

# Simulating radio propagation in ice with the Parabolic Equation method:

## Applications to terrestrial glaciers and ice moons (Enceladus)

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# Enceladus Explorer (EnEx)

## Project Objectives:

- Develop an autonomous melting probe to land safely near an active geyser on Enceladus' south polar region (Tiger Stripes) [4]
- Locate and navigate towards near surface aquifer
- Probe water for microbes

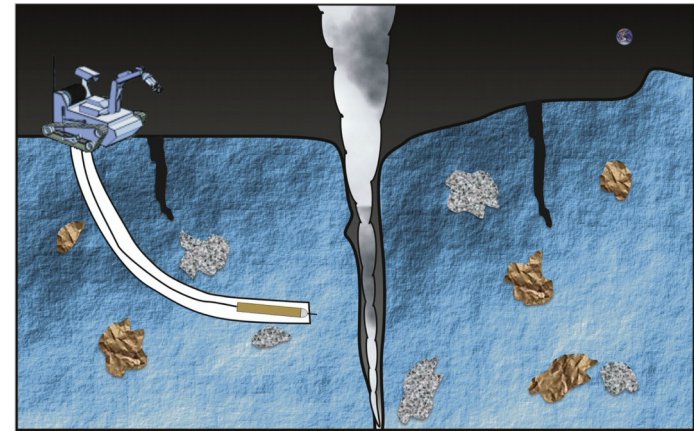
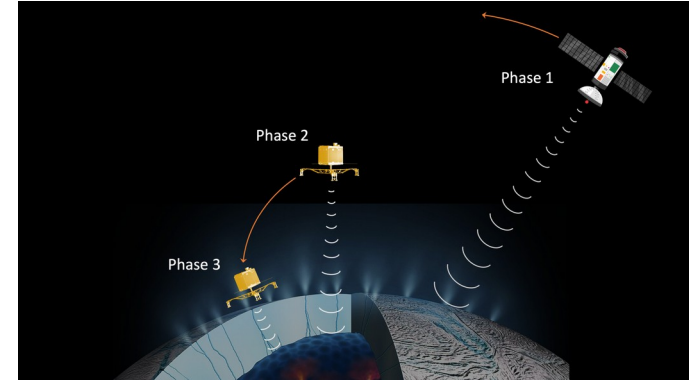
Ice is relatively transparent at Microwave frequencies (0.3 to 2 GHz)

## Radar imaging system:

- Map Enceladus' surface and upper ice crust
- Identification of a safe landing site close to a geyser
- Identification of near-surface aquifers, crevasses and obstacles
- Tracking the melting-probes' position through the ice relative to obstacles'
- Learn about Enceladus' geological history and geophysical processes

**Time of radar echo from water pockets, aquifers depends on surface permittivity profile:**

$$\nabla T(x,y,z) = \epsilon_r(x,y,z) / c^2$$



# Permittivity Model of Enceladus

## South polar terrain is complex

Surface is covered by unconsolidated ice grains from the geyser plumes → 'snow layer'

Depth of layer depends on how long South Polar region has been active ( $< 2$  Myr)[5]

Surface temperature  $\sim 80$  K on average ,  $\sim 140$  K within tiger stripes (near vents) [2]

Ice grains undergo sintering → forming a denser layers

Ice grain types identified by Cassini [1]:

Type I – Salt poor

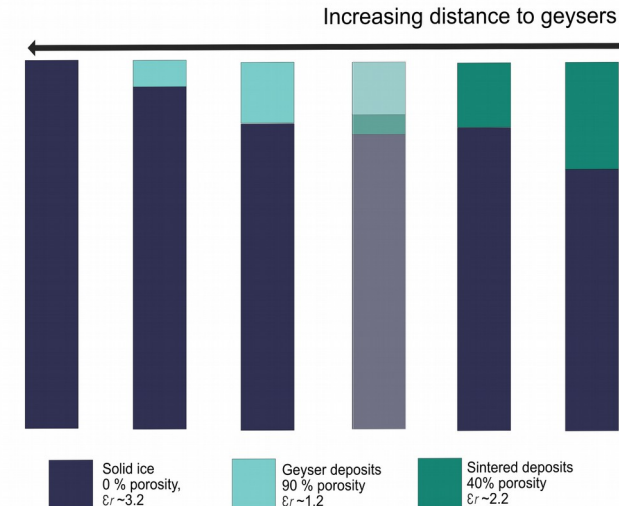
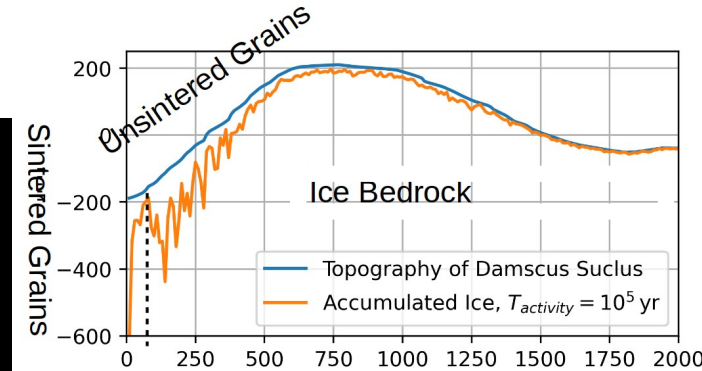
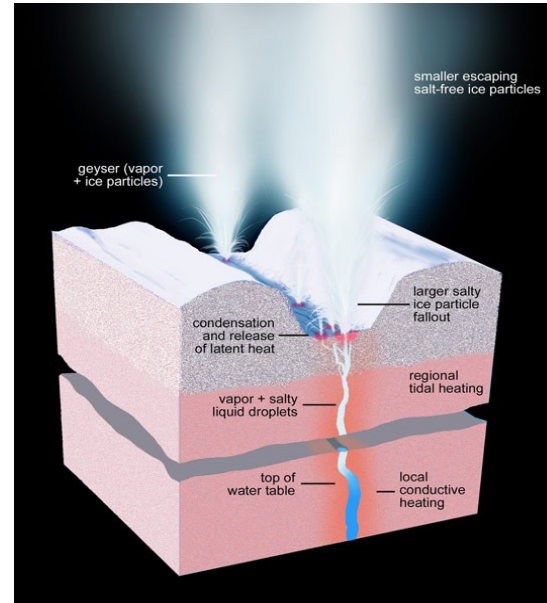
Type II – Organics rich & salt poor

Type III – Salt rich

Type III concentrated near Tiger Stripes → higher conductivity → high attenuation rate

Additional inhomogeneities:

Ice boulders ( $\sim 10$  m), crevasses, meteorites

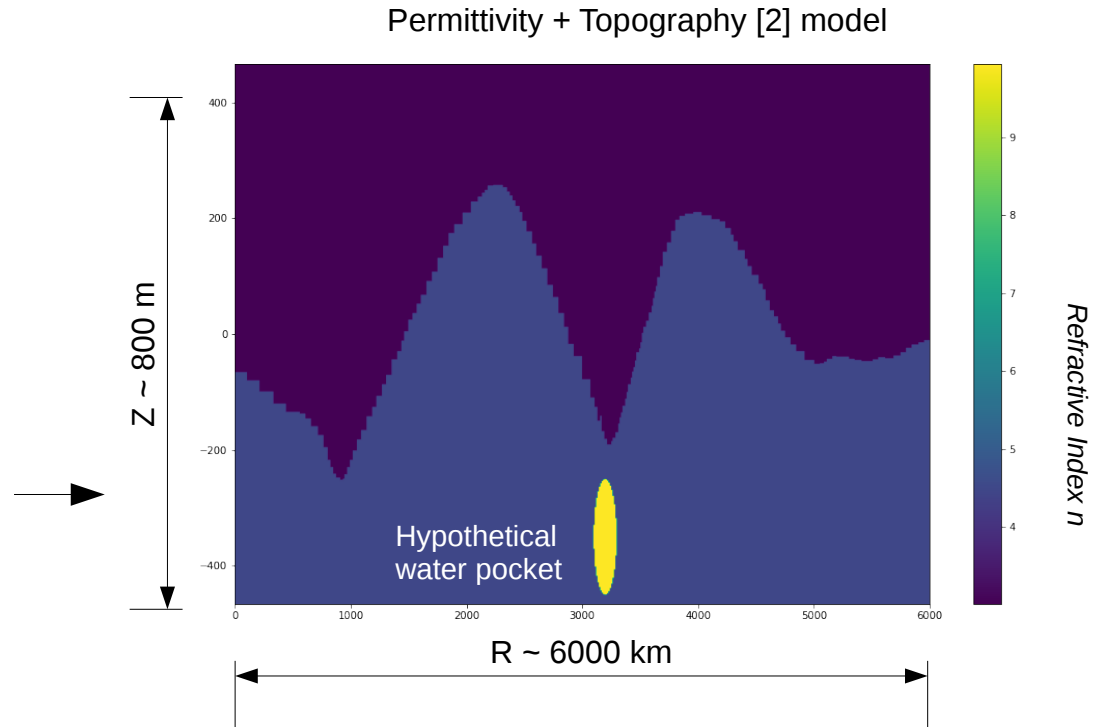
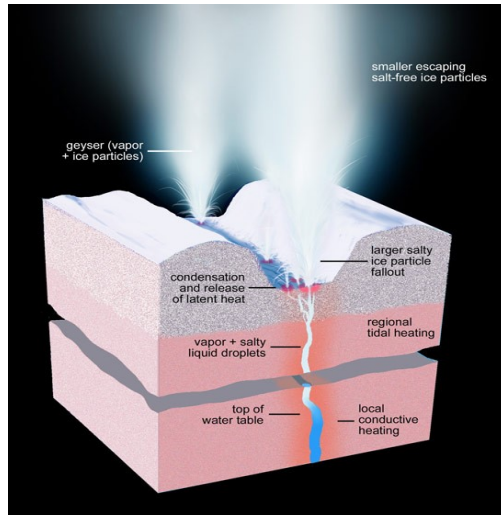


# Simulating radar back-scatter

Accurate radio-propagation simulations needed to predict the radar signature of water pocket given likely Enceladus permittivity models

Enceladus water table at a depth  $\sim 10\%$  of ice crust thickness [7] (5 km – 8 km deep at Enceladus South Pole [6])  $\rightarrow$  water pocket depth 500 m – 800 m

Frequency range:  $100 \text{ MHz} < f < 2000 \text{ MHz}$   
Simulation Dimensions:  $\sim 6000 \text{ m} \times 1000 \text{ m}$



