Climate Ambiguity and Optimal Allocation of Renewable Energy Capacities

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EU wind and solar power potential

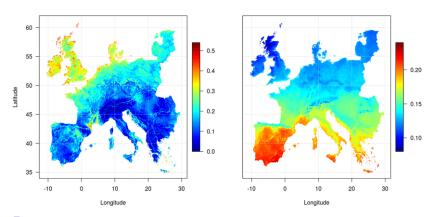


Figure 1: Average capacity factors for wind (left) and photovoltaics (right) in Europe (1995-2015) derived from the regional reanalysis COSMO-REA6 and the satellite-based SARAH 2 dataset, Kaspar et al. (2019).

Source: Kaspar et al. 2019, A climatological assessment of balancing effects and short-fall risks of photovoltaics and wind energy in Germany and Europe, Adv. Sci. Res., 16, 119–128, https://doi.org/10.5194/asr-16-119-2019.

Existing EU power grid and renewables

Renewable sources of energy are

- 1. weather-dependent, hence, intermittent, but at the same time
- 2. persistently rich across certain so called "rich-in-renewable zones" in the EU,

whereas

... existing EU power grid:

- 1. nearly deterministic and centred around natural resources: "buy and sell gas and oil, dig coal, enrich uranium",
- 2. allocations are typically cost-based, i. e. classical energy modelling, deterministic and have "fossil-focus",
- 3. with no long-term (and in practice also no short-term) storage capacities available.

Ambiguity in climate projections

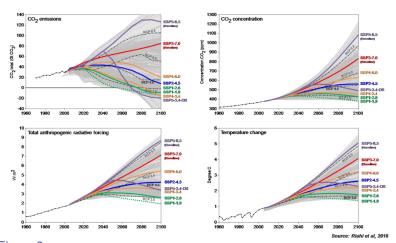


Figure 2: CO2 emissions (a), concentrations (b), anthropogenic radiative forcing (c), and global mean temperature (d) for the 21st century scenarios in the ScenarioMIP design, from Riahi et al. (2016). Concentration, forcing, and temperature outcomes are calculated with a simple climate model (MAGICC version 6.8.01 BETA; Meinshauser et al., 2011a, b). Temperature outcomes include natural forcing in the historical period; projections assume zero volcanic forcing and maintain 11-year solar forcing cycles, consistent with the CMIP5 approach (Meinshausen et al., 2011c). Gray areas represent the range of scenarios in the scenarios database for the IPCC Fifth Assessment Report (Clarke et al., 2014), Figure 3 from Neil et al., 2016, The Scenario Model Intercomparison Project (ScenarioMIP) for CMIP6

Problem outline

Transition to a climate-neutral society according to the goals stipulated in the European Green Deal includes, i. a.:

- (1) the need to take **strategic** decisions on how to invest in renewables in order to reach EU Green Deal 2050 goals,
- (2) the need to take **operational** decisions, i. e. manage (existing, or future) renewable capacities efficiently.

In this work our goal is to show how to reach the Green Deal goals already now by making investment decisions in renewable energy capacities based on so called **Markowitz portfolio** model under **climate ambiguity**.

Portfolio approach in energy modelling

Portfolio approach in energy modelling with respect to wind resources has been considered in:

- ➤ Roques et al., 2010, Optimal wind power deployment in Europe: a portfolio approach, https://doi.org/10.1016/j.enpol. 2009.07.048.
- Santos—Alamillos et al., 2017, Exploring the mean-variance portfolio optimization approach for planning wind repowering actions in Spain, https://doi.org/10.1016/j.renene.2017. 01.041.
- Thomaidis et al., 2022, Handling risk dimensions of wind energy generation, https://papers.ssrn.com/sol3/papers.cfm? abstract_id=4037386.

and in some more.

Problem summary

Research question: How to invest in renewable capacities efficiently under climate ambiguity?

