} G_0(t-u)F(u)du$$



Where the Green's function is:

$$s = \frac{1}{A} \qquad \tau = Cs$$

$$G_0(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}$$

Fractional Energy Balance Equation

- We now generalize the derivative operator to a Weyl fractional derivative:

$$\tau^h {}_{-\infty}D_t^h + T(t) = sF(t)$$

Where the solution again can be represented in terms of a Green's function:

$$T(t) = s \int_{-\infty}^{t} G_{0,h}(t-u)F(u)du$$

The Dirac response Green's function is given as:

$$G_{0,h}(t) = \tau^{-1}\left(\frac{t}{\tau}\right)^{h-1} E_{h,h}\left(-\left(\frac{t}{\tau}\right)^{h}\right)$$

Using the 2-Parameter Mittag-Leffler function:

$$E_{\alpha,\beta} = \sum_{k}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

Model and Forcing Parameters

$$T(t) = s \int_{-\infty}^{t} G_{0,h}(t-u)F(u)du$$

$$F(t) = F_{GHG} + \alpha F_{Aer} + F_{Vol} + \frac{\langle F_{Vol} \rangle}{\langle F_{Vol}^{\nu} \rangle} F_{Vol}^{\nu} + \gamma$$

Model Parameters: h and τ

- h characterizes the memory of the system.
- τ characterizes the approach of the system to equilibrium.

Forcing Parameters: α and ν

- α is the aerosol linear scaling factor that effectively re-calibrates the aerosol forcing.
- N is the volcanic intermittency correction exponent that allows high intermittent volcanic forcing to be used in a linear framework.

Climate Sensitivity Parameter: *s*

- *s* is the climate sensitivity parameter s, refers to the equilibrium change in the annual GMST following a unit change in radiative forcing.

Bayesian Parameter Estimation and Error Models

Fractional Gaussian Noise Correlation Function:

$$R_h^{(fGn)}(\lambda) = \frac{1}{2}\left((\lambda+1)^{2h+1} + (\lambda-1)^{2h+1} - 2\lambda^{2h+1}\right)$$

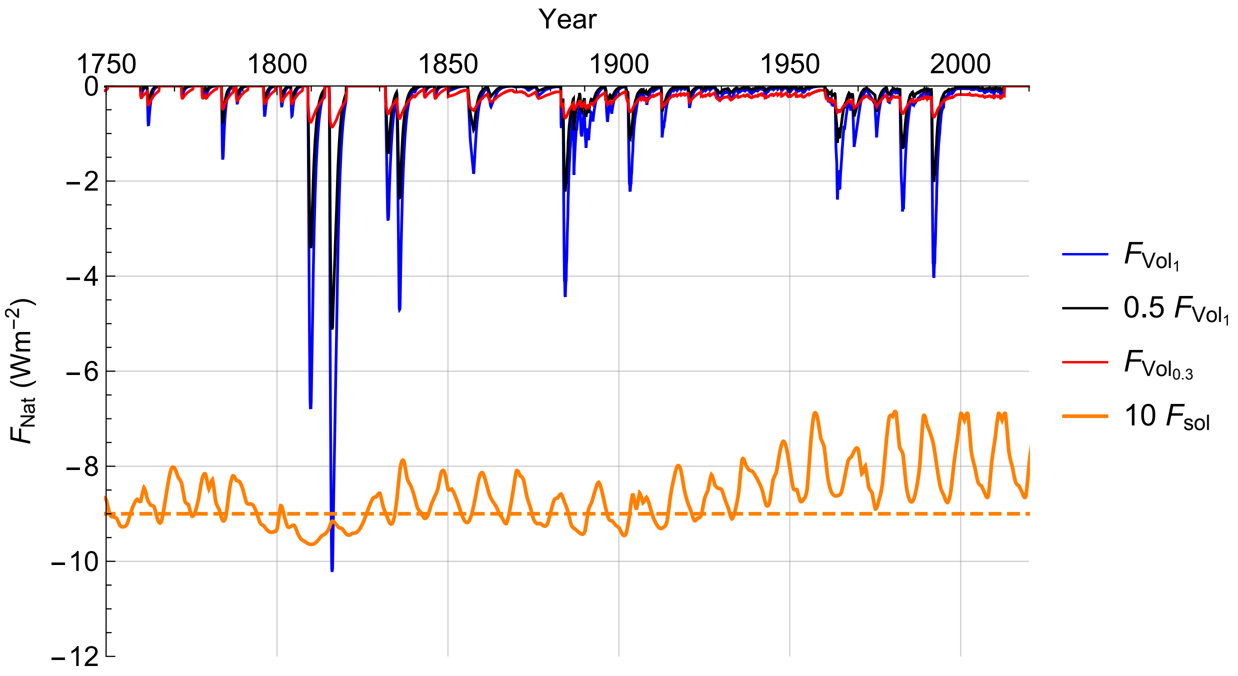
Fractional Relaxation Noise Correlation Function:

$$R_h^{(fRn)}(\lambda) = \frac{\tau^2}{2}\left(V_h\left(\frac{\lambda-1}{\tau}\right) - V_h\left(\frac{\lambda+1}{\tau}\right) - 2V_h\left(\frac{\lambda}{\tau}\right)\right)$$

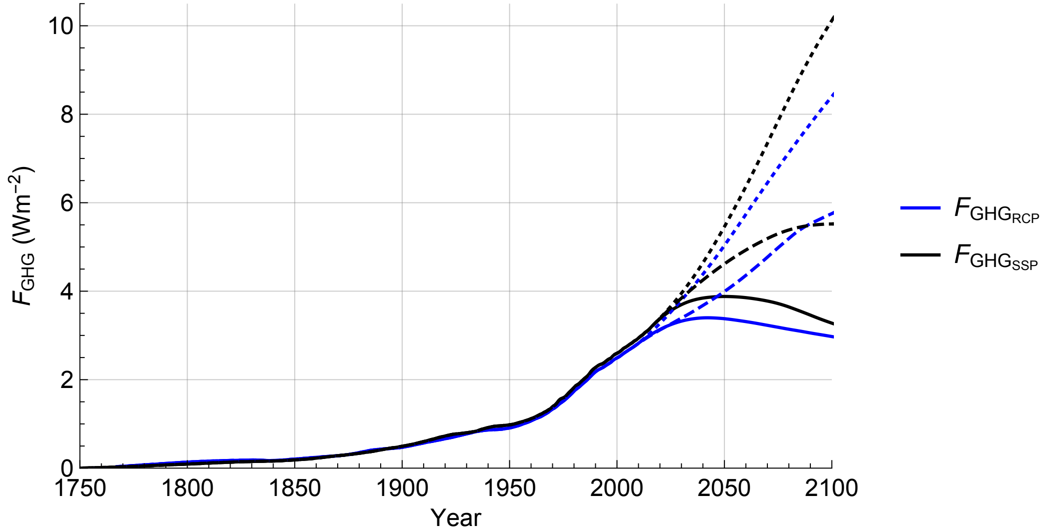
The error model was not ad hoc, rather predicted by the model itself: the internal variability response to white noise internal forcing, a fraction Relaxation noise (fRn).