# Parabolic Wave simulations

Approximation of Maxwell's equations in 'paraxial direction' (within 90° of propagation direction)]

PE solved using marching method → E-field at  $x + \Delta x$  solved from previous step  $x$

More efficient than FDTD methods, more accurate than Ray Tracing (RT) methods

paraPropPython is a FOSS PE solver for ice environments written in python [3]  
<https://github.com/prchyr/paraPropPython.git>

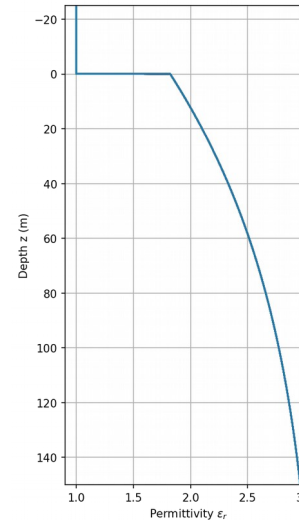
Procedure:

1. Set dimensions
2. Set complex permittivity profile
3. Set source (TX) depth
4. Define source as emitting continuous wave or a pulse
5. Solve PE across entire geometry
6. Sample amplitude at receivers → RX

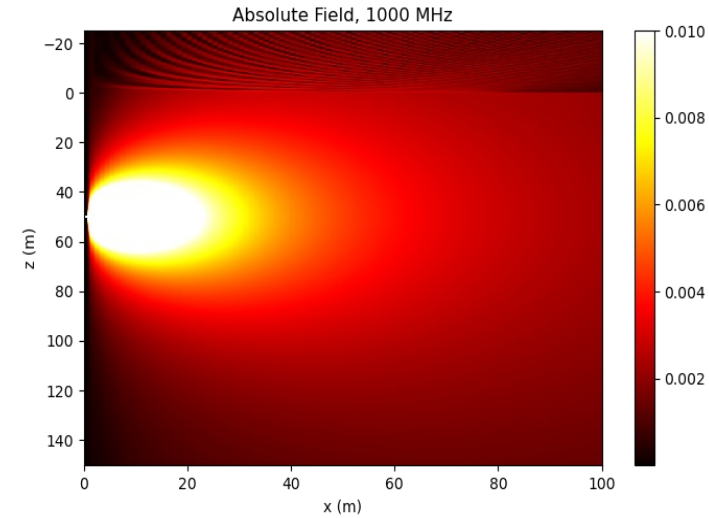
$$E(x + \Delta x) = \exp(-ik_0 \Delta x (1 - Q)) E(x)$$

$$Q_{ice} = \sqrt{1 + \left(\frac{\delta}{\delta z}\right)^2} \frac{1}{k_0^2} + n \sqrt{1 + \frac{1}{n_0^2}} - \sqrt{1 + \frac{n}{n_0}}$$

## Terrestrial South Pole Ice (empirical)



$$\epsilon_r(z) = (A + B e^{-Cz})^2$$



Source  $z = 50$  m,  $f = 1$  GHz.

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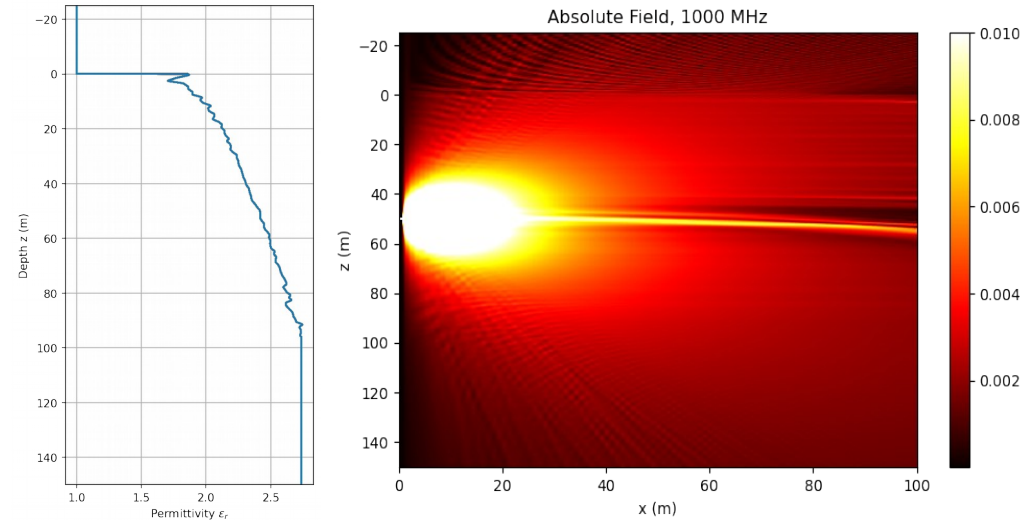
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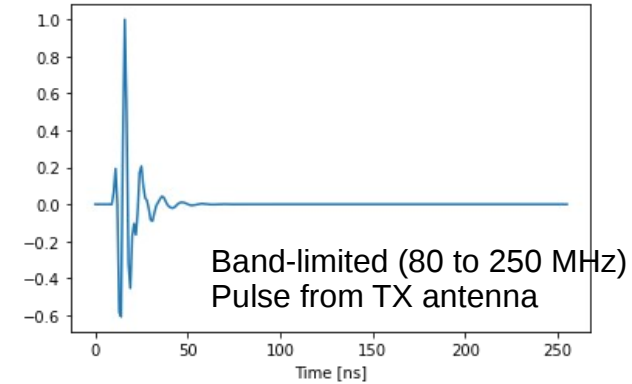
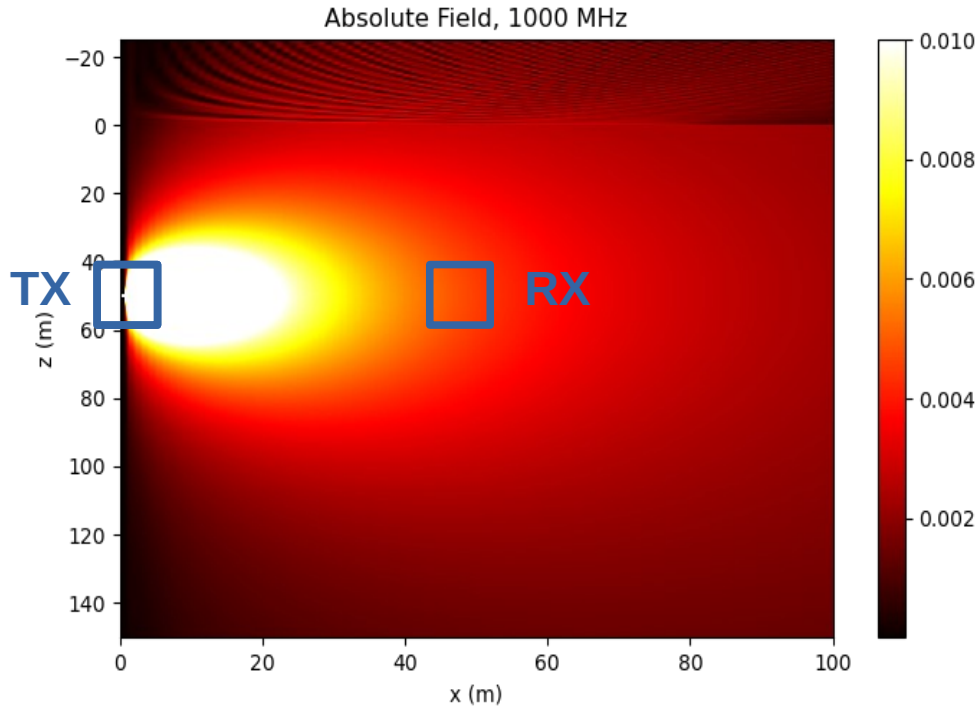
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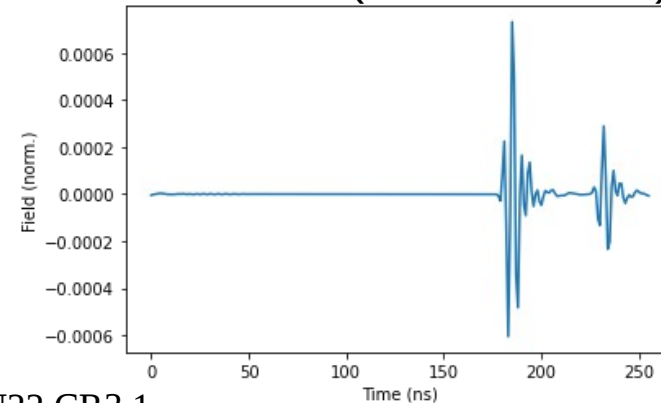


# Parabolic Wave time-domain

Transmitter ( $x = 0, z = 50$ )



Receiver ( $x = 50, z = 50$ )

