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Simulation of pure qP-wave in vertical transversely isotropic (VTI) media

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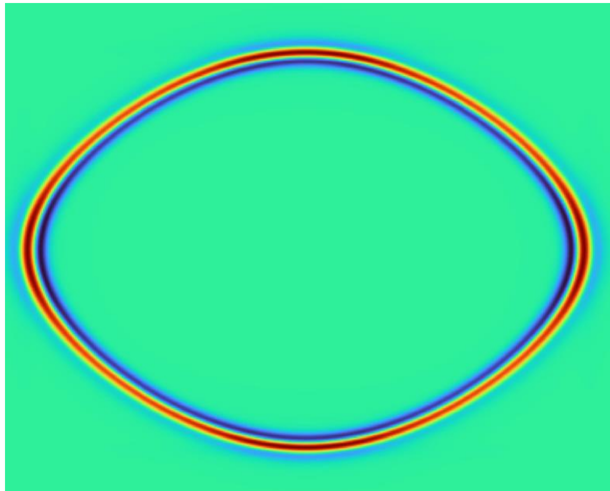


Theory

- P- and S- waves can be separated using Helmholtz decomposition in elastic isotropic media.
- In a 2D elastic VTI medium, the Christoffel equation is given by
$$(G_{P-SV} - \rho v^2 I)A = 0$$
- We have the characteristic equation from the Christoffel wave equation,
$$\det(G_{P-SV} - \rho v^2 I) = v^4 - av^2 + b = 0$$
- Acoustic VTI media can be achieved by
  - Classic: Setting $v_s(\theta = 0) = 0$ (Alkhalifah, 1998; Alkhalifah, 2000), the characteristic equation still has 2 solutions, qP-wave and qSV-wave artefact.
  - New: Setting $v_s = 0$ with $b = 0$, then $(v^2 - \tilde{a})v^2$=0 (Stovas, Alkhalifah, & Waheed, 2020), there is no propagating qSV-wave.

$$v_p(\theta)^2 = A_{11}\sin^2\theta + A_{33}\cos^2\theta - \frac{2\eta A_{11}A_{33}\sin^2\theta\cos^2\theta}{(1+2\eta)A_{33}cos^2\theta + A_{11}\sin^2\theta(1+2\eta\sin^2\theta)}$$

$$A_{mn} = \frac{C_{mn}}{\rho}$$

Numerical solution

- qP-wave equation in (ω, k_1, k_3)

$$(-1i\omega)^2 \hat{P} = -\omega^2 \hat{P} = -A_{11}k_1^2\hat{P} - A_{33}k_3{}^2\hat{P} + \frac{2\eta A_{11}A_{33}(k_1^4k_3^2 + k_1^2k_3^4)}{A_{11}(1+2\eta)k_1^4 + A_{33}(1+2\eta)k_3^4 + (A_{11} + A_{33}(1+2\eta))k_1^2k_3^2}\hat{P}$$

Spatial derivatives - wavenumber domain.
Time derivatives - finite-difference.
A linearization for the anellipticity term is necessary for heterogenous media.

$$1ik_j = \frac{\partial}{\partial x_j}$$

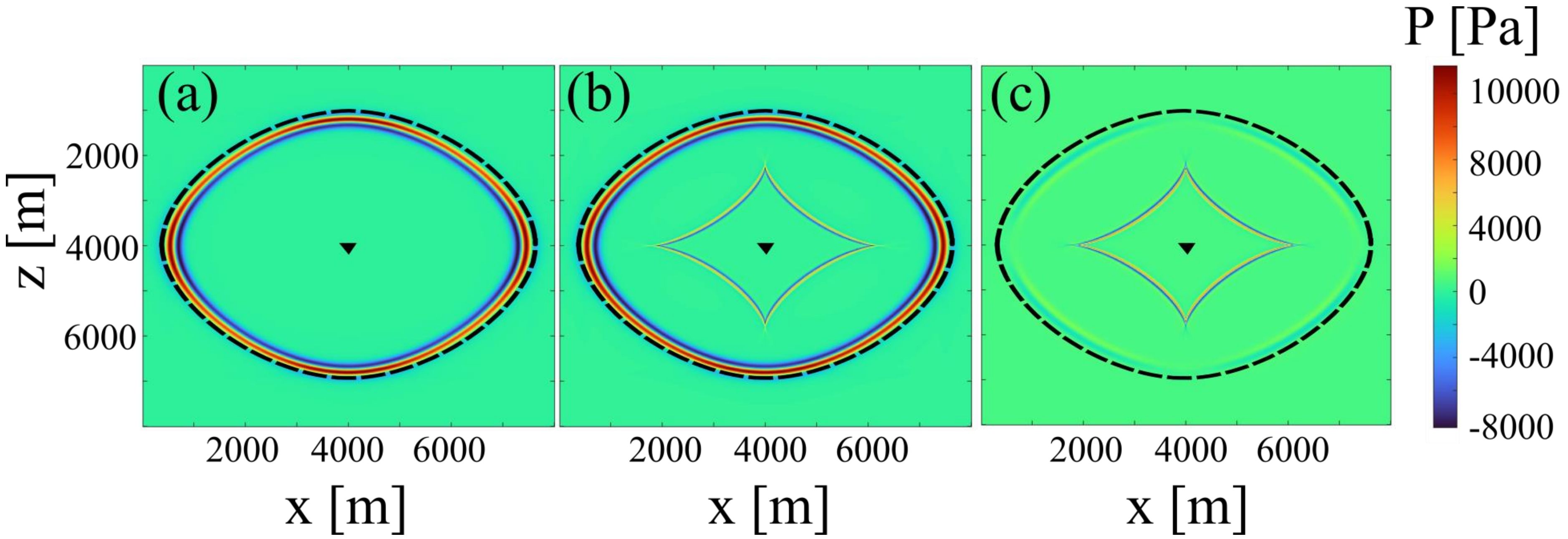
- Time-domain pseudospectral method for $\mathrm{P}(t, x, z)$