The fGn is only the high frequency approximation of the more general fRn.

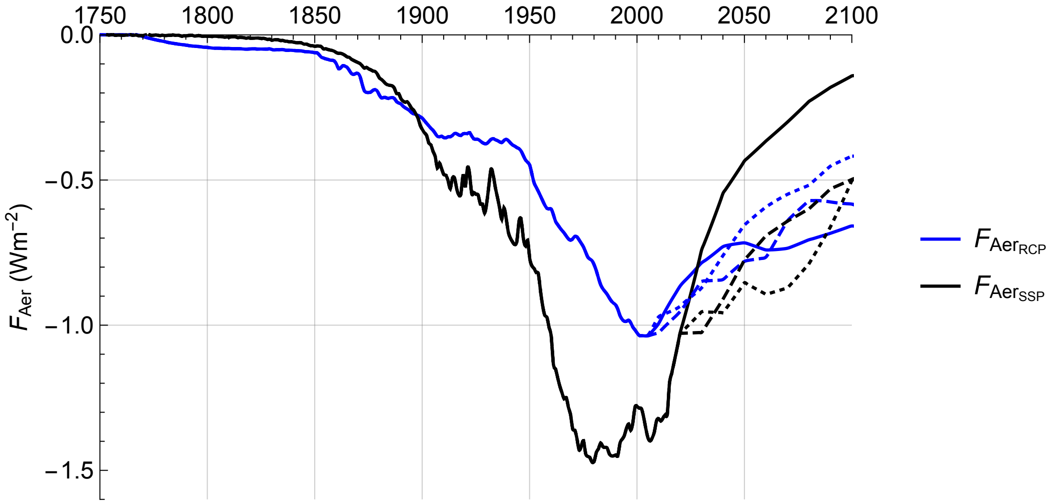
Forcings



Year
1750
1800
1850
1900
1950
2000
0
−2
−4
−6
−8
−10
−12
F_{Nat} (Wm^{-2})
F_{Vol_1}
0.5 F_{Vol_1}
$F_{Vol_{0.3}}$
10 F_{sol}