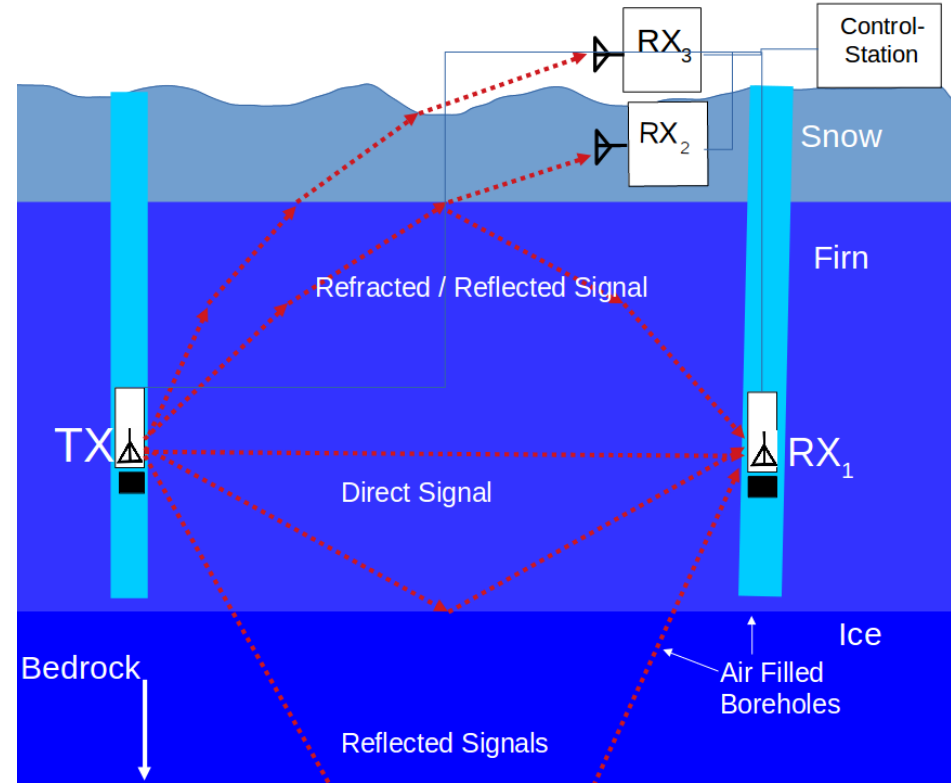


# Applications to Enceladus

PE simulations may be used to radar response to subsurface water pocket

Example on the right → cross borehole transmission through Enceladus ice

- Snow Layer 10 m deep
- Firn Layer 20 m deep
- Water pocket 50 m deep





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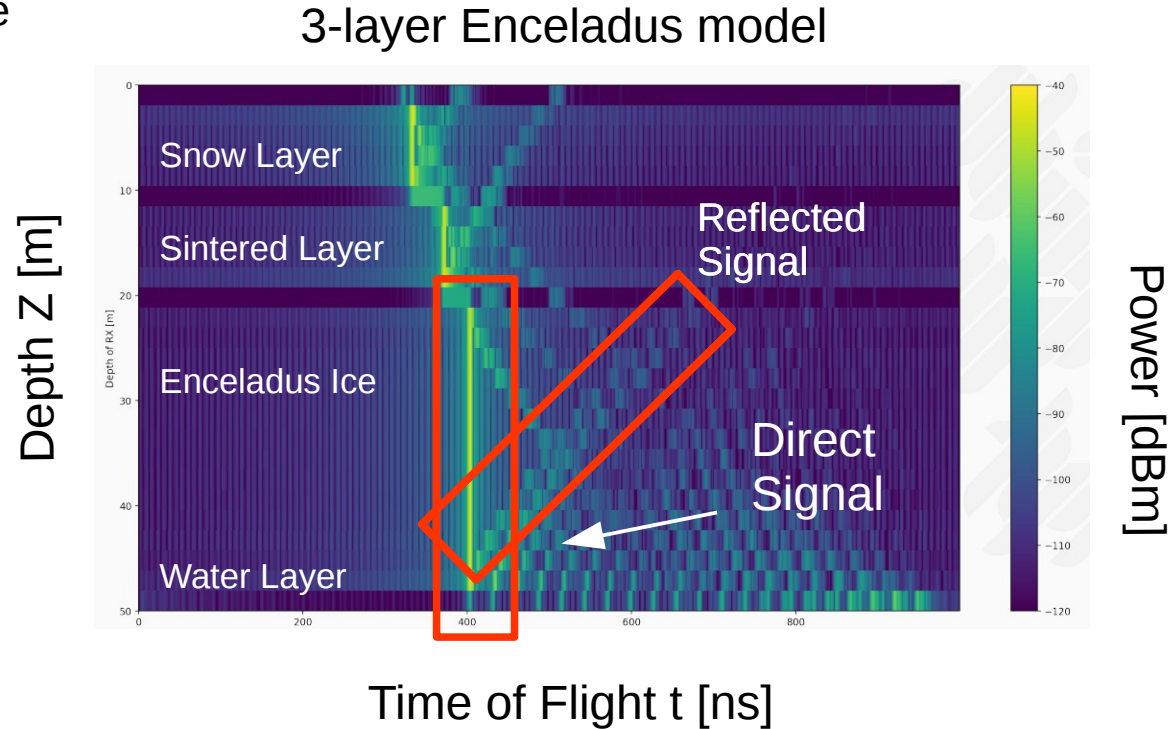
Example on the right → cross borehole transmission through Enceladus ice

- Snow Layer 10 m deep
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## Results:

Simulations shows strong boundary layer reflections from snow-sinter, sinter-ice, ice-boundaries

Strong signal attenuation (assuming Type III particles dominate)  $\alpha \sim 0.8$  dB/m ( $f = 1$  GHz)



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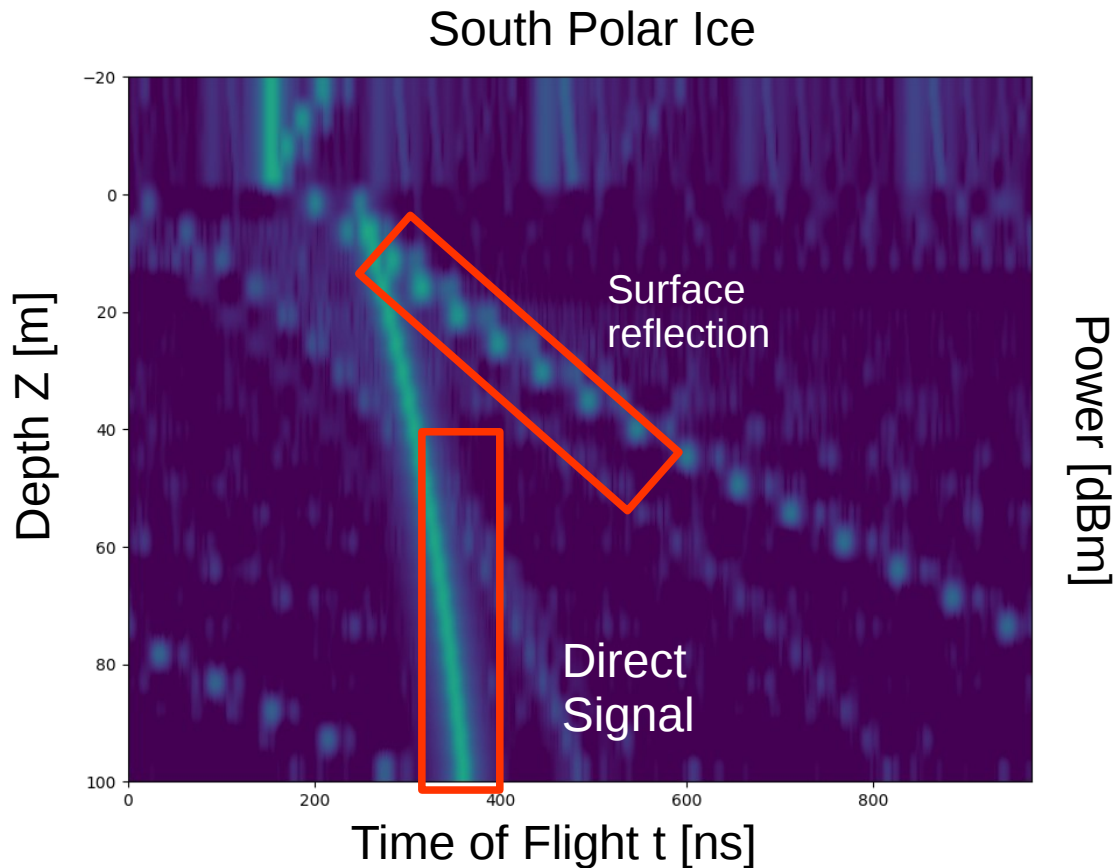
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# Summary

Enceladus ice model: Layer of deposited ice grains in Tiger Stripe region → sintering of ice grains within Tiger Stripes ( $R < 100$  m from source), unconsolidated ice grains outside

Primordial ice beneath it → water pocket ~ 500 m – 1000 m beneath surface → should be accessible to melting probe mission

PE simulations may be used to simulate radar-echo of water pocket while accounting for different ice environment scenarios on icy bodies such as Europa, Mars and Enceladus

Also useful for modelling radio propagation through firn on terrestrial glaciers and ice sheets

## **Future work:**

Verification of PE simulation technique using real-world data → EnEx field campaign in Aletsch Glacier, Switzerland March/April 2022

Perform a comprehensive analysis of radar echo signature of water pockets → using global topography, deposition and temperature data

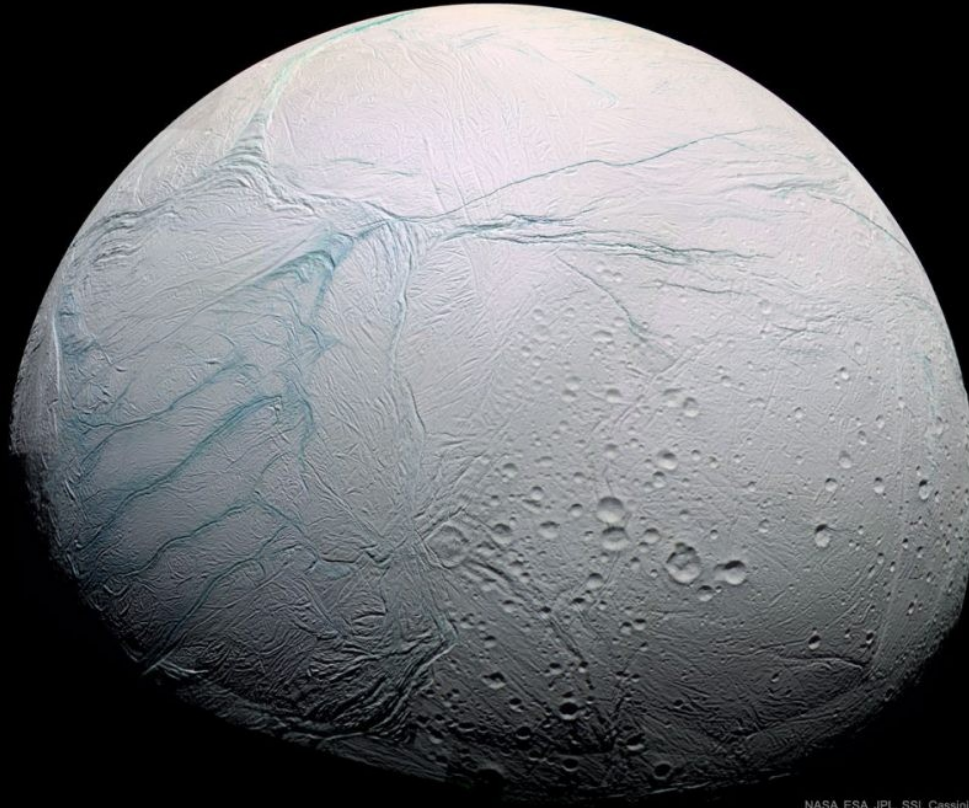
paraPropPython simulation code available at: <https://github.com/prchyr/paraPropPython.git>

