





# Stochastic transport in an idealized ocean-atmosphere coupled system

Long Li1

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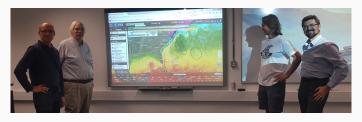
Joint work with Etienne Mémin<sup>1</sup>, Bertrand Chapron<sup>2</sup> and Noé Lahaye<sup>1</sup>



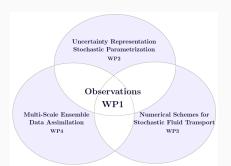
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<sup>&</sup>lt;sup>2</sup> LOPS - Ifremer Brest France

## Stochastic Transport in Upper Ocean Dynamics (STUOD)



https://www.imperial.ac.uk/ocean-dynamics-synergy/



"Our approach accounts for transport on scales that are currently unresolvable in computer simulations, yet are observable by satellites, drifters and floats."

Flow under location uncertainty (LU)

$$d\boldsymbol{X}_{t} = \boldsymbol{u}(\boldsymbol{X}_{t}, t) dt + \underbrace{\sum_{m} \boldsymbol{\xi}_{m}(\boldsymbol{X}_{t}, t) dB_{t}^{m}}_{\text{unresolved}}$$

References: [Bauer et al., 2020a, Li, 2021]

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• Stochastic transport of tracer

$$\mathbb{D}_{t}\Theta := \mathrm{d}_{t}\Theta + \left(\mathbf{u}^{*}\,\mathrm{d}t + \sum_{m} \mathbf{\xi}_{m}\,\mathrm{d}B_{t}^{m}\right) \cdot \nabla\Theta - \underbrace{\frac{1}{2}\,\nabla\cdot\left(\sum_{m} \mathbf{\xi}_{m}\mathbf{\xi}_{m}^{\mathsf{T}}\nabla\Theta\right)\mathrm{d}t}_{\text{eddy diffusion}} = 0$$

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• Pathwise conservation of moments

$$\mathbb{D}_t \Theta^p = 0, \quad d_t \int_{\Omega} \Theta^p \, d\mathbf{x} = 0$$

References: [Bauer et al., 2020a, Li, 2021]

# Application to Q-GCM [Hogg et al., 2003]

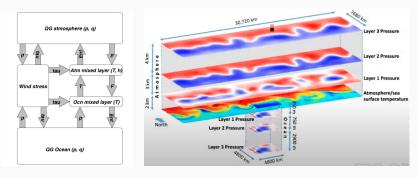


Figure 1: Model illustration from http://www.q-gcm.org

• Idealized midlatitude coupled model emphasizing ocean dynamics

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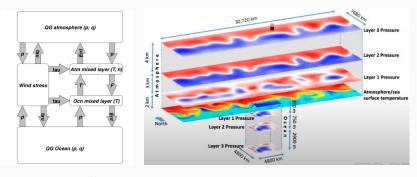


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- Classical Ekman model with wind-current-depended stresses

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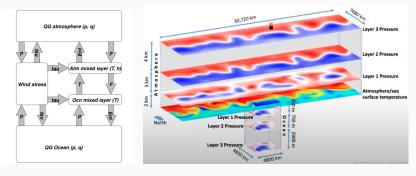


Figure 1: Model illustration from http://www.q-gcm.org

- Idealized midlatitude coupled model emphasizing ocean dynamics
- Classical Ekman model with wind-current-depended stresses
- Our applications: stochastic transport of PV and SST for both ocean and atmosphere

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- Code available on:

```
https://github.com/matlong/qgcm_lu
https://github.com/louity/qgm_pytorch
```

#### Numerical results

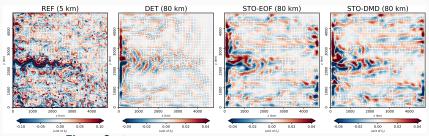


Figure 2: Snapshots of ocean upper layer relative vorticity.

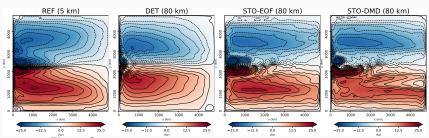


Figure 3: 40-years averages of ocean upper layer streamfunction.

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- More realistic configurations: surface buoyancy conditions [Lapeyre and Klein, 2006], North Atlantic circulation (from FFT to multigrid)
- Ensemble forecasting and data assimilation

Thank you very much!
Questions?

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