10
8
6
4
2
0
F_{GHG} (Wm^{-2})
1750
1800
1850
1900
1950
2000
2050
2100
Year
$F_{GHG_{RCP}}$
$F_{GHG_{SSP}}$

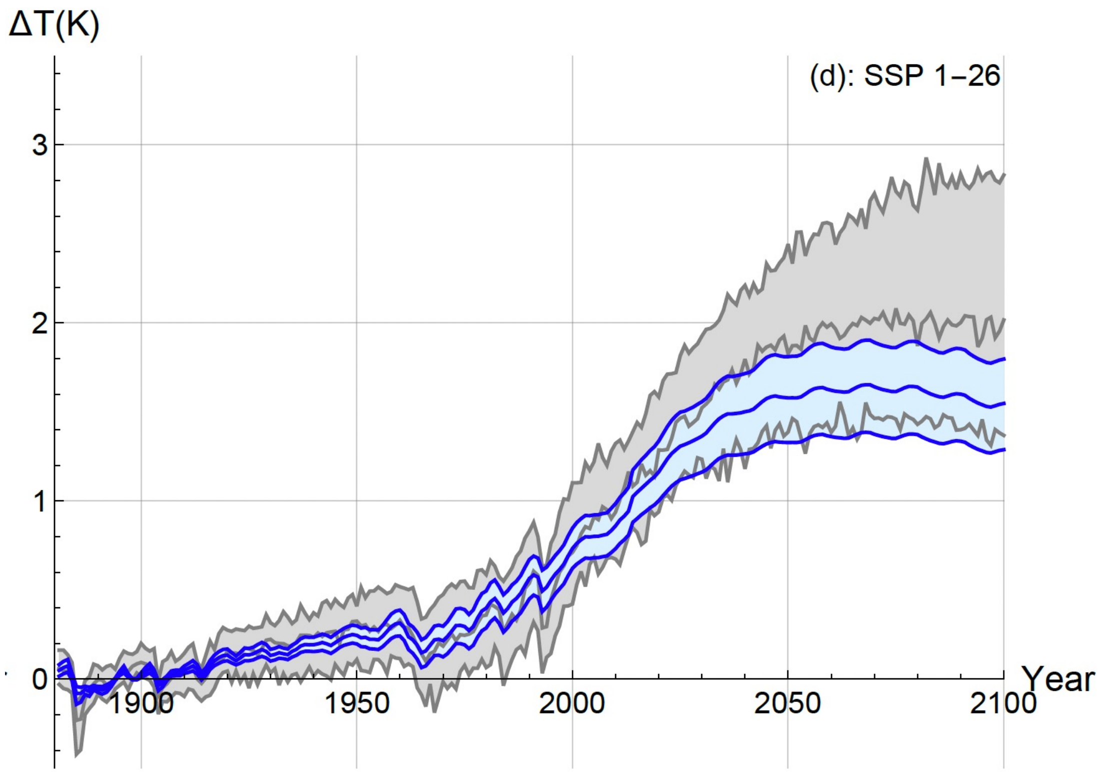


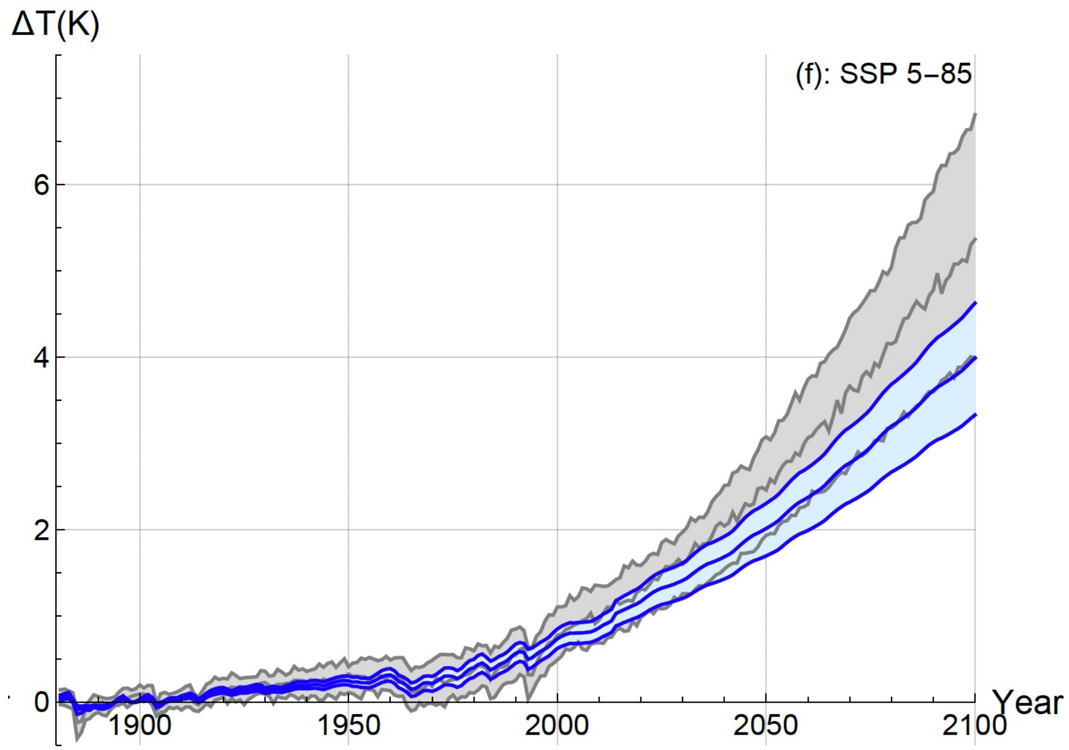
1750
1800
1850
1900
1950
2000
2050
2100
0.0
−0.5
−1.0
−1.5
F_{Aer} (Wm^{-2})
$F_{Aer_{RCP}}$
$F_{Aer_{SSP}}$

Global Parameter Estimates

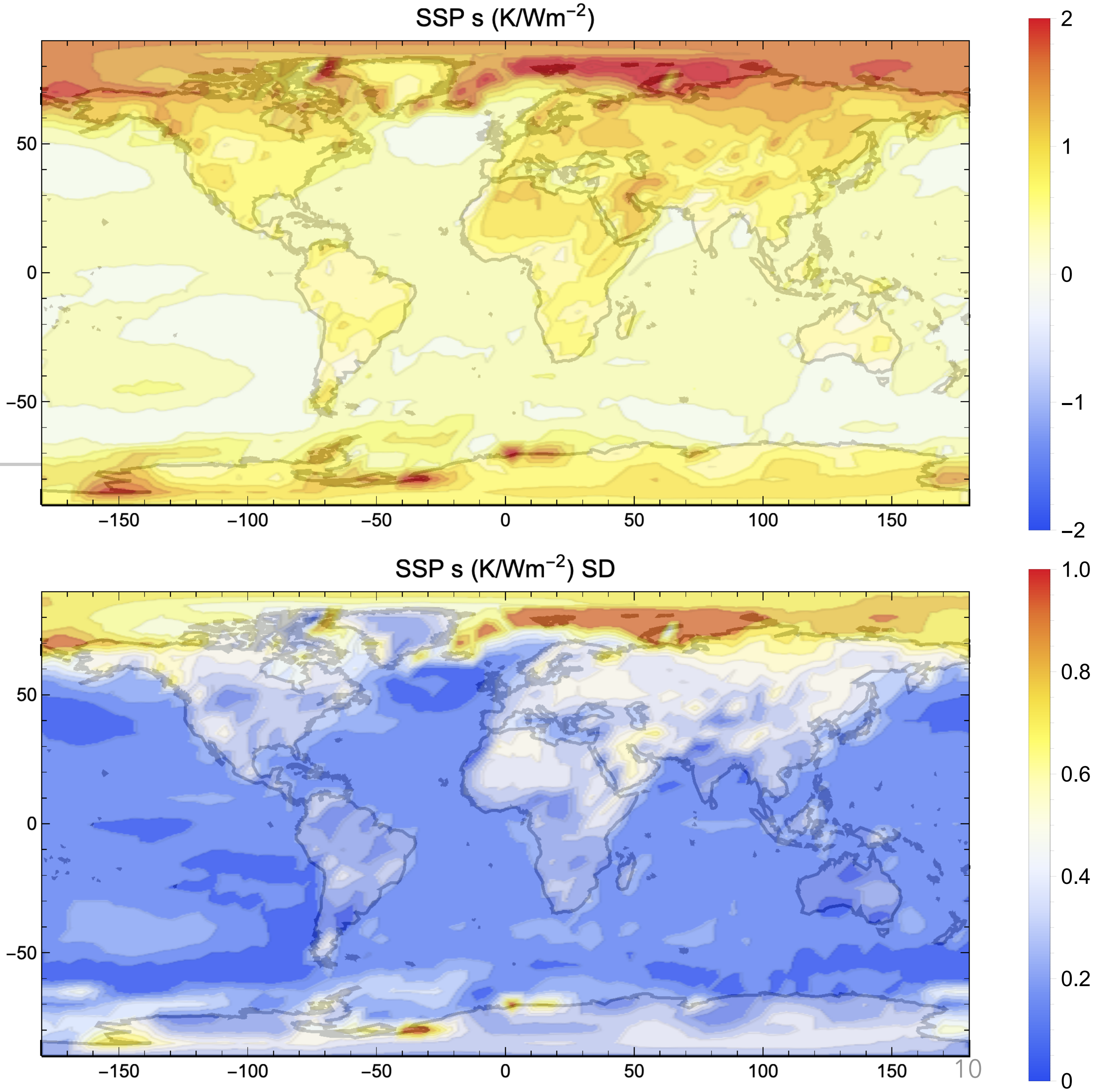
	h	τ (years)	α	ν	ECS
Median	0.38	4.7	0.33	0.28	1.8
90% CI	[0.32, 0.44]	[2.4, 7.0]	[0.05, 0.61]	[0.16, 0.40]	[1.5, 2.2]