Assumptions:

- 1. perfect transmission network within EU,
- cost-free/technological-constraints-free operation of renewable capacities,
- 3. only wind and solar energy as renewables,
- 4. no (further) technological progress in wind and solar energy production,
- current capacity factors are good approximation to whatever capacity factors are actually now and might be in the future,
- 6. weak-stationarity stationarity and other statistical assumptions for a portfolio model.

Outline

1. Motivation and problem outline

2. Mathematical model

- Markowitz portfolio
- Distributionally robust portfolio
- Mathematical assumptions
- On distributional robustness

3. Data analysis

- Historical data
 - Computing capacity factors and renewable potentials
 - Renewable power production and demand: hourly vs daily
- Future data
 - Climate projections
 - Approximating robustness parameters
- Results summary

4. Summary and outlook

Portfolio model: basics

Denote by ${\bf x}$ the returns on M assets and by ${\bf \omega} \in \mathbb{R}^M$ portfolio weights. Define portfolio moments as follows:

$$m_1(\boldsymbol{\omega}) = \mathbb{E}(\boldsymbol{\omega}^\top \mathbf{x}) = \boldsymbol{\omega}^\top \boldsymbol{\mu},$$

$$m_2(\boldsymbol{\omega}) = \mathbb{E}(\boldsymbol{\omega}^\top \mathbf{x} \mathbf{x}^\top \boldsymbol{\omega}) = \boldsymbol{\omega}^\top \Sigma \boldsymbol{\omega},$$

where $\Sigma = \mathbb{E}(\mathbf{x}\mathbf{x}^{\top})$ is the covariance matrix. In our case \mathbf{x} denotes the production of renewable energy at certain locations.

Note: Bold small letters stand for vectors, bold capital letters stand for matrices and plain letters denote scalars.

Markowitz portfolio

The classical Markowitz mean-variance portfolio is obtained by solving the following problem:

$$\min_{\boldsymbol{\omega}} \ \boldsymbol{\omega}^{\top} \ \boldsymbol{\Sigma} \ \boldsymbol{\omega} \ \text{ such that } \ \boldsymbol{\omega}^{\top} \boldsymbol{\mu} \geq \boldsymbol{r}, \ \boldsymbol{\omega}^{\top} \boldsymbol{1} = \boldsymbol{1}, \ \boldsymbol{\omega} \in \Omega,$$

where r is some target return and Ω is the set of feasible portfolio weights.

Note:

$$\Sigma = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_{1,2} + \sigma_{1,3} + \sigma_{1,4} + \dots,$$

where $\sigma_{i,j} \in \mathbb{R}$ denotes covariance between *i*-th and *j*-th assets.

Markowitz portfolio: correlation matrix

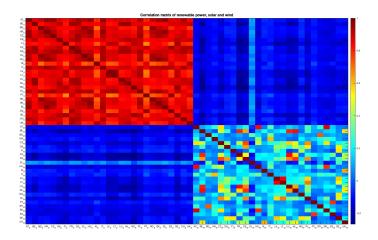


Figure 3: Correlation matrix for solar and wind capacity factors, time series for 27 EU countries, 2019, data: https://researchdata.reading.ac.uk/272/.

Markowitz portfolio: efficiency frontier

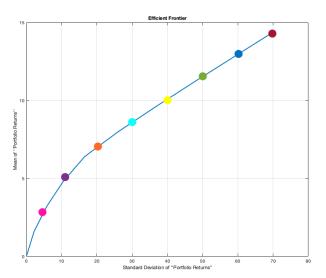


Figure 4: Example of efficiency frontier based on 2019 weather data, dots in different colours denote different efficient allocations in terms of "portfolio" mean and standard deviation.

