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Scaling invariance behaviour of thermal fluxes from an extensive green roof

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NP3.2 Climate Variability Across Scales and Multifractals in Urban Geosciences

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HYDROLOGY METEOROLOGY and COMPLEXITY

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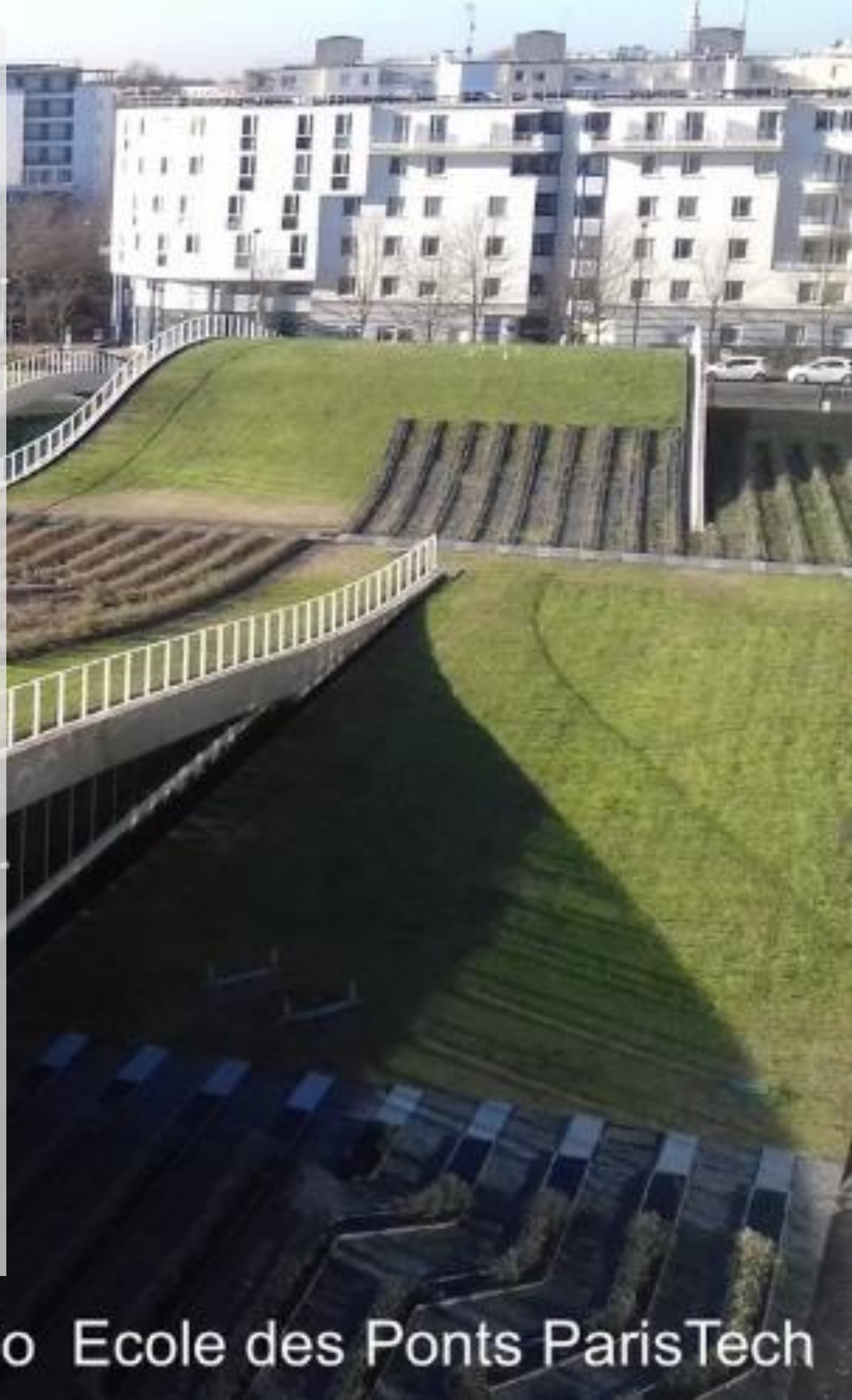
Context

Analysis of the **scale-invariant properties of heat fluxes** related to the evapotranspiration (ET) flux, such as the structure function parameter of the refraction index of air ($\boldsymbol{C_n}^{\boldsymbol{2}}$) and the air temperature ($\boldsymbol{T}$) in the Blue Green Wave, **using the Universal Multifractal framework**.