Global Projections

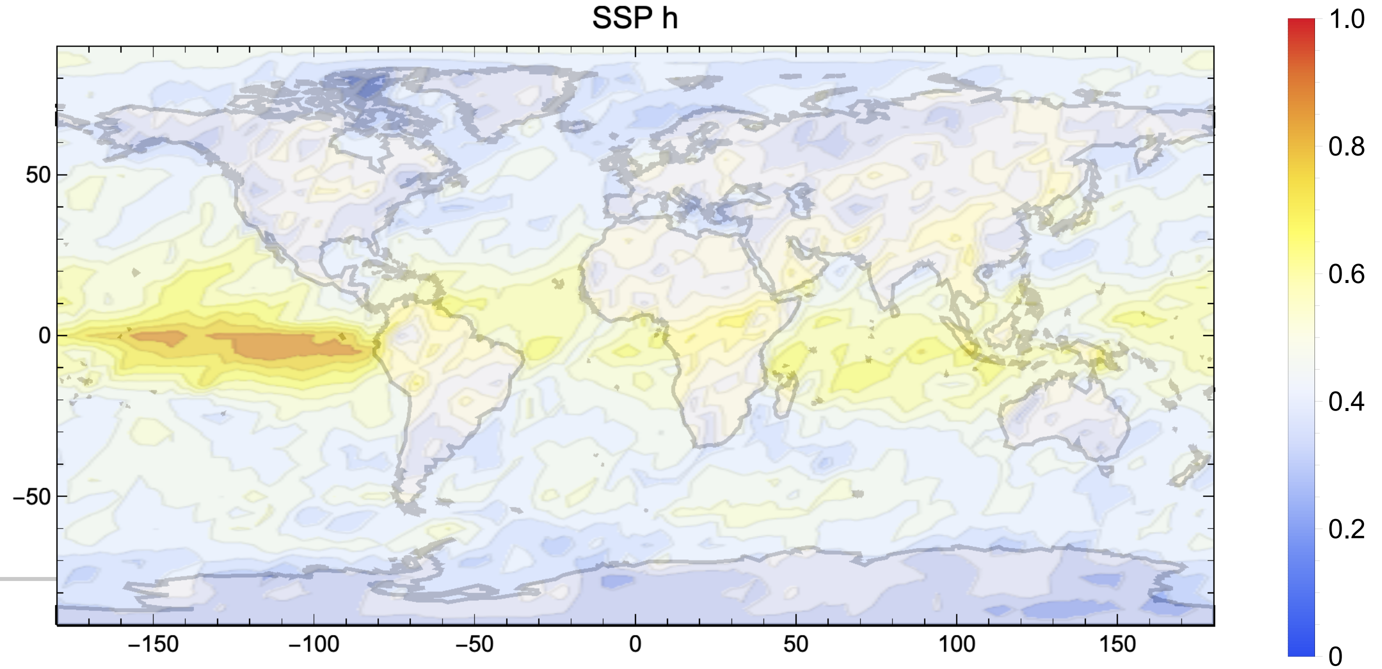


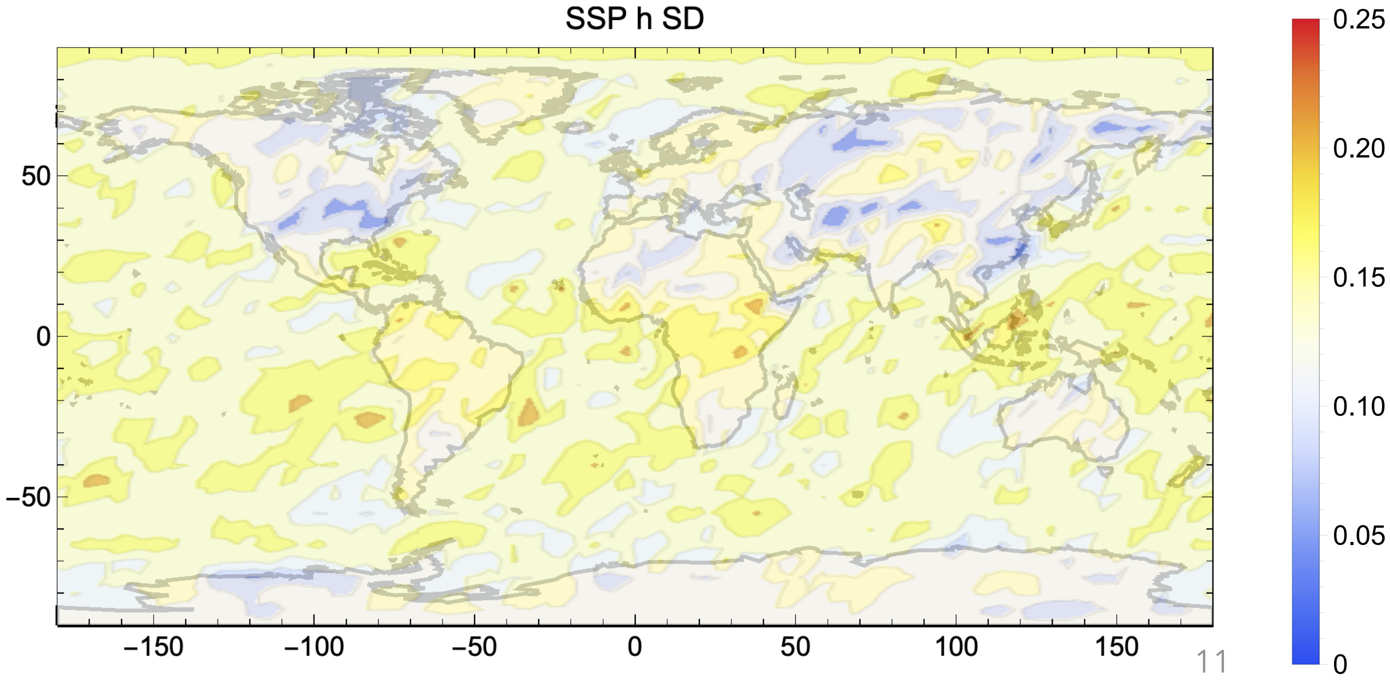


Regional Parameters: Climate Sensitivity

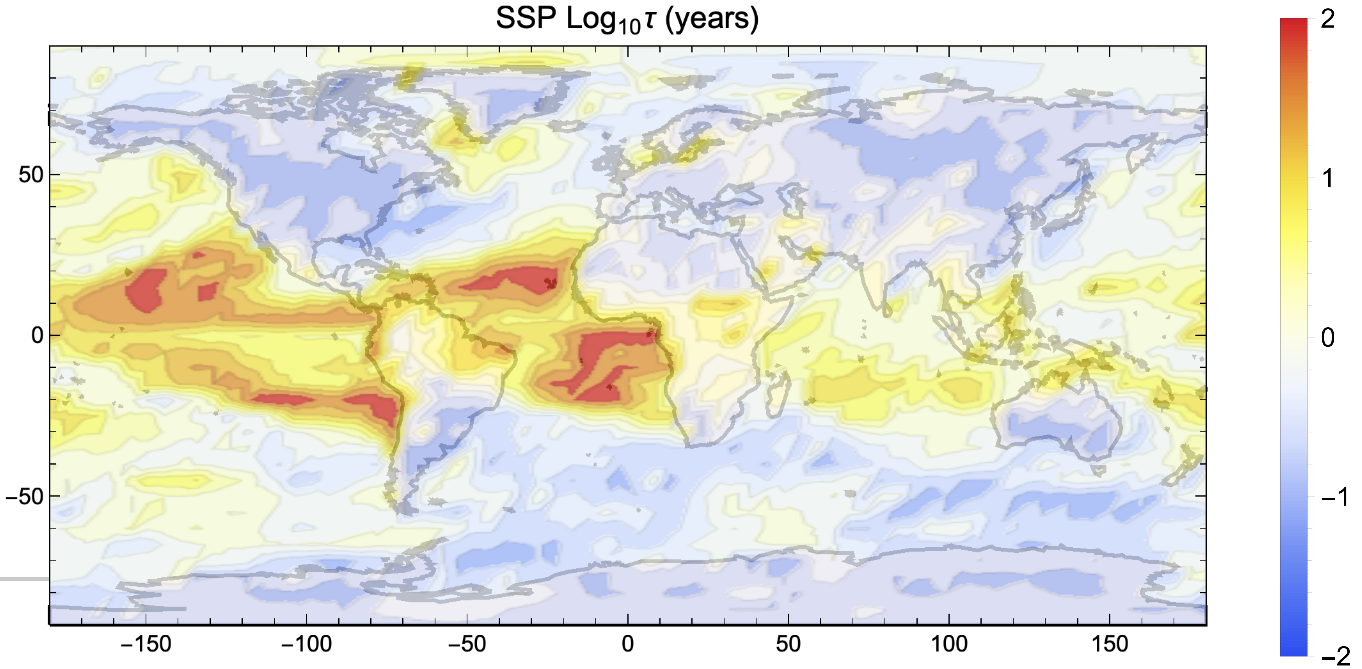


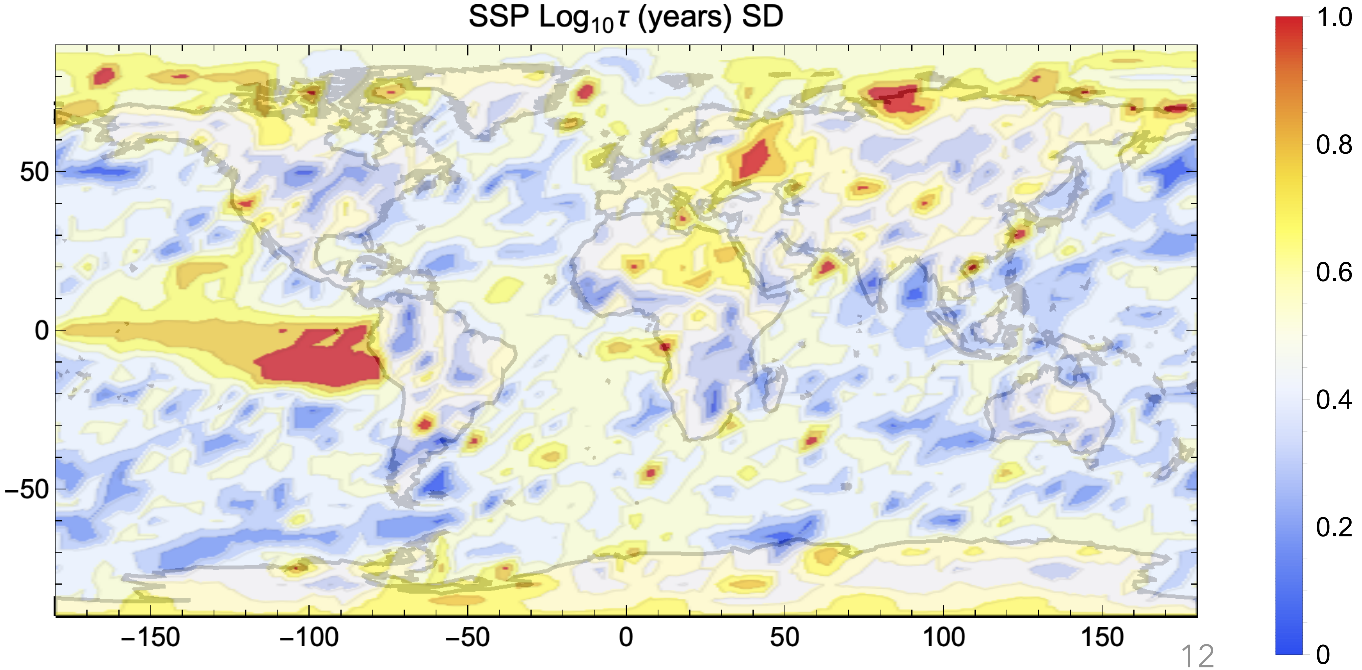
Regional Parameters: h Parameter



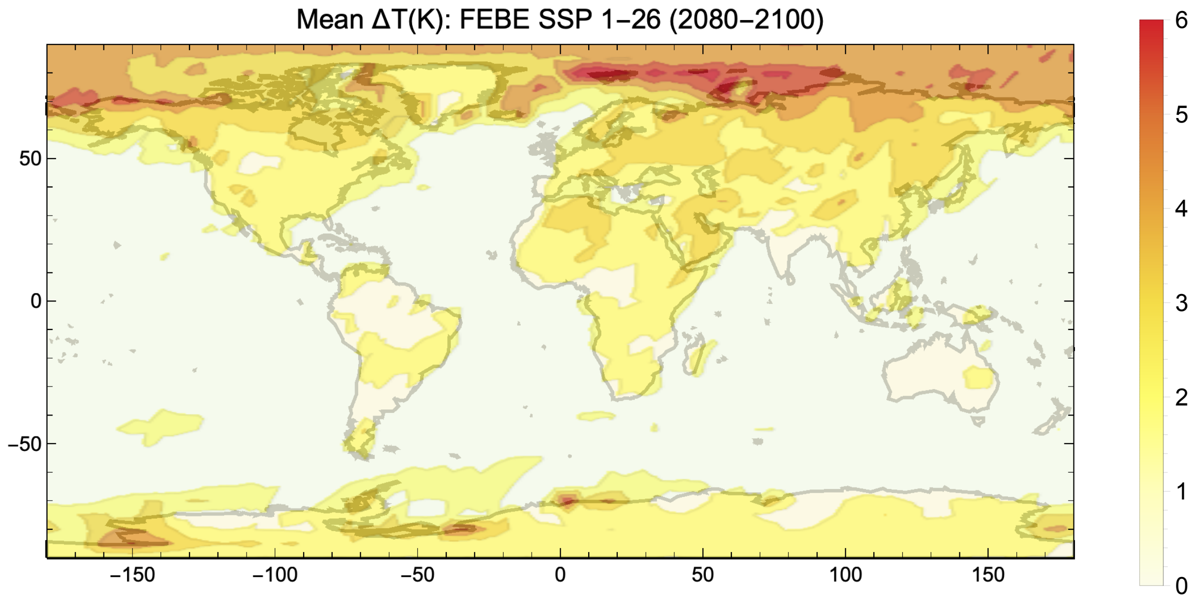