**Contact: [kyriacou@uni-wuppertal.de](mailto:kyriacou@uni-wuppertal.de)**

# References:

1. Postberg et al (2018b) ***Plume and Surface Composition of Enceladus, Enceladus and the Icy Moons of Saturn***
2. Abramov et al. (2015), ***Temperature of vents within Enceladus Tiger Stripes, LPSC 2015***
3. Prohira et al (2021) ***Modeling in-ice radio propagation with parabolic equation methods***
4. Konstantinides et.al. (2015), 'Enceladus explorer (ENEX): A lander mission to probe subglacial water pockets on Saturn's moon enceladus for life'
5. Porco et al. (2006) ***Cassini Observes the Active South Pole of Enceladus***
6. Kang and Flierl ***Spontaneous formation of geysers at only one pole on Enceladus's ice shell***
7. Spencer (2018) ***Plume Origins and Plumbing (Ocean to Surface), Enceladus and the Icy Moons of Saturn***

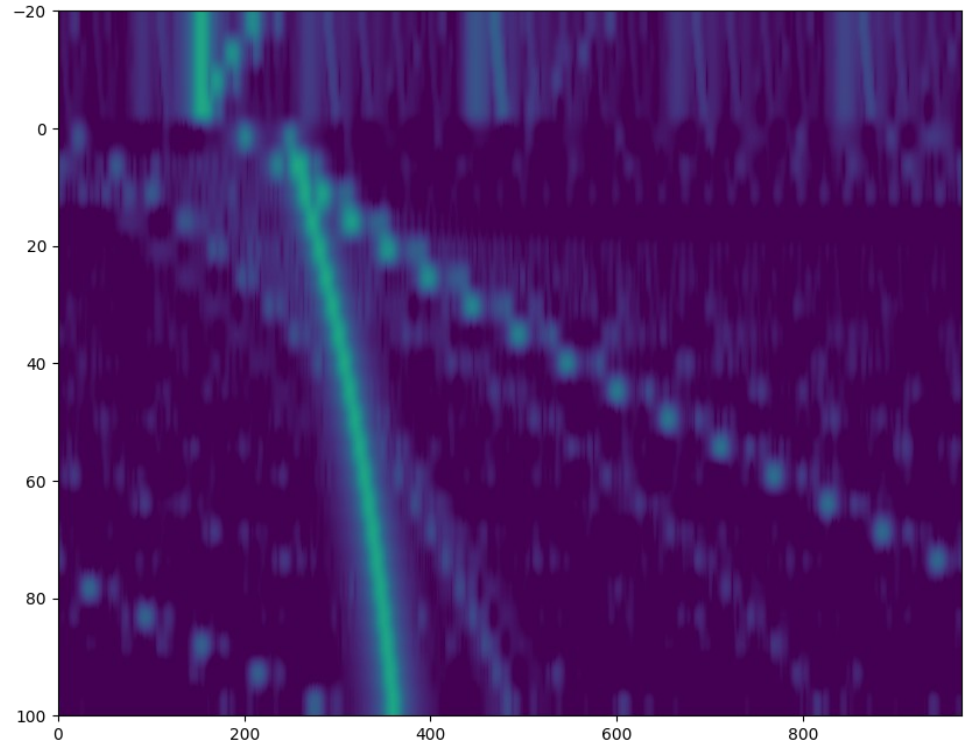
# Backup Slides



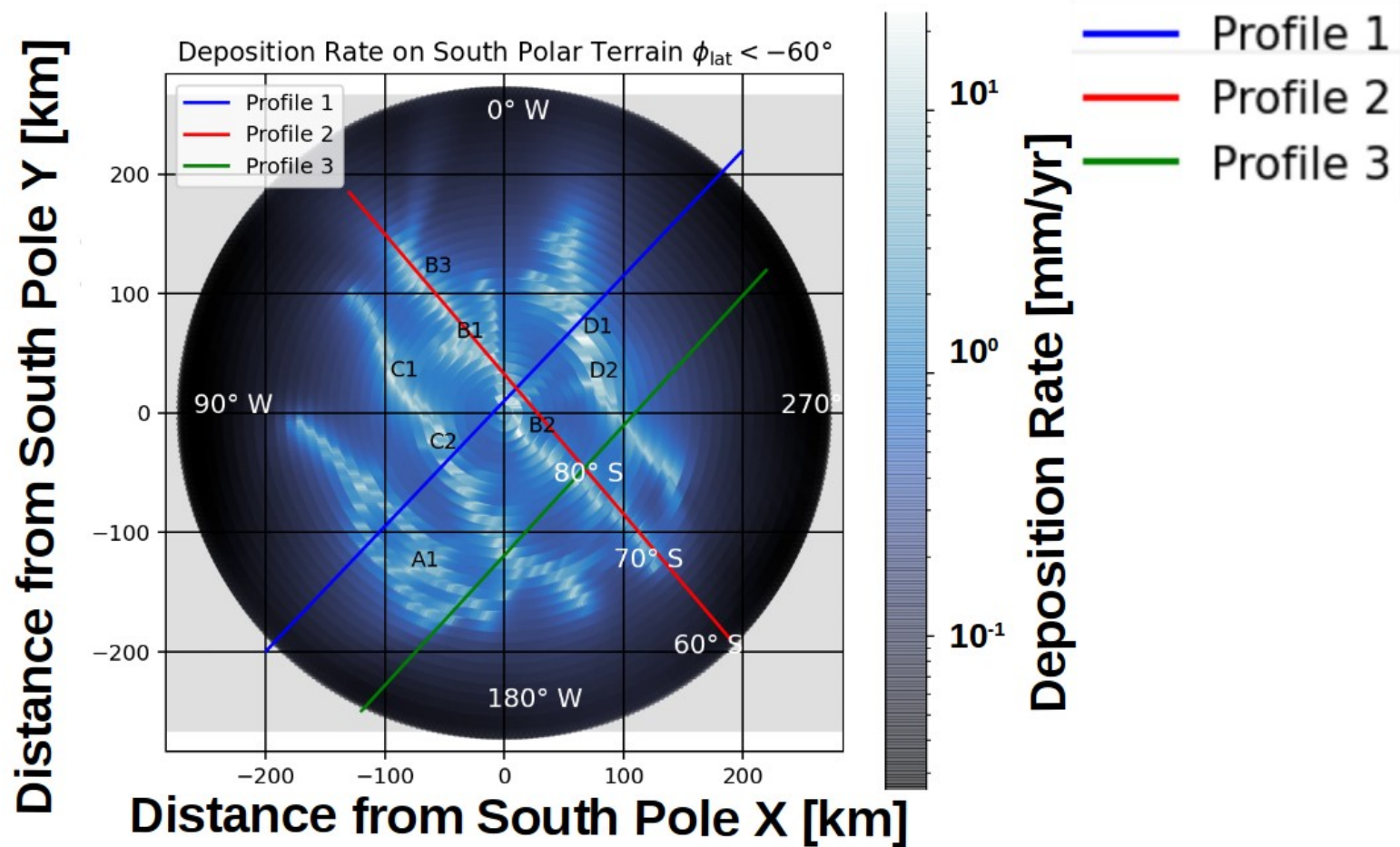
NASA, ESA, JPL, SSI, Cassini

# B-Scans – Example with South Poalr Ice

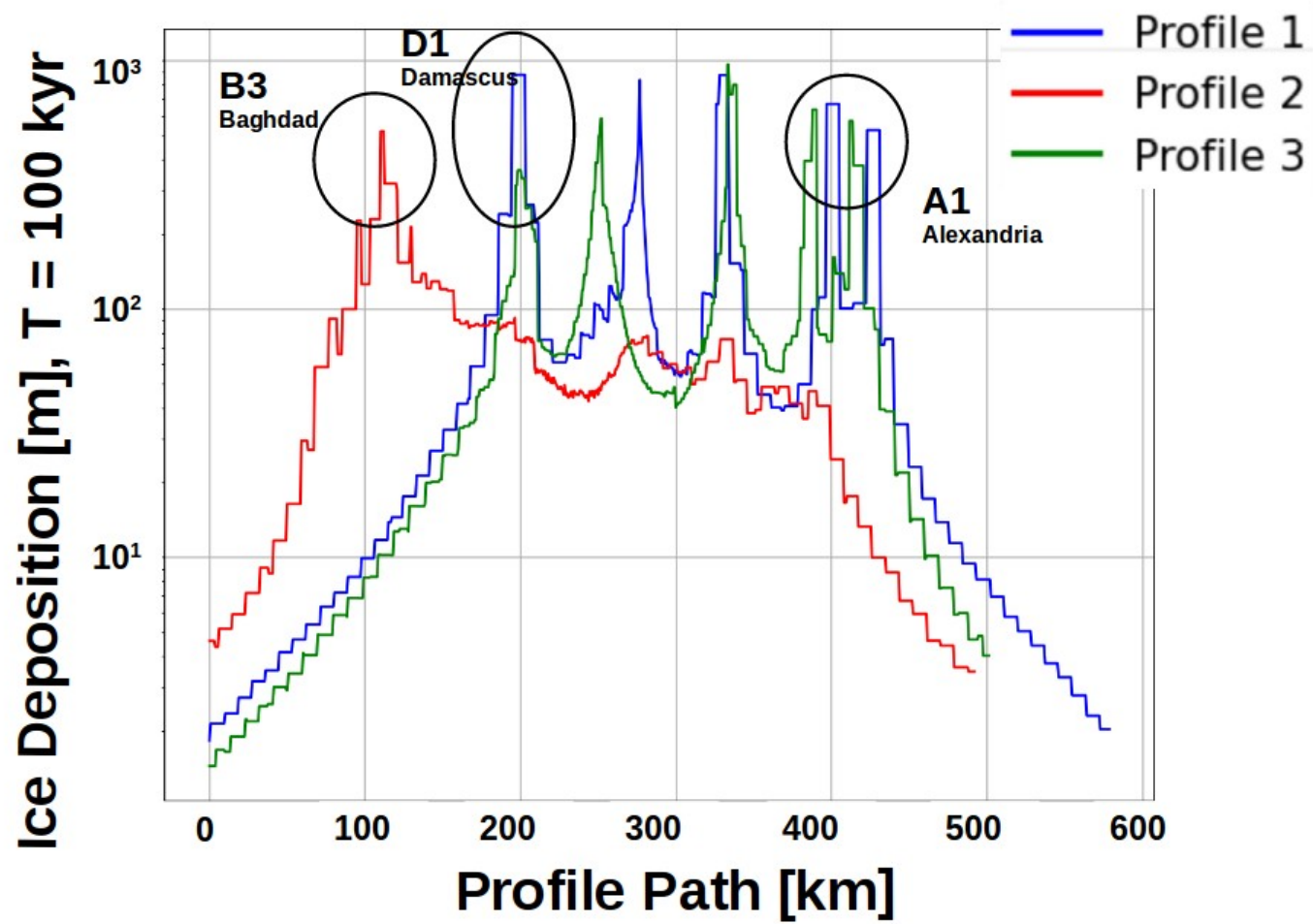
- A B-Scan is an image formed by combining signals measure at multiple different antenna positions
- Two antennas (TX and RX) at constant range  $R = 50$  m move down through the ice
- TX produces a pulse recorded by RX
- Images shows two features, direct propagating signal, and a reflection signal (from the ice surface interface)