Markowitz portfolio: efficient allocations

allocation colour	countries and weights
magenta	FR/S 0.0391, ES/S 0.0276, DK/W 0.2416,
	FR/W 0.1642, NO/W 0.2040, UK/W 0.3235
	FC /C 0 000F DI/ /M 0 100F
lila	ES/S 0.0965, DK/W 0.1927,
ilia	FR/W 0.0099, UK/W 0.7009
orange	ES/S 0.2068, UK/W 0.7932
light blue	ES/S 0.3654, UK/W 0.6346
yellow	ES/S 0.5241, UK/W 0.4759
green	ES/S 0.6827, UK/W 0.3173
blue	ES/S 0.8414, UK/W 0.1586
red	ES/S 1

Table 1: Allocation weights depending on the mean return on a "portfolio" for the efficiency frontier as on the previous slide, allocations are based on 2019 weather data.

Markowitz portfolio: critique

- ► Markowitz solutions are too 'extreme': Michaud, 1989, The Markowitz Optimization Enigma: Is 'Optimized' Optimal? Financial Analysts Journal, Vol. 45, No. 1, pp. 31-42.
- Return estimates are often very noisy: Bonami and Lejeune, 2017, An Exact Solution Approach for Portfolio Optimization Problems Under Stochastic and Integer Constraints, Operations Research, Vol. 57, No. 3 (May - Jun., 2009), pp. 650-670.
- ▶ Return distribution is not perfectly known: Wozabal, D., and Pflug, G., Ambiguity in Portfolio Selection, Quantitative Finance, 7:4, pp. 435-442.

Distributionally robust mean-variance portfolio (DRMV)

Distributionally robust mean-variance portfolio is given as

$$\min_{\boldsymbol{\omega} \in \mathcal{F}_{\delta,\bar{\boldsymbol{\alpha}}(n)}} \max_{\mathbb{P} \in \mathcal{U}_{\delta}(\mathbb{P}_n)} \{ \boldsymbol{\omega}^{\top} \; \boldsymbol{\Sigma}_{\mathbb{P}} \; \boldsymbol{\omega} \}, \tag{1}$$

where \mathbb{P}_n is the empirical probability derived from historical information of the sample size n. The ambiguity set, $\mathcal{U}_{\delta}(\mathbb{P}_n)$, is defined as

$$\mathcal{U}_{\delta}(\mathbb{P}_n) := \{\mathbb{P} : \mathcal{D}_{c}(\mathbb{P}, \mathbb{P}_n) \leq \delta\},$$

where $\mathcal{D}_c(\cdot,\cdot)$ is the notion of discrepancy between two probability measures based on a suitably defined Wasserstein distance. The feasible region of portfolios is denoted as

$$\mathcal{F}_{\delta,\bar{\boldsymbol{\alpha}}}(\boldsymbol{n}) = \{\boldsymbol{\omega}: \boldsymbol{\omega}^{\top}\boldsymbol{1} = 1, \boldsymbol{\omega} \in \mathbb{R}_{\geq 0}, \min_{\mathbb{P} \in \mathcal{U}_{\delta}(\mathbb{P}_{\boldsymbol{n}})} \mathbb{E}_{\mathbb{P}} \ \boldsymbol{\omega}^{\top}\boldsymbol{\mu} \geq \bar{\boldsymbol{r}}\},$$

where \bar{r} is the target return given the ambiguity set.

Wasserstein ambiguity sets

Wasserstein ambiguity sets are represented as

$$\begin{split} &\{\mathbb{P}: d(\mathbb{P},\widehat{\mathbb{P}}_n) \leq \delta\} = \\ &\{\mathbb{P}: \text{ there is a bivariate probability } \mathcal{K}(.,.) \text{ such that } \\ &\int_{v} \mathcal{K}(u,dv) = \mathbb{P}(u), \int_{u} \mathcal{K}(v,du) = \widehat{\mathbb{P}}_n(v) \text{ and } \\ &\int_{u} \int_{v} ||u-v||_{p} \mathcal{K}(du,dv) \leq \delta, \} \end{split}$$

where δ is the so called allowed maximum distance between distributions. The bivariate probability K has the interpretation as the solution of Monge's transportation problem. For p=1 these sets are known as Kantorovich ambiguity sets.