$$P_{i,j,l+1} = \delta t^2(-A_{11}\mathbf{F}^{-1}(k_1^2\mathbf{F}(P_{i,j,l})) - A_{33}\mathbf{F}^{-1}(k_3^2\mathbf{F}(P_{i,j,l})) + \mathbf{F}^{-1}(\frac{2\eta A_{11}A_{33}(k_1^4k_3^2 + k_1^2k_3^4)}{A_{11}(1+2\eta)k_1^4 + A_{33}(1+2\eta)k_3{}^4 + (A_{11} + A_{33}(1+2\eta))k_1^2k_3^2}\mathbf{F}(P_{i,j,l}))) - P_{i,j,l-1} + 2P_{i,j,l}$$

- Boundary condition: Hybrid scheme for absorbing periodization by pseudospectral method (Liu & Sen, 2010).
    - Weighted one-way propagator method

Benchmark

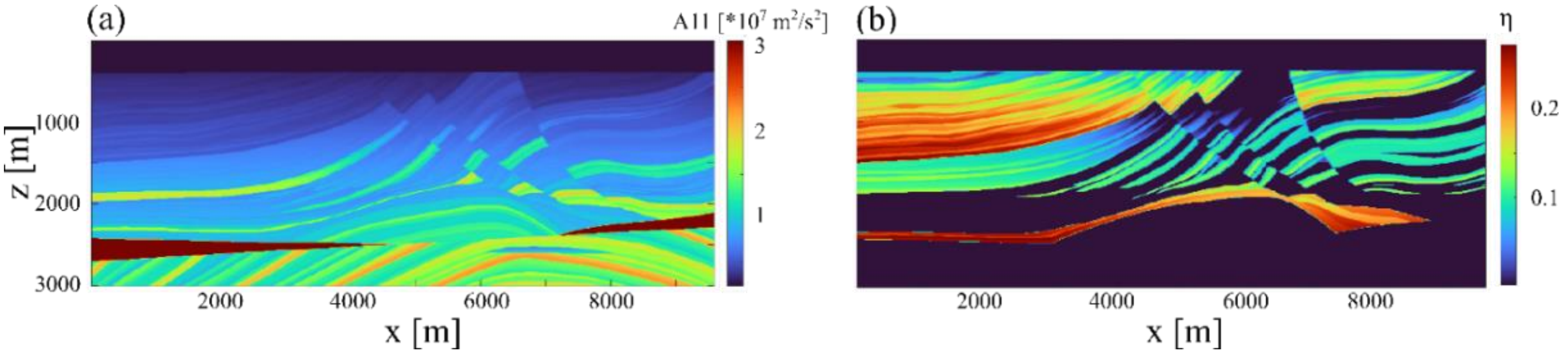
- Wavefield is simulated in a homogenous anisotropic medium, $A_{11} = 1.447 * 10^7\ m^2/s^2,\ A_{33} = 9.57 * 10^6\ m^2/s^2, \eta = 0.341.$