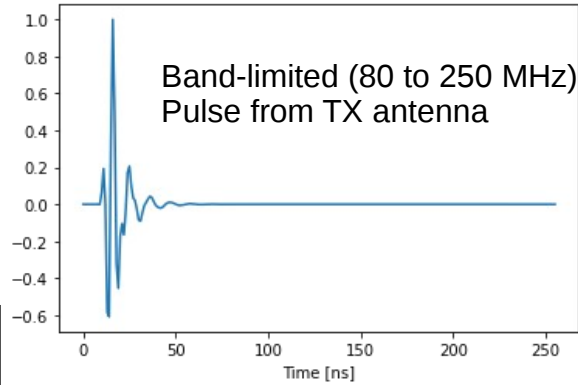




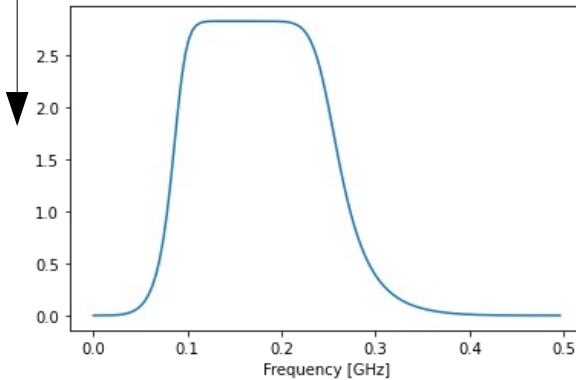




## Transmitter ( $x = 0, z = 50$ )



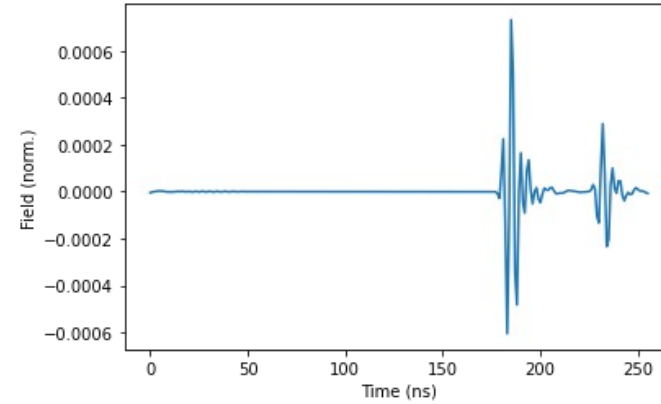
1. Fourier transform



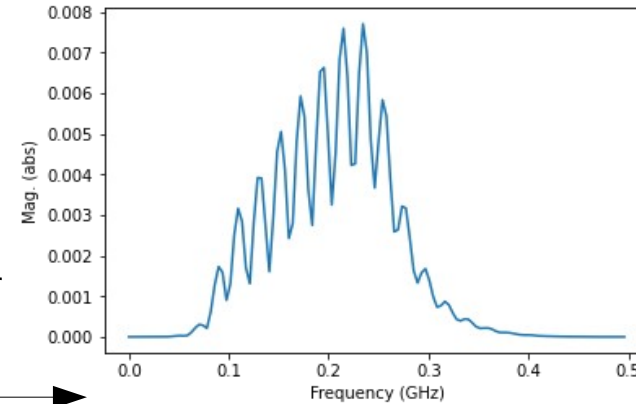
2. Run PE solver  
Define source amplitude at each frequency using the pulse spectrum

3. Sample field amplitude at RX for each frequency

## Receiver ( $x = 20, z = 50$ )

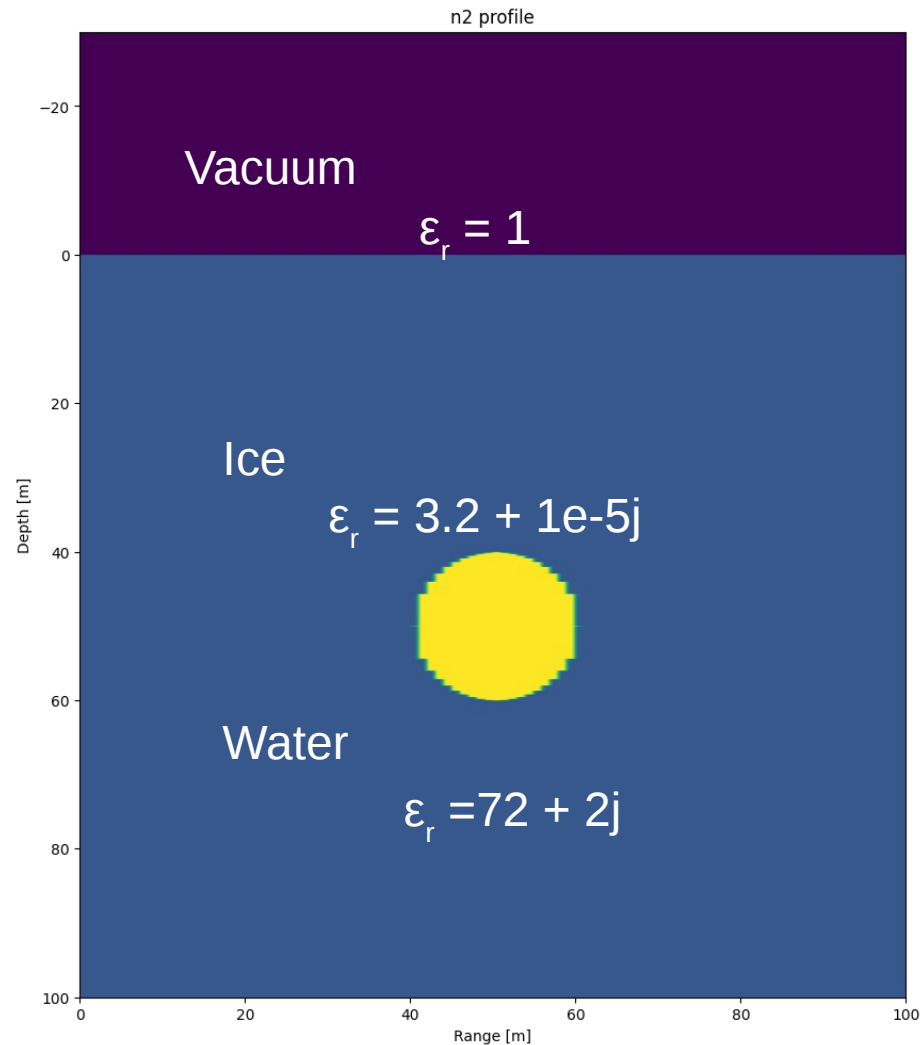


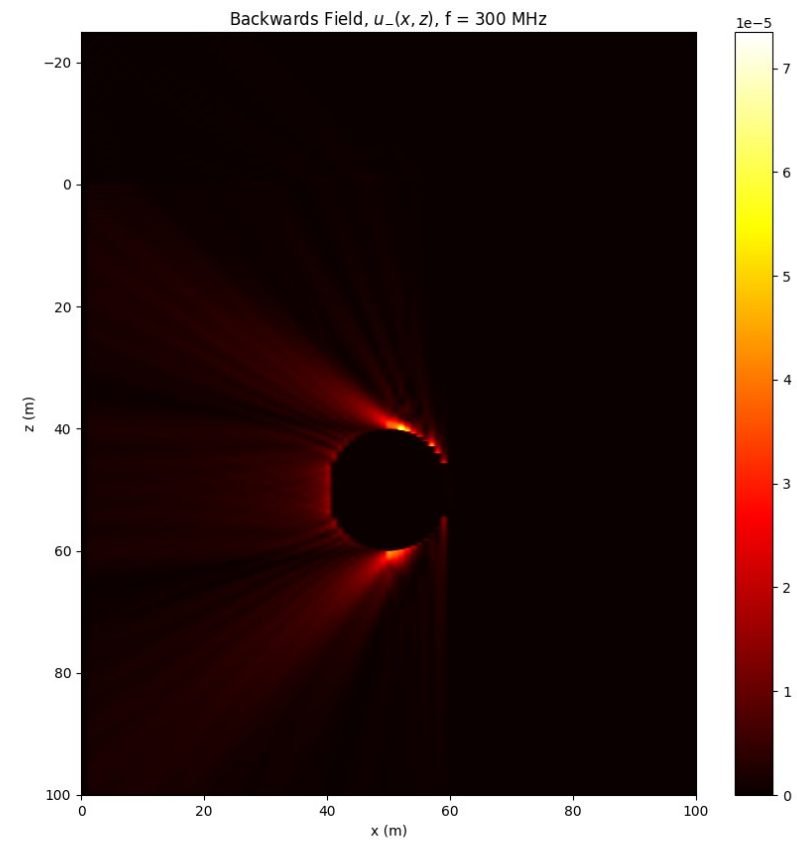
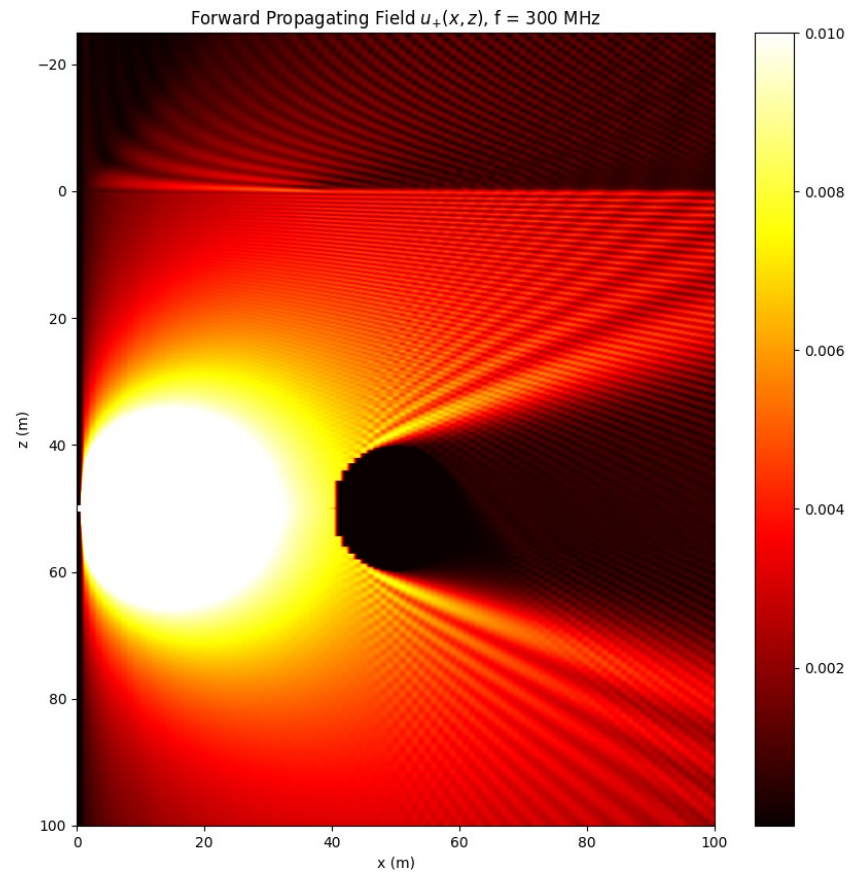
4. Inverse Fourier transform spectrum to reconstruct receiver waveform