More: Pflug and Pichler, 2014, Multistage stochastic optimization, Springer.

DRMV: optimal transport

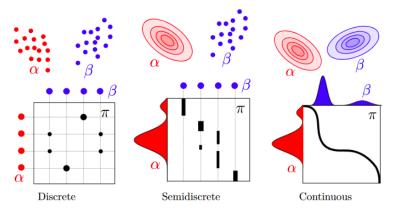


Image: Gabriel Peyré and Marco Cuturi. Computational Optimal Transport.

Figure 5: Illustration of optimal transport problem, Pavel Dvurechensky lectures on Optimal transport, https://indico.cern.ch/event/845380/attachments/1915103/3241592/Dvurechensky_lectures.pdf.

DRMV: solution approaches

Existing solution methods can be broadly classified into three caterogiries:

- Successive convex programming: Wozabal, D., and Pflug, G., 2007, Ambiguity in Portfolio Selection, Quantitative Finance, 7:4, pp. 435-442.
- ▶ Dualization of minmax problem: Esfahani, P. M., and Kuhn, D., 2018, Data-driven distributionally robust optimization using the Wasserstein metric: performance guarantees and tractable reformulations, Mathematical Programming volume 171, pp. 115–166.
- ▶ Proving equivalence to a regularised problem: Blanchet, J., Chen, L., Zhou, X. Y., 2021, Distributionally Robust Mean-Variance Portfolio Selection with Wasserstein Distances, Management Science, Articles in Advance, 1–29.

DRMV: equivalence to a regularised problem

The primal formulation as in (1) is equivalent to the following dual problem (Thm. 1 in Blanchet et al. (2021)):

$$\begin{split} \min_{\boldsymbol{\omega} \in \mathbb{R}_{\geq 0}^{M}} \sqrt{\boldsymbol{\omega}^{\top} \boldsymbol{\Sigma}_{\mathbb{P}_{n}} \boldsymbol{\omega}} + \sqrt{\delta} ||\boldsymbol{\omega}||_{p}, \\ \text{subject to } \boldsymbol{\omega}^{\top} \boldsymbol{1} = 1, \ \boldsymbol{\omega} \in \mathbb{R}_{\geq 0}^{M}, \\ \mathbb{E}_{\mathbb{P}_{n}} (\boldsymbol{\omega}^{\top} \boldsymbol{\mu}) \leq \bar{r} + \sqrt{\delta} ||\boldsymbol{\omega}||_{p} \end{split} \tag{2}$$

in the sense that the two problems, (1) and (2), have the same optimal solutions and optimal value.

DRMV: Choosing robustness parameters, δ and \bar{r}

Penalty, δ , should be chosen such that the set

$$\mathcal{U}(\mathbb{P}_n) = \{\mathbb{P} : \mathcal{D}_c(\mathbb{P}, \mathbb{P}_n) \leq \delta\}$$

contains all the probability measures that are plausible variations of the data presented by \mathbb{P}_n . Define

$$\Lambda_{\delta}(\mathbb{P}_n) = \cup_{\mathbb{P} \in \mathcal{U}_{\delta}(\mathbb{P}_n)} \Omega_{\mathbb{P}}$$

as the set of all the corresponding plausible estimates of the weights ω^* . $\Lambda_{\delta}(\mathbb{P}_n)$ forms a confidence region for ω^* , where δ should be chosen as the smallest number δ_n^* such that ω^* belongs to this region with a given confidence region. Namely,

$$\delta_n^* = \inf\{\delta \ge 0 : \mathbb{P}^*(\omega^* \in \Lambda_{\delta}(\mathbb{P}_n)) \ge 1 - \delta_0\},\tag{3}$$

where $1 - \delta_0$ is a user-defined confidence level, e. g. 95 %.