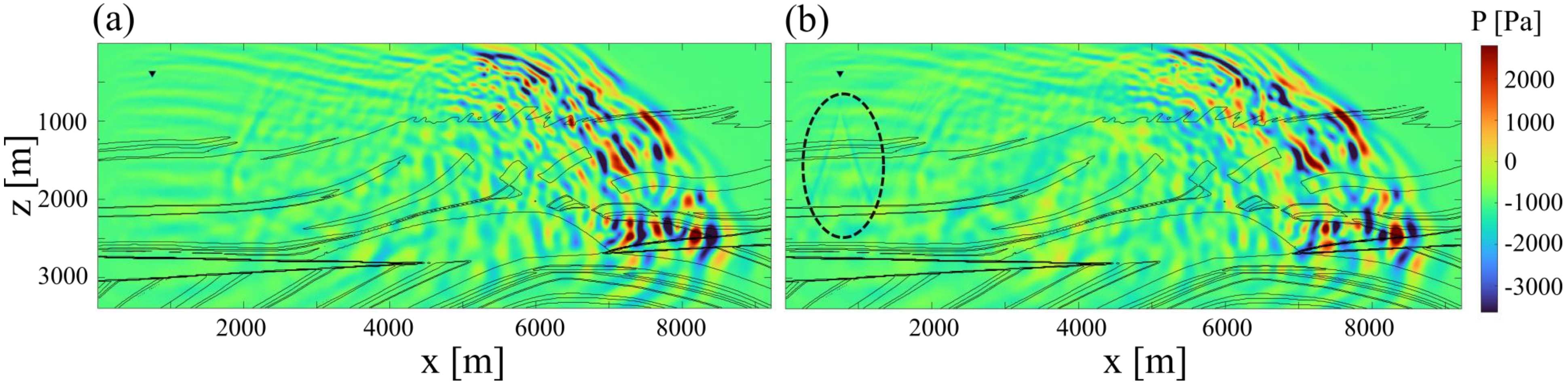
New Classic New - Classic

Analytical group velocity is denoted by the dashed line

Wavefield simulation



VTI Marmousi model



t = 3.2 s

qP-wave (New)

qP-wave (Classic)

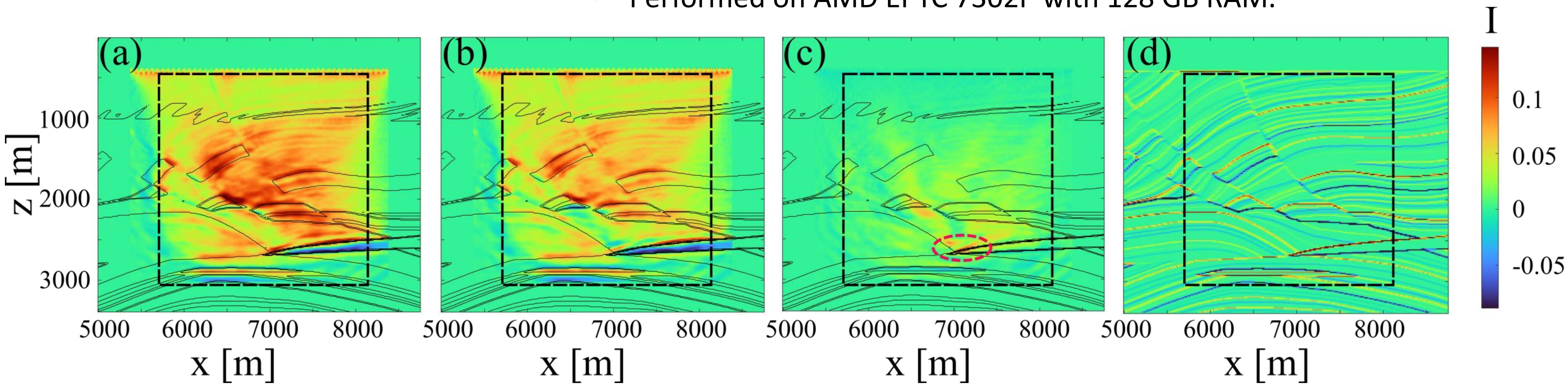
Reverse time migration

qP-wave Equation	Time [s]
New	876
Classic	1125

- Imaging condition

$$I(x,z) \approx \frac{1}{N_s}\sum_{s=1}^{N_s}\frac{\int_t P_{up}(x,z,t,s)P_{down}(x,z,t,s)dt}{\int_t P_{down}(x,z,t,s)^2 dt}$$

- 49 Receivers were placed evenly from 387.5 m to 3387.5 m in the x-direction with z = 387.5 m.
- Sources were shot every 125 m from 387.5 m to 3387.5 m in the x-direction with z = 387.5 m.
- Ricker wavelet 20 Hz.
- The simulation time is 3 s with 3000 time steps. Grid dimension: 201 * 180.
- Performed on AMD EPYC 7302P with 128 GB RAM.



New | Classic | New - Classic | True vertical reflectivity

Conclusions

- The new acoustic approximation for VTI media is not suffered from S-wave artefact with a high preservation of qP-wave.
- The wavefield simulation on VTI Marmousi model shows that the classic approximation generates a strong S-wave artefacts in the high anellipticity region.
- In our RTM application, however, both the classic and the new acoustic approximations show robust results. However, the new approximation has the performance advantage.

References

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