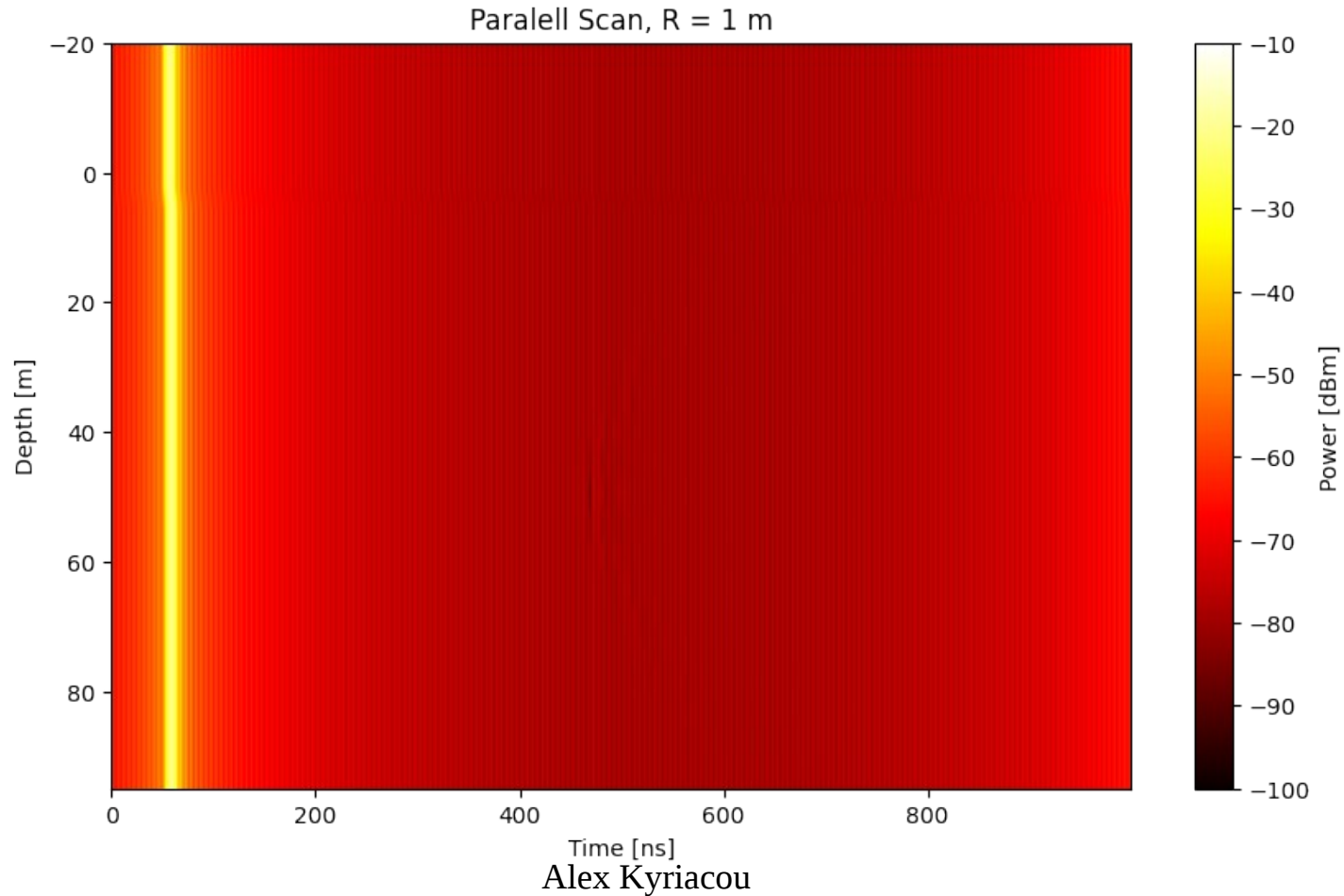
## Single Frequency Simulation

Simple example, with  
homogenous ice and a  
circular water pocket (radius  
of 10 m)