DRMV: Choosing robustness parameters, δ and \bar{r}

The confidence region $\Lambda_{\delta}(\mathbb{P}_n)$ can be approximated by so called Robust Wasserstein Profile (RWP) function, $\mathcal{R}_n(\Sigma_n, \mu_n)$, i. e. (3) can be rewritten as

$$\delta_n^* = \inf\{\delta \geq 0 : \mathbb{P}^*(\mathcal{R}_n(\Sigma_n, \mu_n)) \geq 1 - \delta_0\},\$$

where a central limit result can be proven for distribution of $\mathcal{R}_n(\Sigma_n, \mu_n)$, see Thm. 2 in Blanchet et al. (2021), Blanchet et al. (2019). Thus, δ_n^* can be found as the quantile corresponding to the $1-\delta_0$ percentile of the asymptotic distribution of $\mathcal{R}_n(\Sigma_n, \mu_n)$.

The idea to select \bar{r} is based on a similar central limit result, see Proposition 1 in Blanchet et al. (2021), such that the chosen \bar{r} is large enough to make sure that the inclusion $\omega^* \in \mathcal{F}_{\delta,\bar{r}}(n)$ is with a given confidence level chosen by the user is not excluded.

Data analysis

Historical dataset:

ERA5 derived time series of European country-aggregate electricity demand, wind power generation and solar power generation: 1979-2019 dataset contains hourly time series for EU 28 countries of

- ▶ 100 m wind speed, surface shortwave radiation and 2 m temperature,
- solar and wind capacity factors as well as an econometric model for electricity demand.

Source: https://researchdata.reading.ac.uk/272/.

What is happening with climate: 2 m temperature?

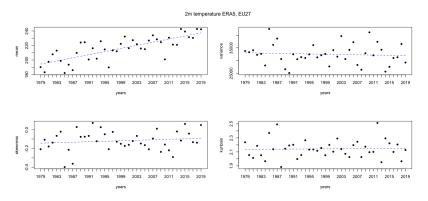


Figure 6: Mean, variance, skewness and kurtosis of irradiation time series for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

What is happening with climate: irradiation?

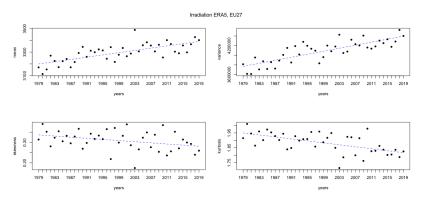


Figure 7: Mean, variance, skewness and kurtosis of irradiation time series for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

What is happening with climate: 100 m wind speed?

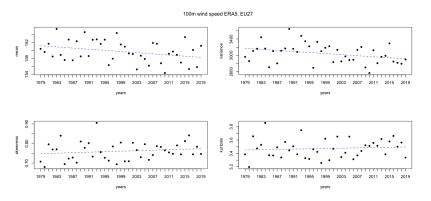


Figure 8: Mean, variance, skewness and kurtosis of irradiation time series for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Solar power conversion

Let r(t) denote instantaneous solar irradiance at time t, η_r photovoltaic cell efficiency, β_r the fractional decrease of cell efficiency per unit temperature increase, $T_c(t)$ cell temperature and T_r reference temperature. Further, g_r stands for the reference or standard test conditions. Then the solar power produced at time point t, $p_s(t)$, is given as follows:

$$p_s(t) = \eta_r (1 - \beta_r (T_c(t) - T_r)) \frac{r(t)}{g_r},$$

where, e. g., $T_r=25^{\circ}C$, $g_r=1000~Wm^{-2}$, $\beta_r=0.005~{\rm and}$ $\eta_r=0.9$.

Source: Evans, D.L. and L.W. Florschuetz (1977). Cost studies on terrestrial photovoltaic power systems with sunlight concentration. Solar Energy, 19, 255-262.