- The Blue Green Wave is an extensive green roof of 1 ha, located in the Ecole des Ponts ParisTech (France)

- Measuring of path-averaged ${C_n}^2$ over the 190 m with a Large Aperture Scintillometer LAS MKI every 10 min. The path-length between transmitter and receiver of LAS MKI pass over the BGW.

- The path-averaged ${C_n}^2$ measured by the LAS MKI is mainly due to temperature fluctuations (Moene, 2003): Monitoring of the air temperature next to the receiver unit with a CNR4 radiometer.



Methodology

1. The power spectral density

The power spectral density corresponds to the second-order moment statistics.

$$\boldsymbol{E(f) \sim f^{-\beta}}$$

f is the frequency
β is the spectral exponent

$$\boldsymbol{\beta = 1 + 2H - K(2)}$$

H is the Hurst exponent
$K(2)$ is the scaling moment function

2. The structure function

The structure function enables to fully characterise the variability of the flux across scales over several orders of statistical moments q.

$$\boldsymbol{S_q(r) = \langle |u(x+r) - u(x)|^q \rangle}$$

u velocity increments

$$\boldsymbol{S_q(r) = \epsilon^{-q/3} r^{q/3} \propto r^{\zeta(q)}}$$

$\zeta(q)$ is the scaling exponent function

The curve $\zeta(q)$ vs. q:

- For a monofractal process: $\zeta(q)$ follows the power law qH.
- For a multifractal process: $\zeta(q)$ does not follow the power law qH, which is affected by the nonlinearity of $K(q)$ from intermittent and multifractal processes.

$K(q)$ is the scaling moment function that can be characterised by the Universal Multifractal framework.

3. Universal Multifractal (UM)

The UM framework enables the characterisation of $K(q)$ with three scale invariant parameters α, C_1 and H (Schertzer & Lovejoy, 1987):

$$K(q) + Hq = \begin{cases} \dfrac{C_1}{\alpha - 1}(q^{\alpha} - q) & \alpha \neq 1 \\ C_1 q \ln q & \alpha = 1 \end{cases}$$

Lévy index α, indicates the extent of multifractality ($0 \leq \alpha \leq 2$):

- Monofractal fields, $\alpha = 0$
- Maximal occurrence of extremes (log normal multifractal), $\alpha = 2$

Mean singularity C_1, measures the clustering of the average field intensity and fractal codimension of the average field:

- $C_1 = 0$ for uniform field

Hurst exponent H, measures the degree of non-conservation of the average field:

- $H = 0$ conservative field

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$K(q)$ and the scale invariant parameters α, C_1 for a conservative field ($H = 0$) can estimated through Trace Moments or Double Trace Moments Technique :

4. Trace Moments (Schertzer & Lovejoy, 1987)

The statistical moments of order q of a scale invariant field ϵ at a given resolution λ, scales with the resolution λ:

$$\langle \boldsymbol{\epsilon}_{\boldsymbol{\lambda}}^{\boldsymbol{q}} \rangle \approx \boldsymbol{\lambda}^{\boldsymbol{K(q)}}$$

$$C_1 = K'(1)$$

$$\alpha = C_1 / K''(1)$$

$$\lambda = \frac{outler\ scale}{observation\ scale}$$

5. Double Trace Moments (Lavallée et al. 1993)

Trace Moments analysis on a renormalised ηth power of the field ϵ_λ.