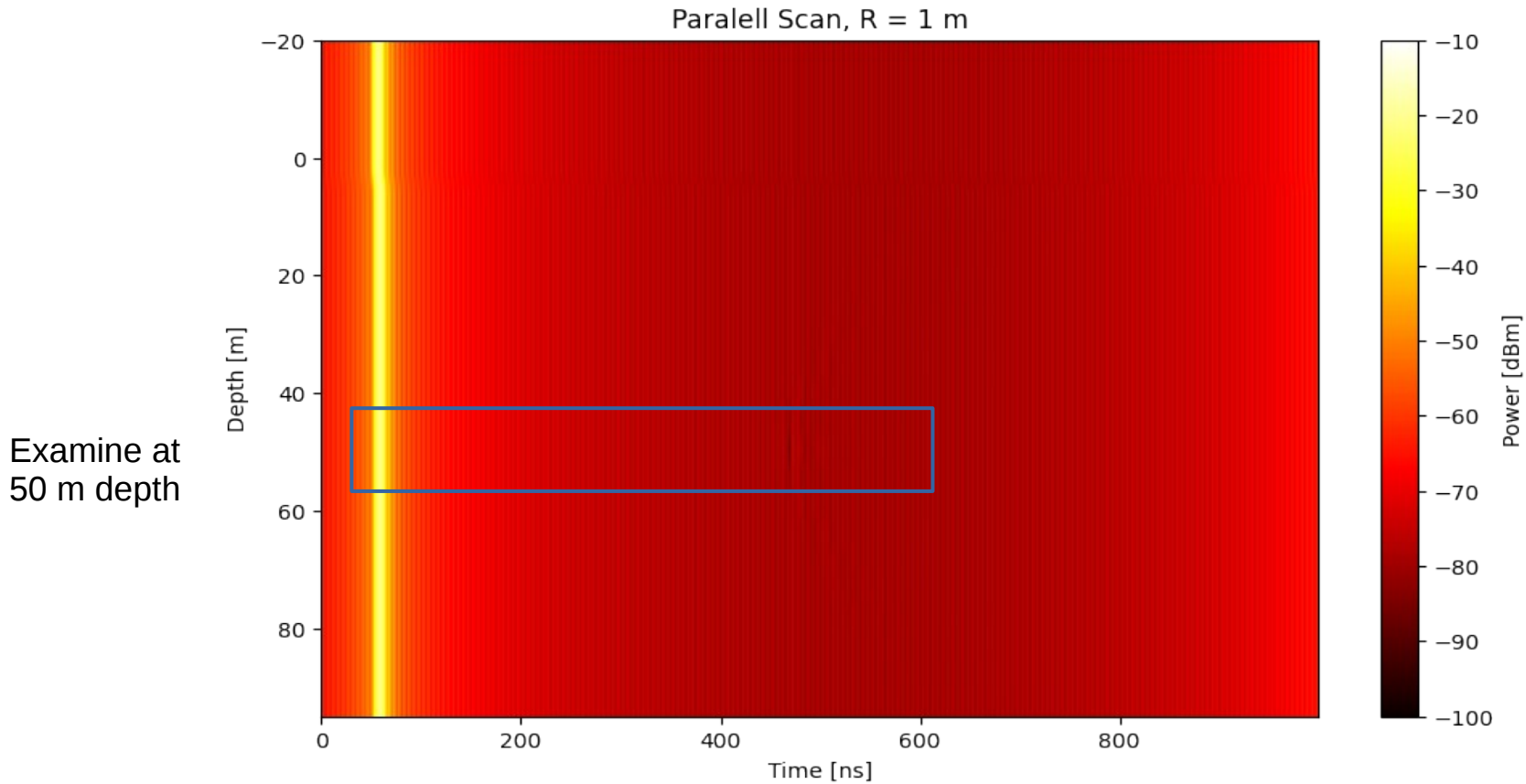




# Result



# Result



# Example:

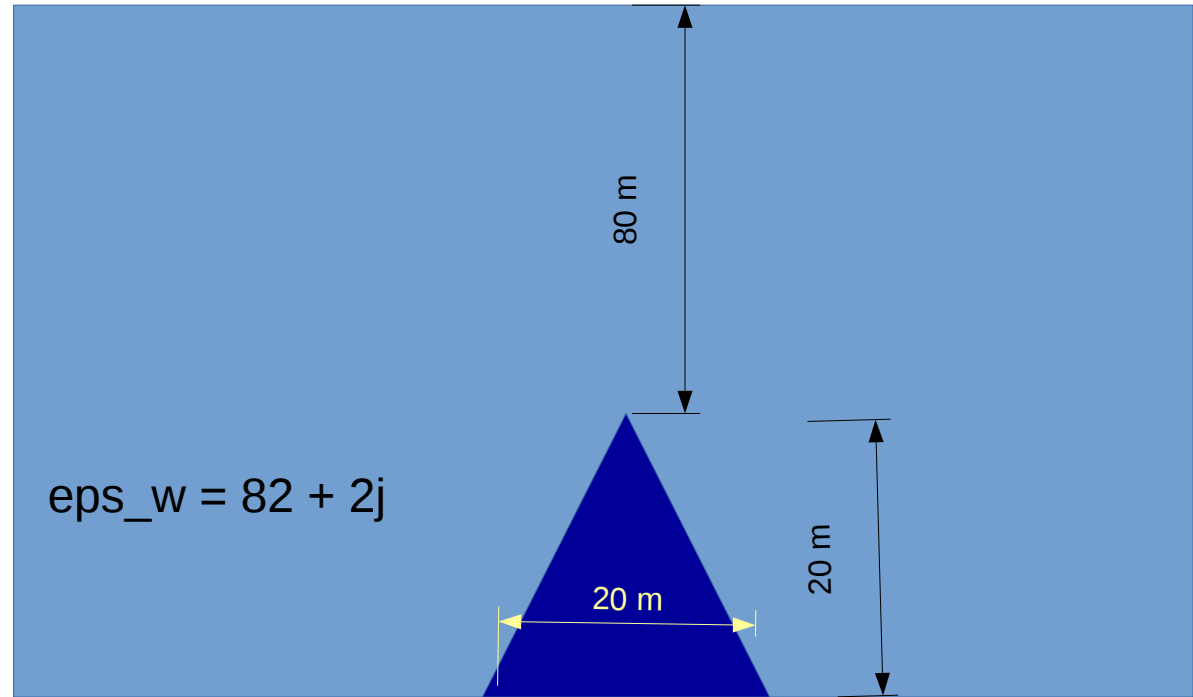
n-prof: South Pole analytical profile

Single-source depth: 20 m

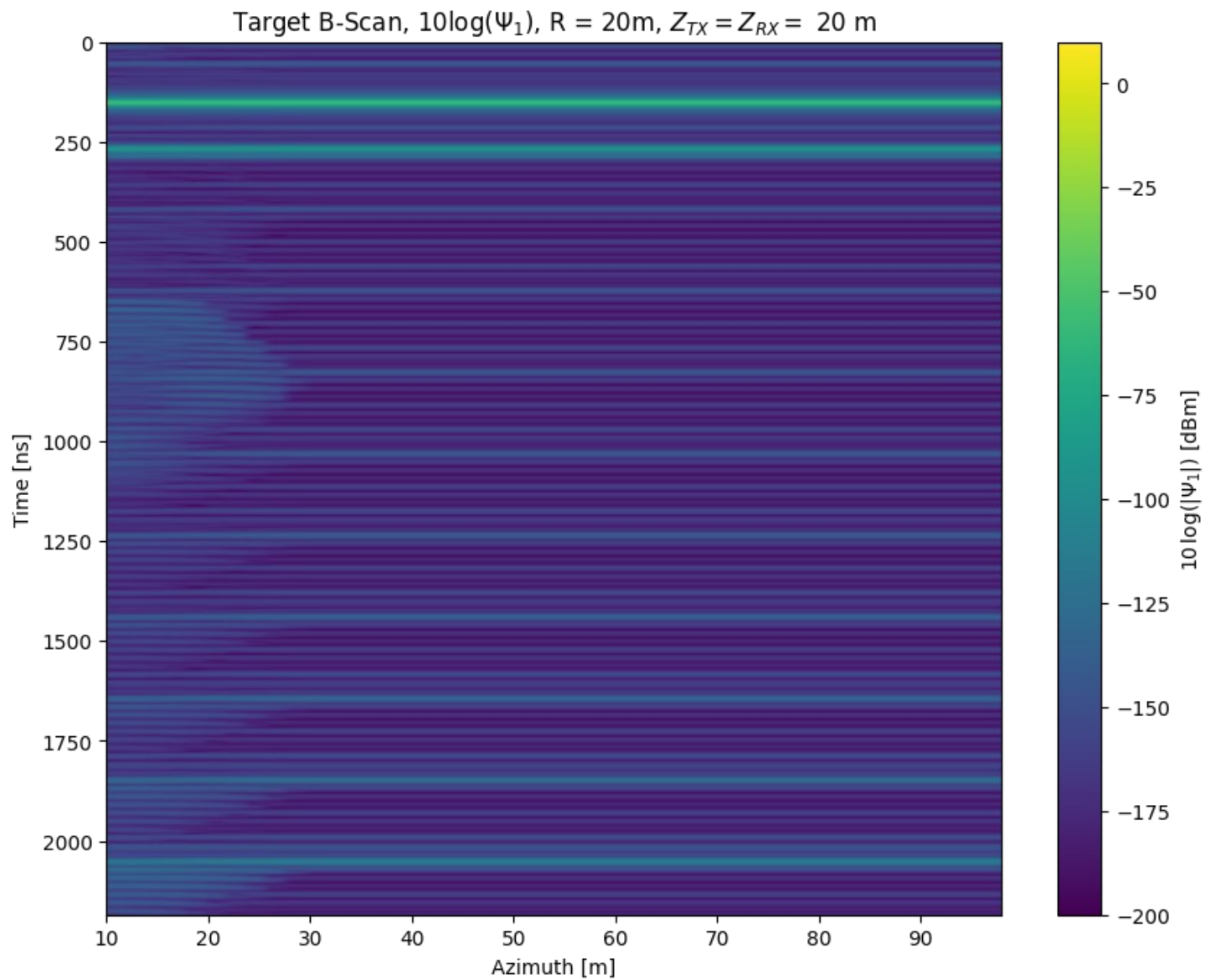
„Mexican hat“ pulse → central frequency 500 MHz