Wind power conversion

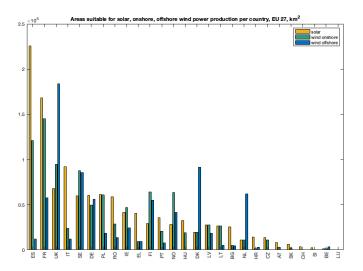
Let u(t) be an instantaneous wind speed at time t that is uniform throughout a rotor disk of diameter d, ρ is the air density and c_p be the so called power coefficient, or the fraction of energy that a wind turbine captures. Then the power produced by a wind turbine, $p_w(t)$, at time point t is given as follows:

$$p_w(t) = c_p \frac{1}{2} \rho \pi \frac{d^2}{4} u(t)^3,$$

where c_p is obtained via wind power curve.

Source: many papers on wind power estimation.

Total area suitable for wind and solar power production



Source: Zeyringer, M., and Price, J. (2020); Corine Land Cover: https://land.copernicus.eu/pan-european/corine-land-cover.

Capacity factor and renewable power potential

- Capacity factors are stochastic variables derived based on solar and wind power conversion formulas.
- ▶ Renewable power potential is the capacity factor multiplied by the area available for solar and wind power production in each country, respectively.

Climate change and solar power production

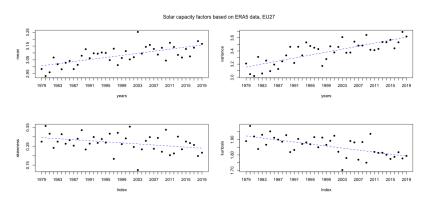


Figure 9: Mean, variance, skewness and kurtosis of solar power capacity time series for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Climate change and wind power production

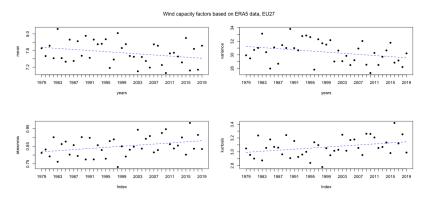


Figure 10: Mean, variance, skewness and kurtosis of wind power capacity time series for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Mean annual solar power capacities

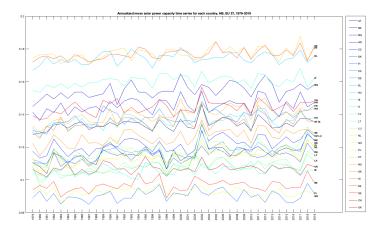


Figure 11: Annualized mean time series for solar power capacity factors across 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Mean annual wind power capacities

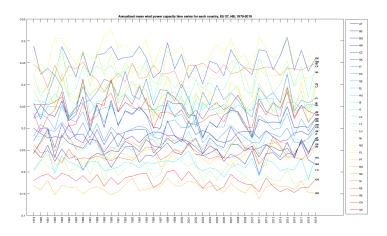


Figure 12: Annualized mean time series for wind power capacity factors across 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Solar power: outliers

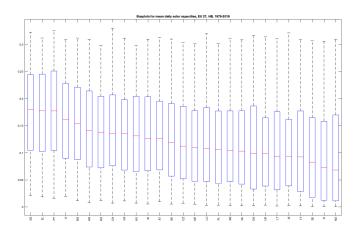


Figure 13: Boxplots for daily mean solar power capacities per each EU 27 country ordered by the corresponding median value, data: https://researchdata.reading.ac.uk/272/. Note that the daily mean for solar capacities is taken over 24 hours.

Wind power: outliers

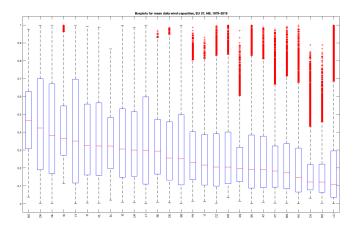


Figure 14: Boxplots for daily mean wind power capacities per each EU 27 country ordered by the corresponding median value, data: https://researchdata.reading.ac.uk/272/.