$$\langle \boldsymbol{\epsilon}_{\boldsymbol{\lambda}}^{\boldsymbol{(\eta)q}} \rangle \approx \boldsymbol{\lambda}^{\boldsymbol{K(q,\eta)}}$$

$$K(q,\eta) = K(q\eta) - qK(\eta)$$

$$K(q,\eta) = \eta^\alpha K(q)$$

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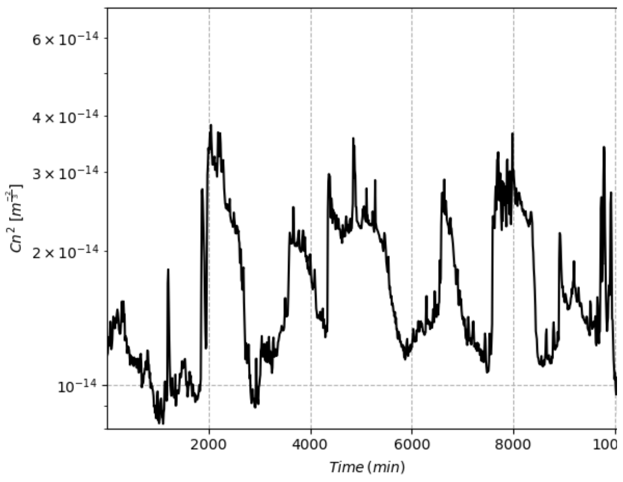
The data-set:

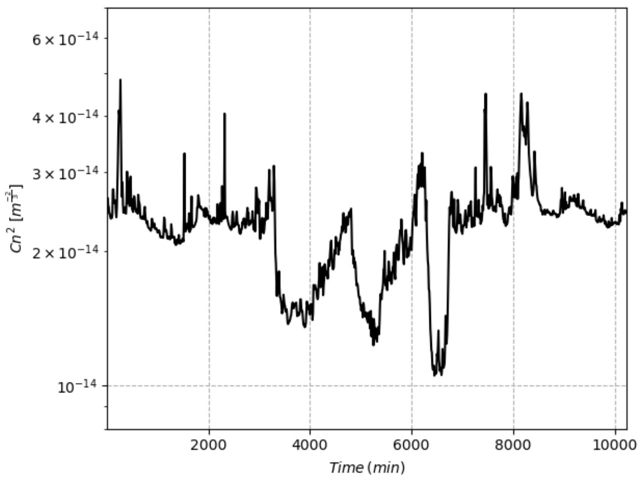
Measurements of ${C_n}^2$ and T with a LAS MKI and a CNR4 respectively, in December 2019 and January 2020.

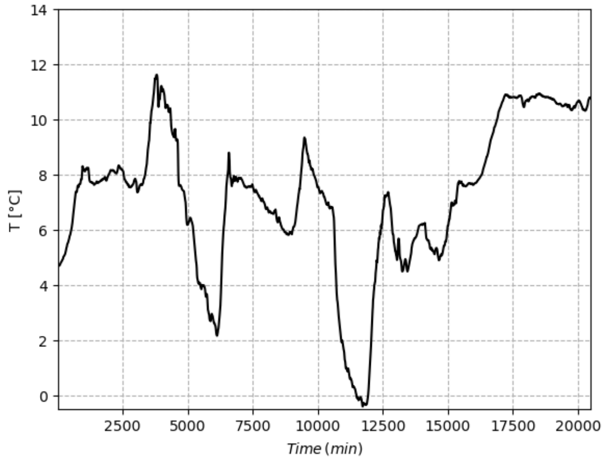
Heat Flux	Collection time	Sample length	Sample frequency
${C_n}^2$	December 2019	1024	1 Hz, 10 min average
	January 2020		
T	December 2019	2048	1 Hz, 5 min average
	January 2020		