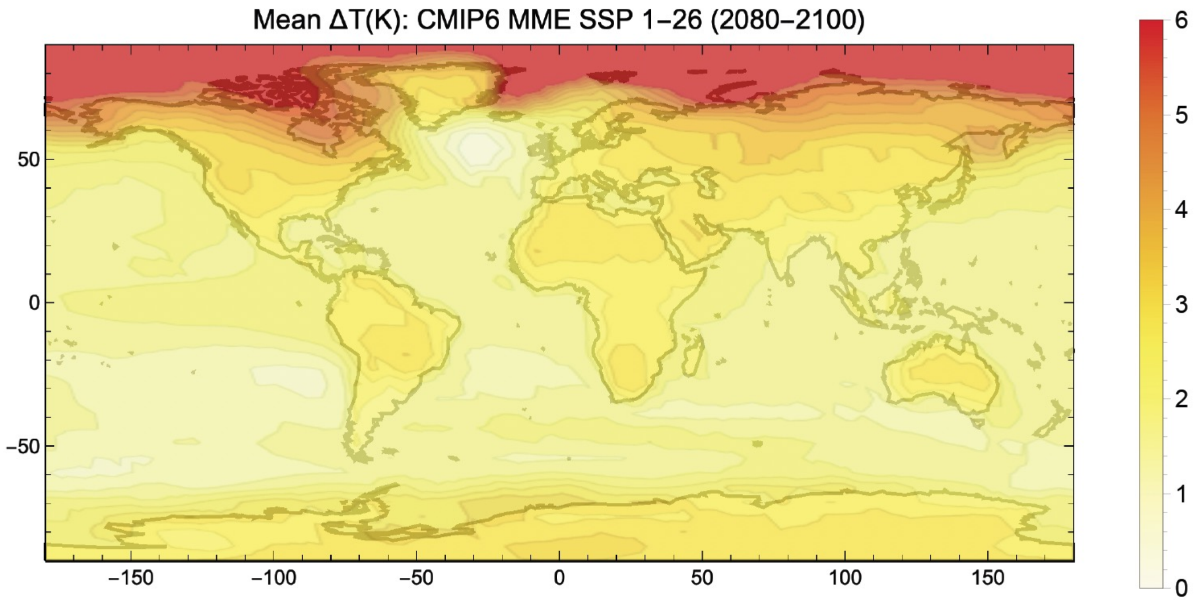
Regional Parameters: Relaxation Time



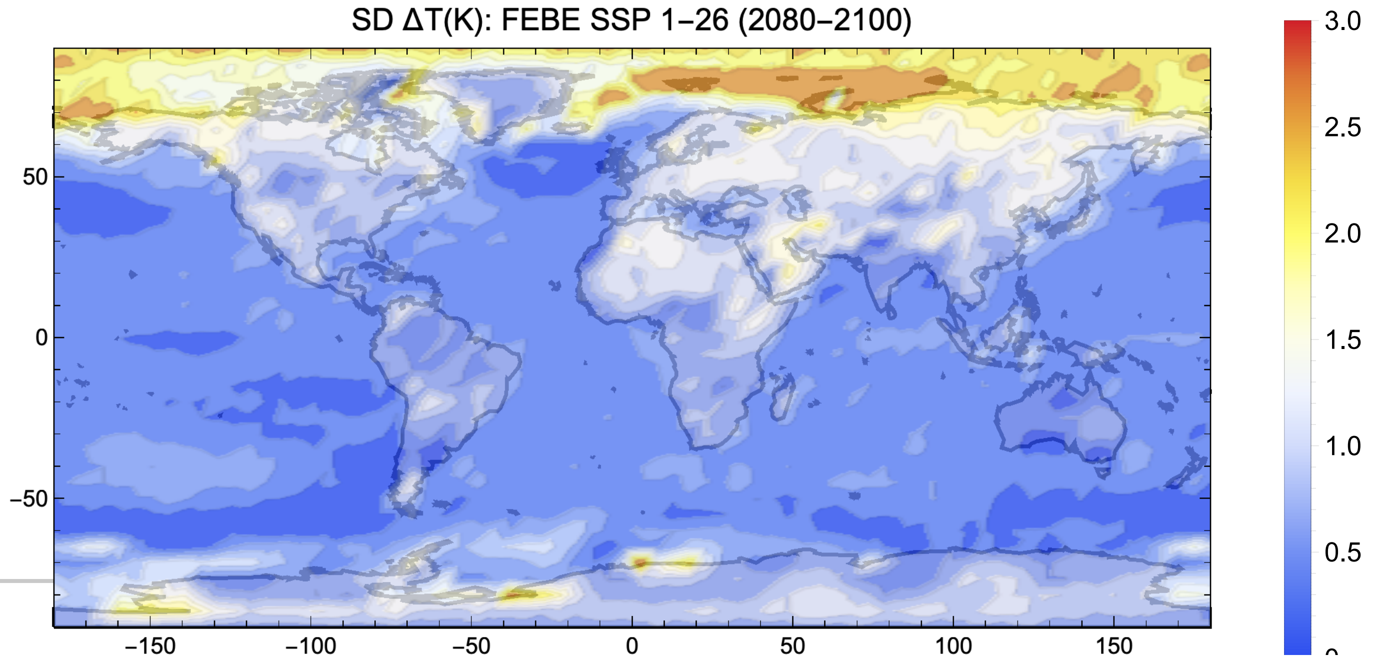


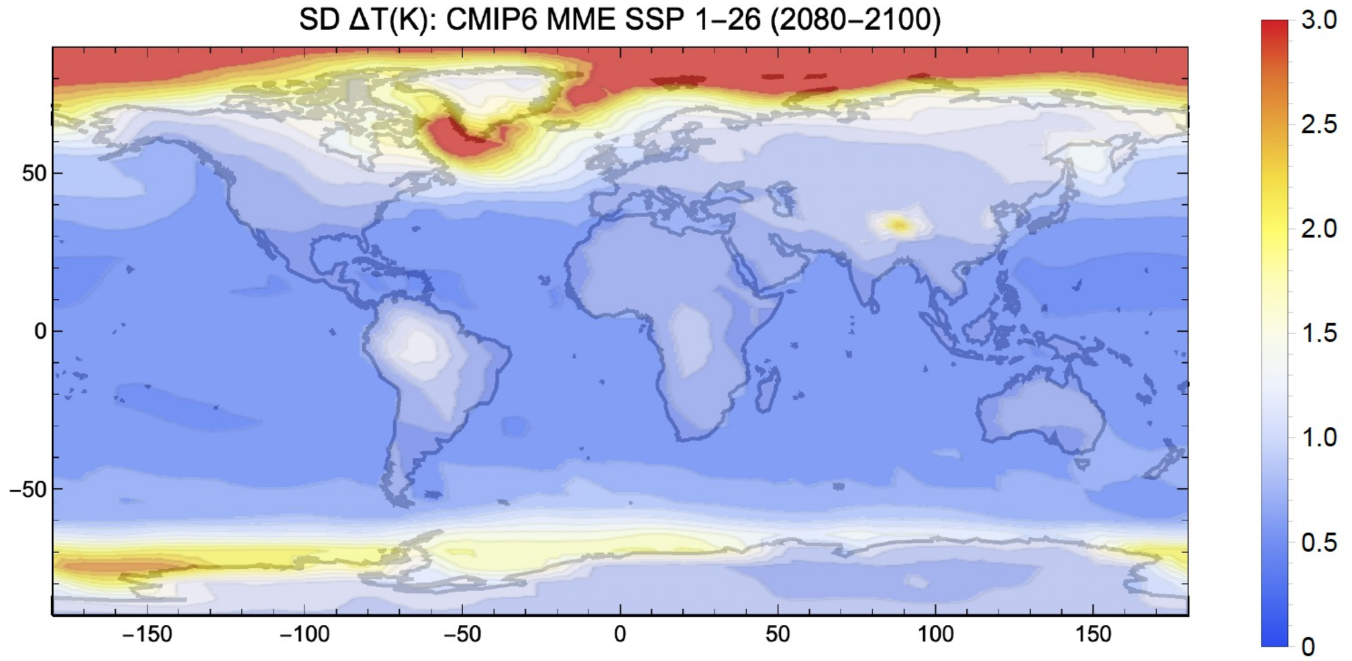
Climate Projections: SSP 1-26



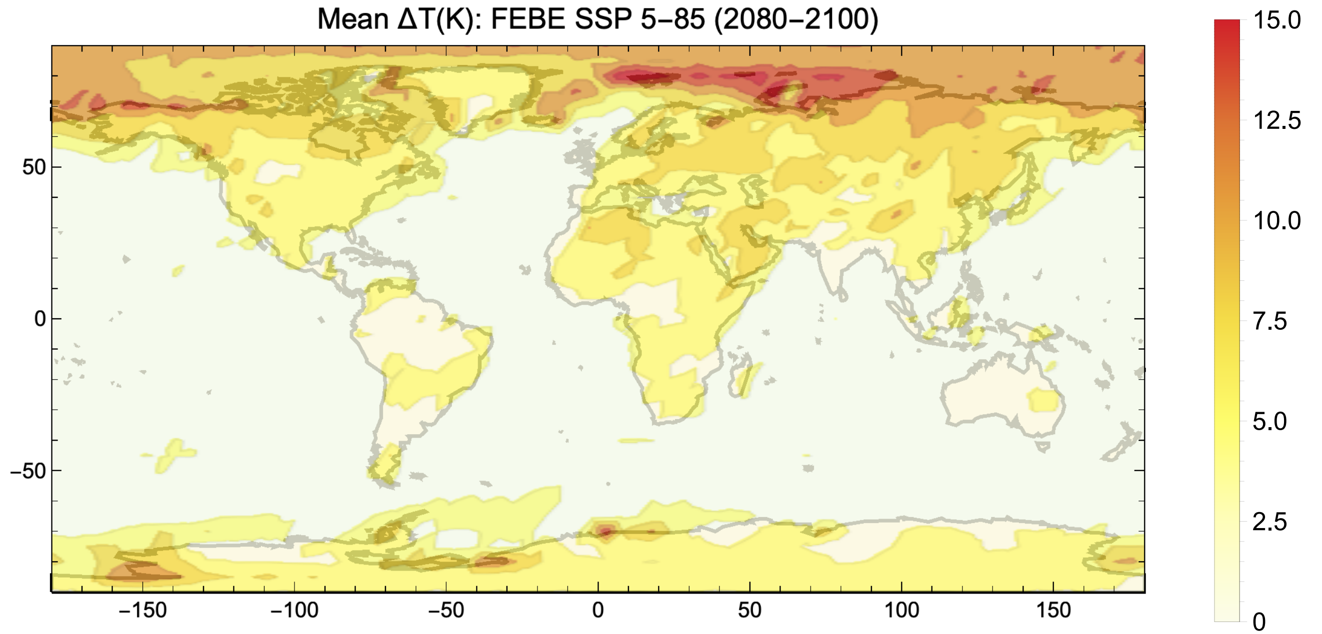


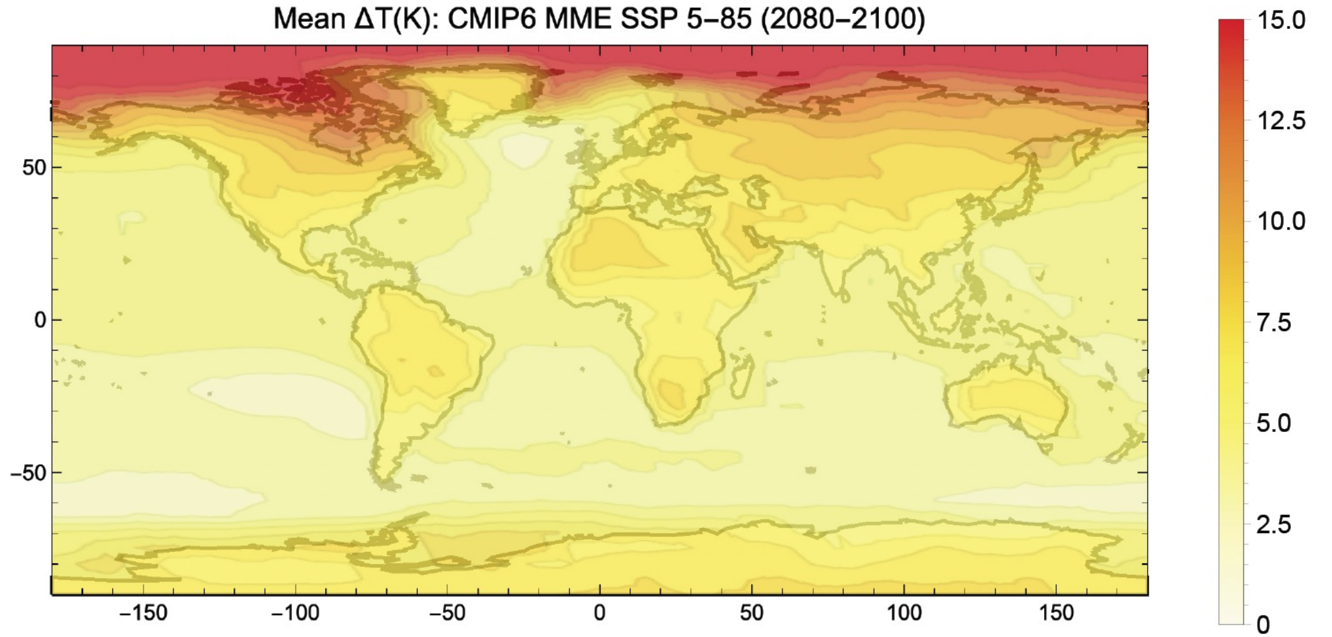