Hourly solar and wind power potentials vs hourly demand

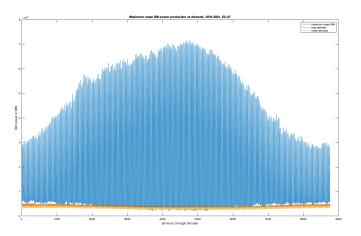


Figure 15: Hourly max mean of solar and wind potentials vs mean and max demand for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Day time (only) solar power allocations, 10:00 - 18:00

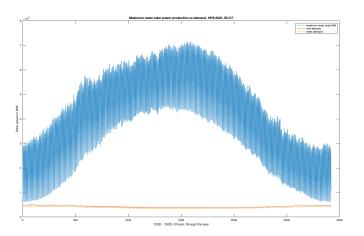


Figure 16: Hourly max mean of solar potentials vs mean and max demand for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Night time (only) wind power allocations, 18:00 - 10:00

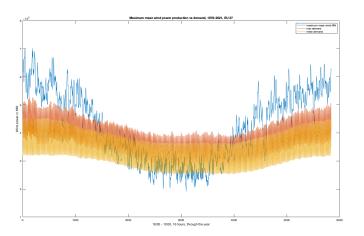


Figure 17: Hourly max mean of wind potentials vs mean and max demand for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Daily solar and wind power potentials vs daily demand

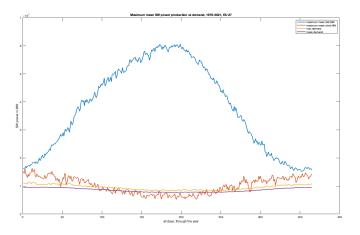


Figure 18: Daily max mean of solar and wind potentials vs mean and max demand for 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Historical data analysis: summary and limitations

- Several compromises have to be made when computing capacity factors, i. e. with respect to 'technology' coefficients, bias correction, the way the data is gathered, and so on.
- ▶ Based on historical data certain trends in climate can be verified which also has an effect on future power production.
- ► There exists countries rich in certain resources, sun or wind, and these patterns are stable over time despite climate change trends, i. e. 'weak stationarity assumption' is satisfied at least in colloquial sense.
- ▶ Hourly allocation, common in energy modelling, are too volatile and are not feasible for some hours when wind production drops (e. g., in summer), therefore daily allocations are preferred which imply, however, existence of storage.
- Solar and wind are complementary to each other, whereby wind power production is less reliable (from a statistical point of view, since wind is prone to outliers compared to solar).



Future data

Future dataset:

ERA5 derived time series of European country-aggregate wind power generation and solar power generation: 1950-2020 dataset contains hourly time series for European countries of

- ▶ 100 m wind speed, surface shortwave radiation and 2 m temperature and
- solar and wind capacity factors

adjusted for the impacts of the climate change from five models, EC-EARTH3P-HR and EC-EARTH3P by European Community, and MOHC-MM-1hr (ensembles 2 and 3) and MOHC-HH-3hr from UK Met Office.

Source: https://researchdata.reading.ac.uk/321/.

Future data: 2 m temperature

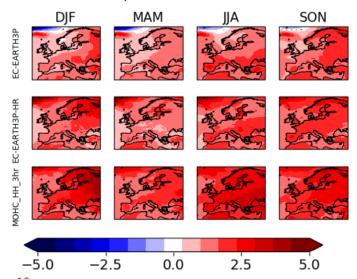


Figure 19: Seasonal-mean difference in 2m temperature between 1980-2010 and 2020-2050 for the 3 different climate model simulations used in this study (see Table 1 for further details of the models). Stippling shows grid points where no statistically significant climate change signal was found when using a 2-sample t-test (Wilks, 2011), Figure A.1. from Bloomfield (2021).