December 2019



January 2020



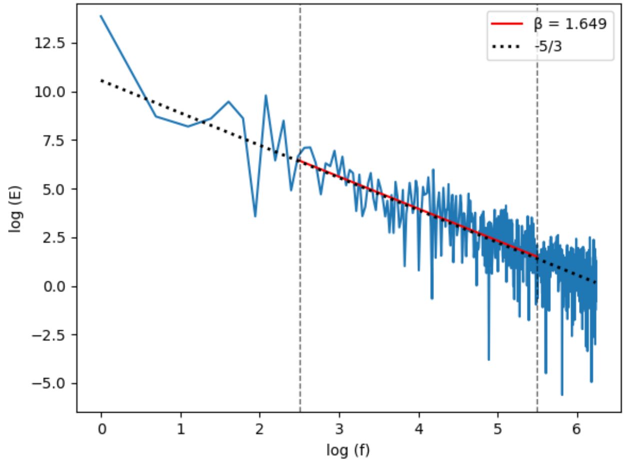


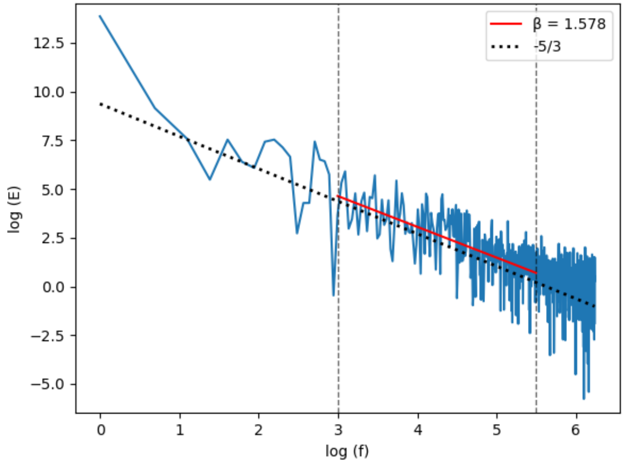
Results

The power spectral density

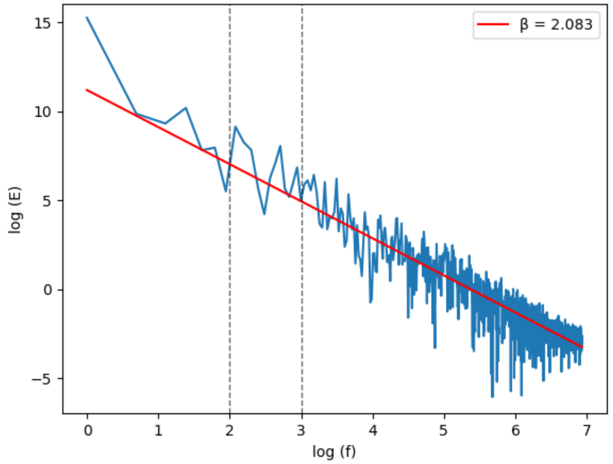
December 2019 | **January 2020**

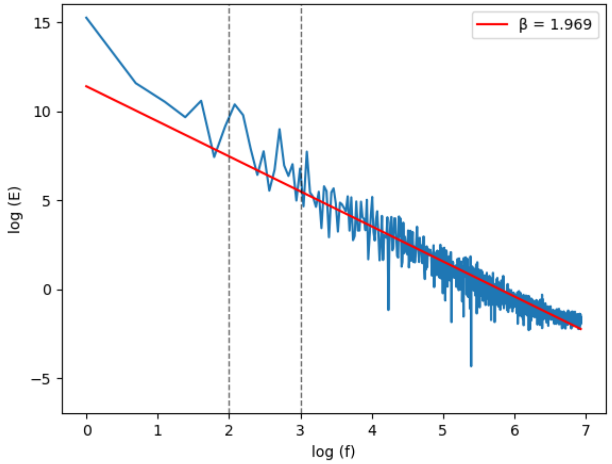
$C_n{}^2$





T





- Fluctuations of $C_n{}^2$ and T in both periods exhibit a linear scaling behaviour.
- Spectral slope β of $C_n{}^2$ roughly equal to 1.6 (~5/3 of isotropic and homogeneous turbulence, Kolmogorov 41)
- Spectral slope β of T roughly equal to 2 (~11/5 of scaling stable stratified turbulence, Bolgiano-Obukhov).