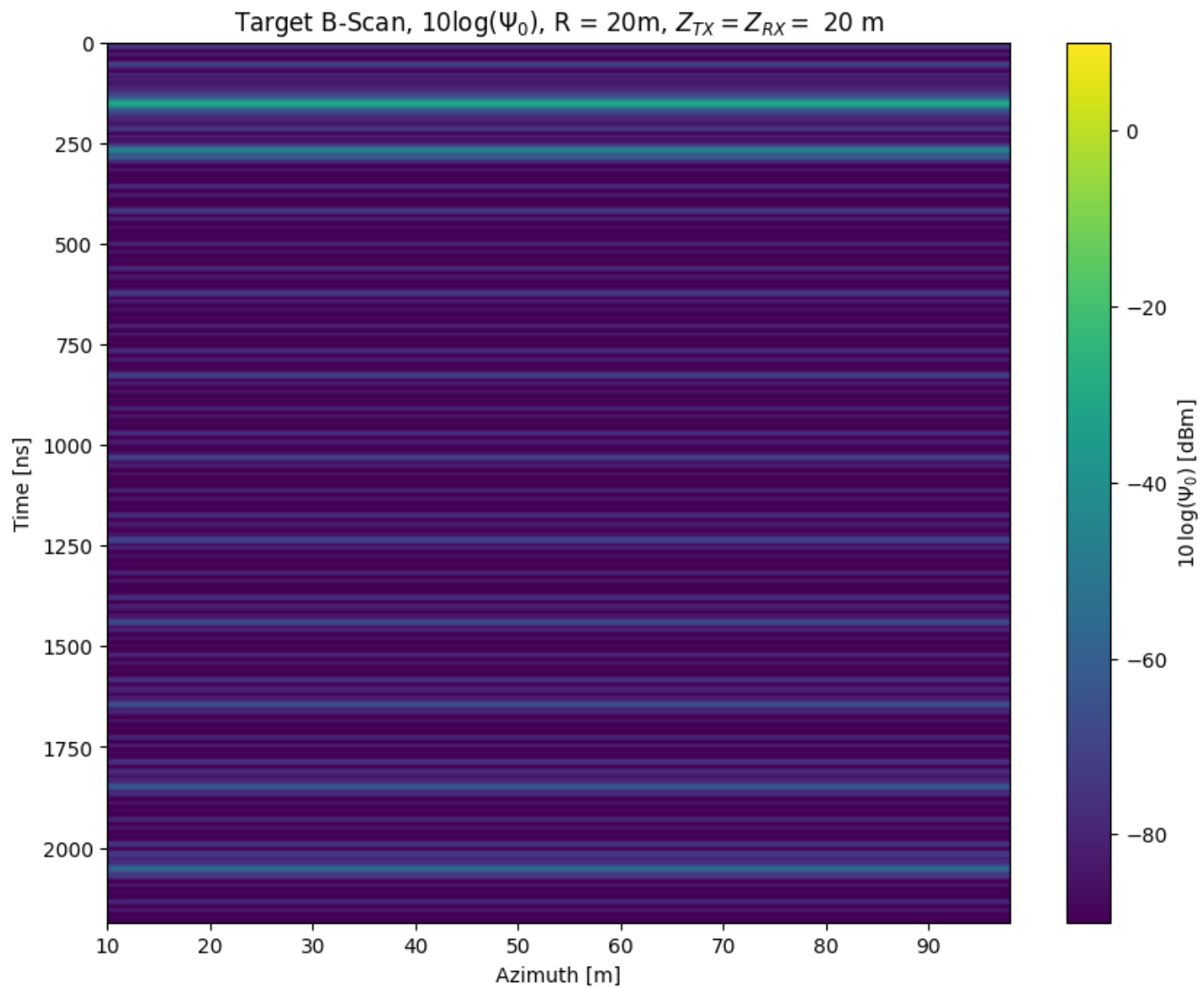
Ice dimensions: 100 m deep, 100 wide

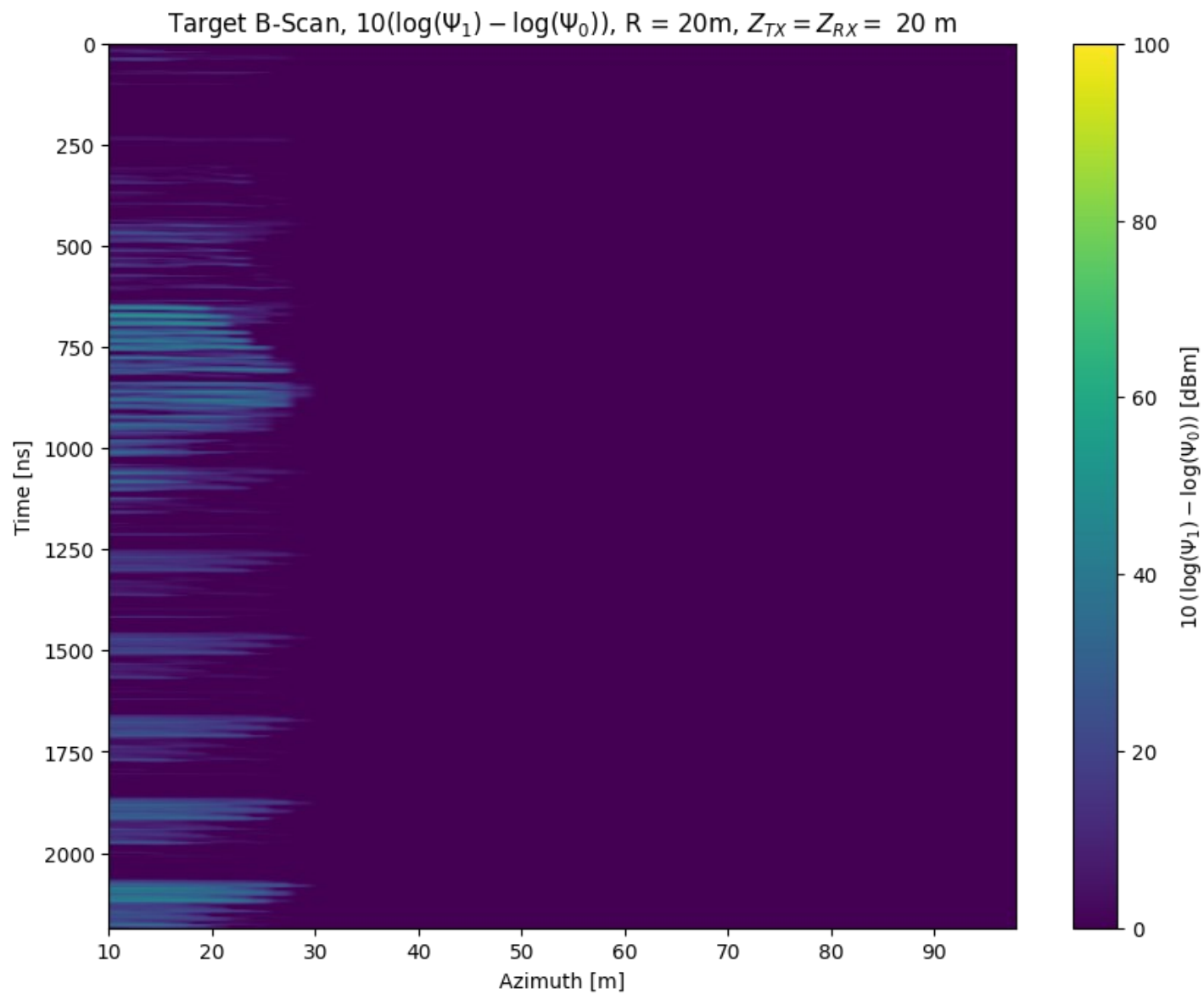
Aquifer dimensions:  
80 m below surface

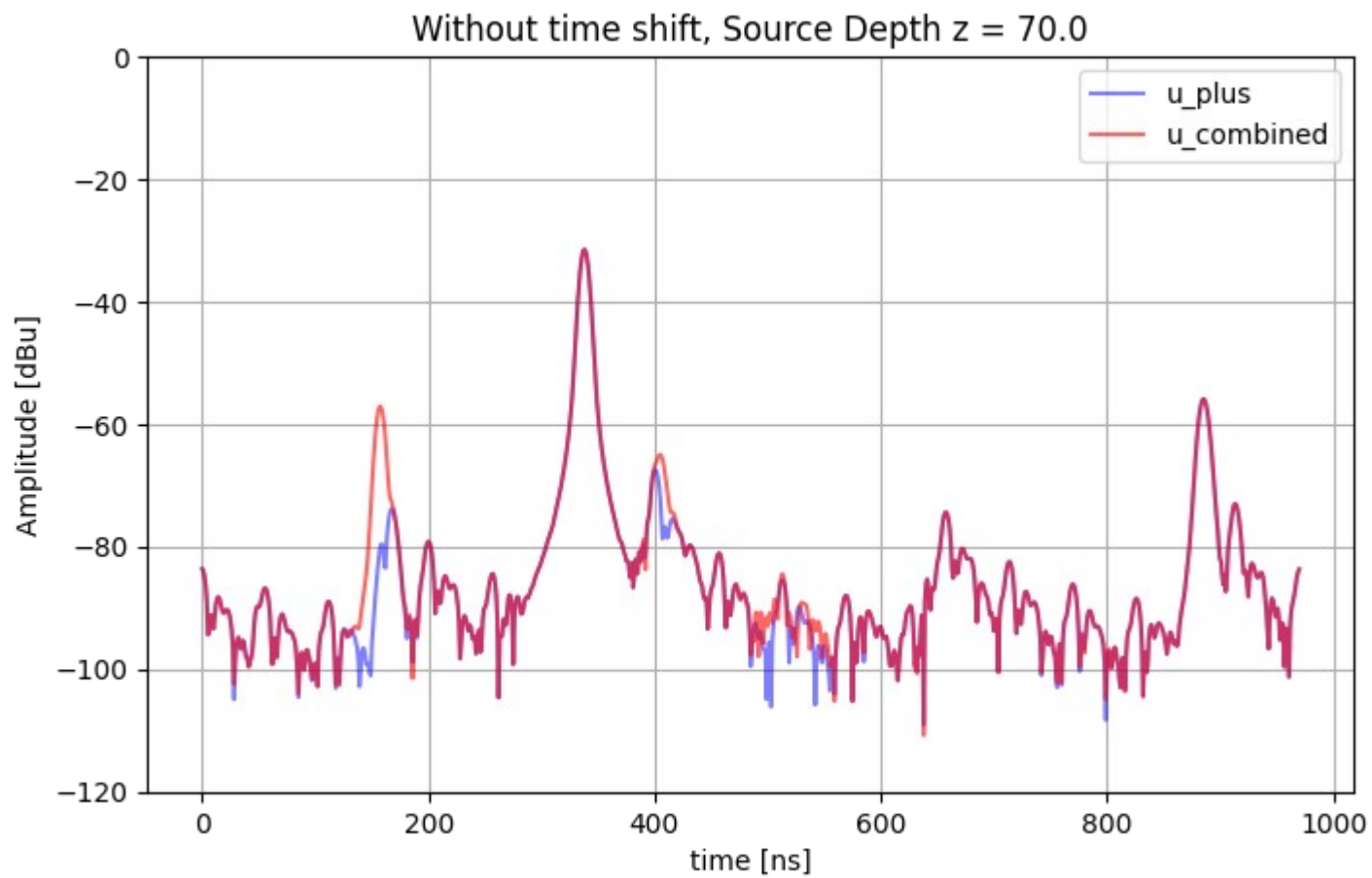




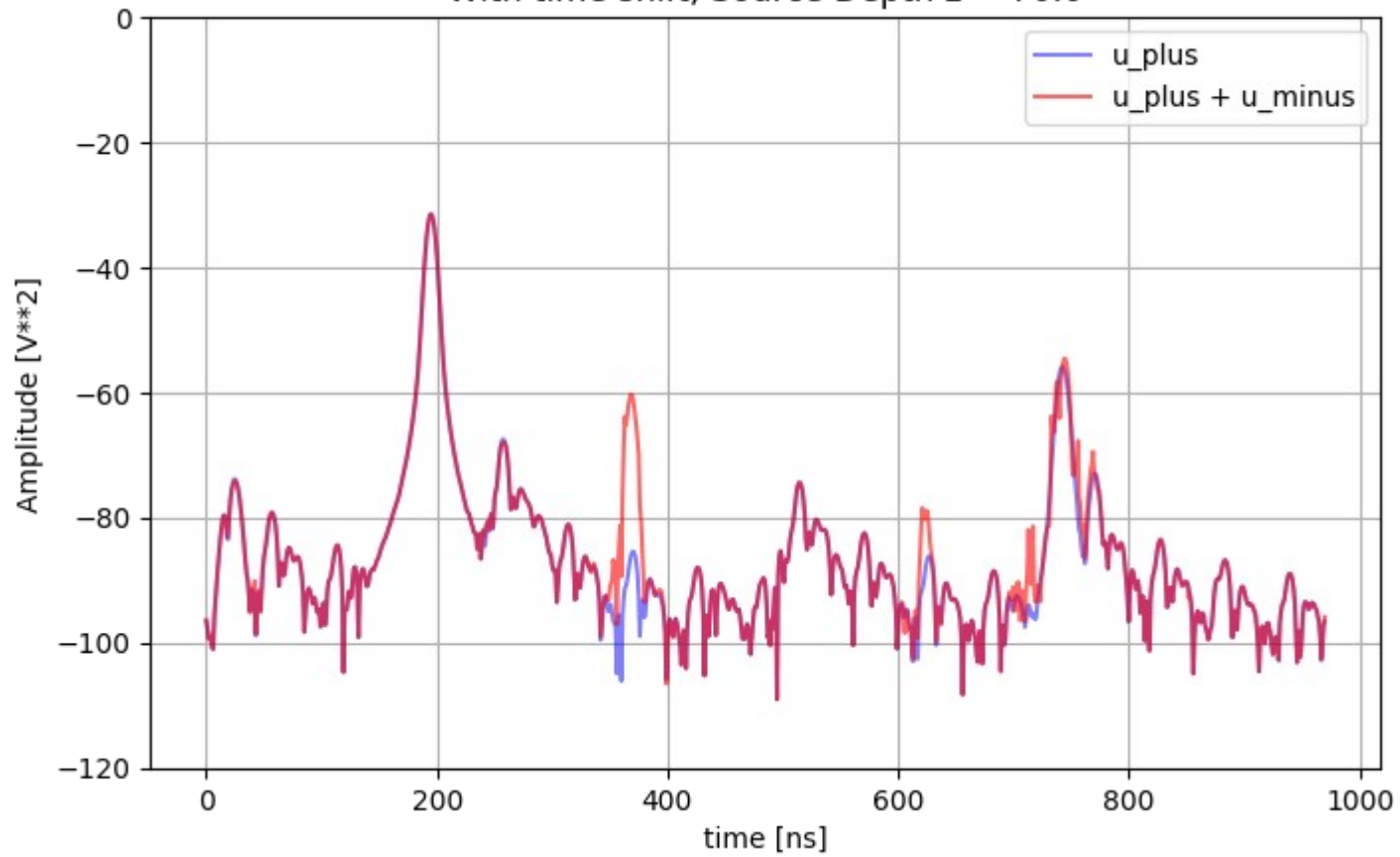








With time shift, Source Depth  $z = 70.0$



# Real World example: Aletsch Glacier

Field Test for the EnEx project in Aletsch Glacier  
March-April 202

Measurement of permittivity profile upper 15 m of firn with bi-static FMCW radar

- direct signal component → the largest signal component
- Reflected signals may arrive earlier or later

Qualitative agreement between simulation and data