Future data: 10 m wind speed

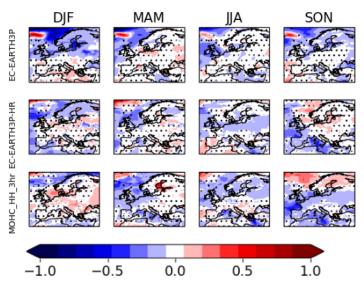


Figure 20: Seasonal-mean difference in 10m wind speed between 1980-2010 and 2020-2050 for the 5 different climate model simulations used in this study (see Table 1 for further details of the models). Stippling shows grid points where no statistically significant climate change signal was found when using a 2-sample t-test (Wilks, 2011), Figure A.2. from Bloomfield (2021).

Future data: irradiation

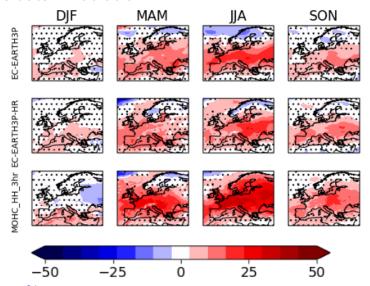


Figure 21: Seasonal-mean difference in surface solar irradiance between 1980-2010 and 2020-2050 for the 5 different climate model simu- lations used in this study (see Table 1 for further details of the models. Stippling shows grid points where no statistically significant climate change signal was found when using a 2-sample t-test (Wilks, 2011), Figure 3 from Bloomfield (2021).

Future data: summary and limitations

- ► There is no unanimous agreement on how future climate will evolve: quantitatively, the spread between the models is too large to draw definite qualitative conclusions.
- Instead of evaluating each model separately it is practical to summarise climate ambiguity in a statistical "robustness" concept.
- "Robustness" parameters are derived based on asymptotic results with separate confidence regions, however, an approximate result seems to be sufficient for the purposes of the analysis.

MV results

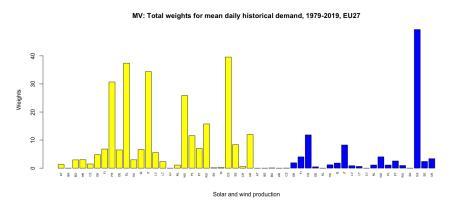


Figure 22: MV portfolio weights, annual total, 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

DRMV EC results

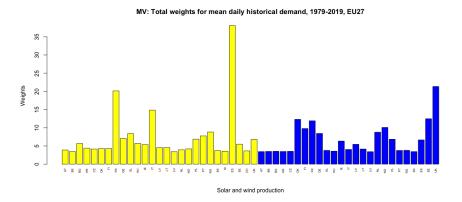


Figure 23: DRMV EC portfolio allocations, annual total, 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

DRMV UK results

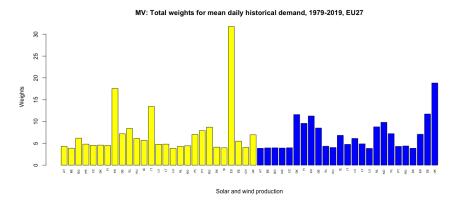


Figure 24: DRMV portfolio allocations, annual total, 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

DRMV with an extreme δ

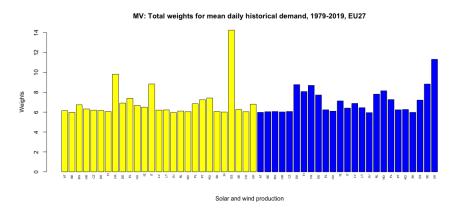


Figure 25: DRMV portfolio allocations, total annual production, 27 EU countries, 1979 - 2019, data: https://researchdata.reading.ac.uk/272/.

Summary and outlook

- Portfolio framework provides 'less optimistic' view on EU renewable potential than deterministic models because stochasticity of underlying weather variables is taken into account.
- Robust portfolio framework suggests even more 'cautious' allocations than the standard mean-variance model: allocations are more spread across different locations instead of relying on traditionally renewable-rich-zones. However, it is not always straightforward to determine the appropriate degree of robustness.
- ► Solar and wind power are complementary to each other. However, both standard- and robust-portfolio-based allocations imply existence of at least a half-a-day storage facilities.

Main references

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