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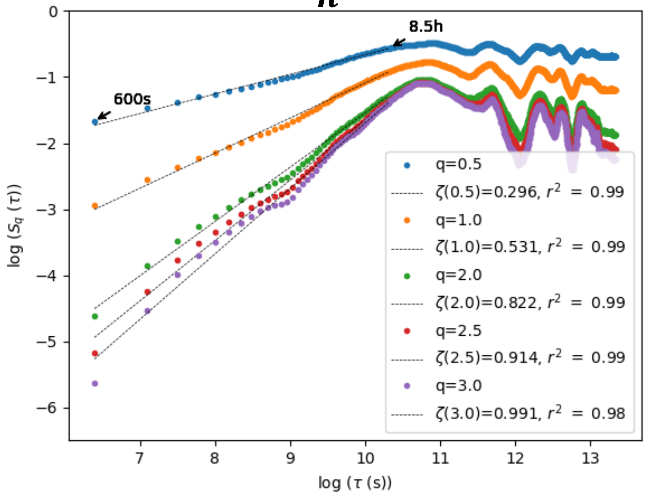
Results

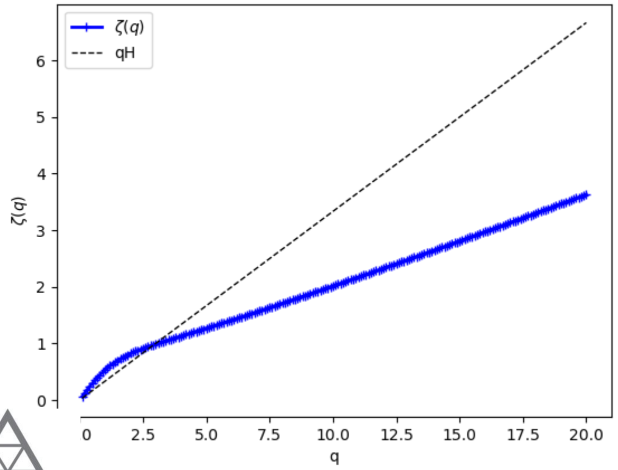
The structure function of temporal fluctuations

$$S_q(\tau) = \tau^{\zeta(q)}$$

December 2019

$C_n{}^2$





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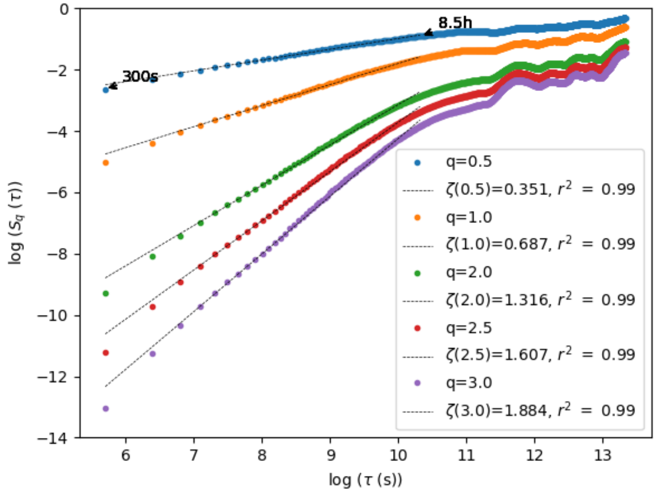
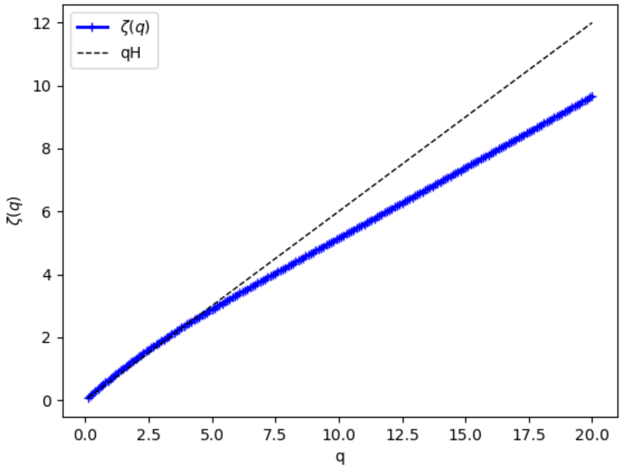
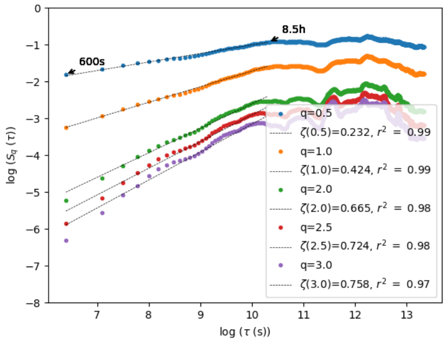