Climate Projections SD: SSP 1-26



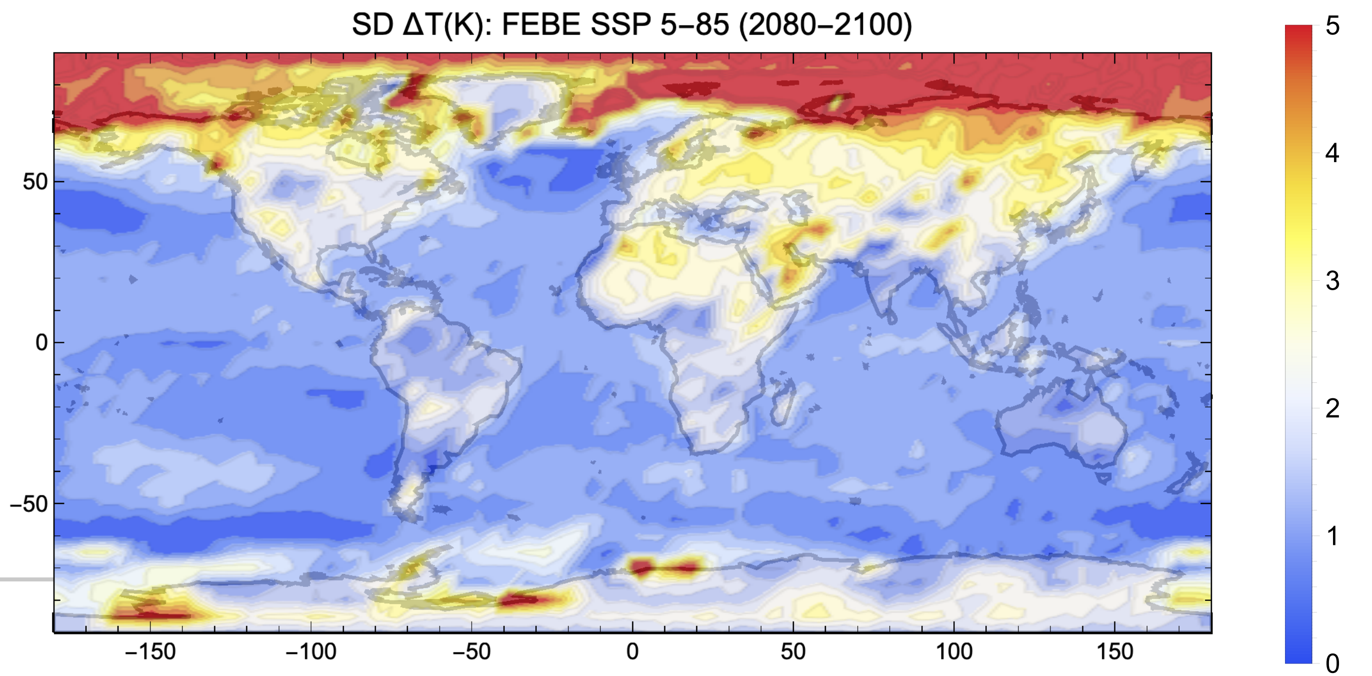


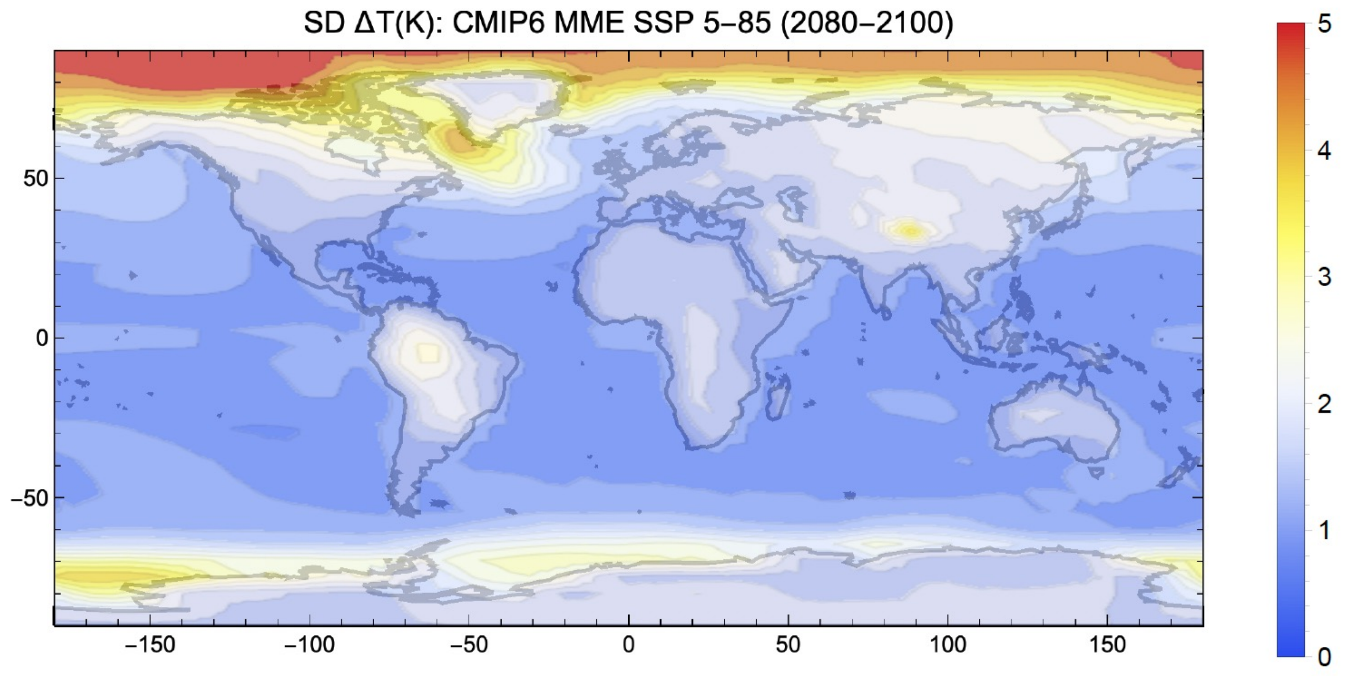
Climate Projections: SSP 5-85





Climate Projections SD: SSP 5-85





Future Work

- Investigate generational differences between CMIP models by calibrating using the historical runs of the CMIP models in order to perform a feedback analysis.
- Updating our parameter estimates from calibrations on GCMs allows for GCM-FEBE hybrid projections.
- The regional FEBE can be expanded to allow for pixel-to-pixel interaction through energy transfers.
- Investigate non-linear feedbacks such as the ice-albedo feedbacks.
- Extensions to precipitation at global and regional scales since the FEBE model is consistent with space-time scaling processes in historical precipitation data.

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Thank you

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