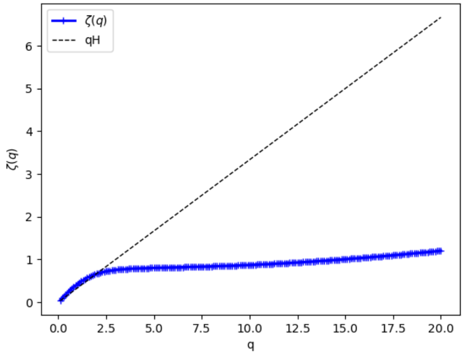


- Scaling behaviour of the $C_n{}^2$ and T, between 600 s and 8.5 h, and 300 s and 8.5 h respectively.
- Nonlinearity of $\zeta(q)$ given by $K(q)$.

January 2020

$C_n{}^2$





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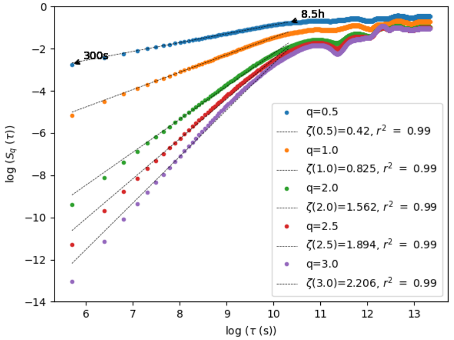

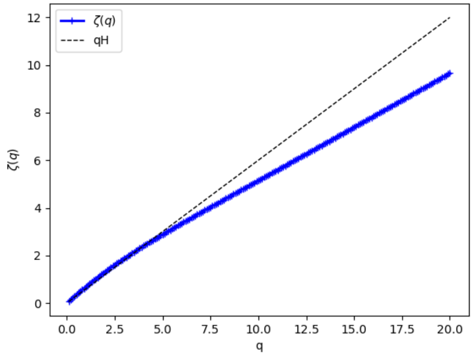

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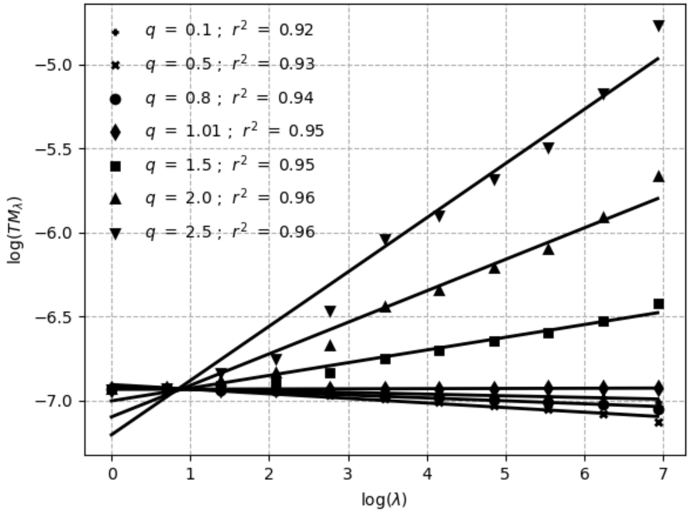
Results

Trace Moments/Double Trace Moments

December 2019

$C_n{}^2$



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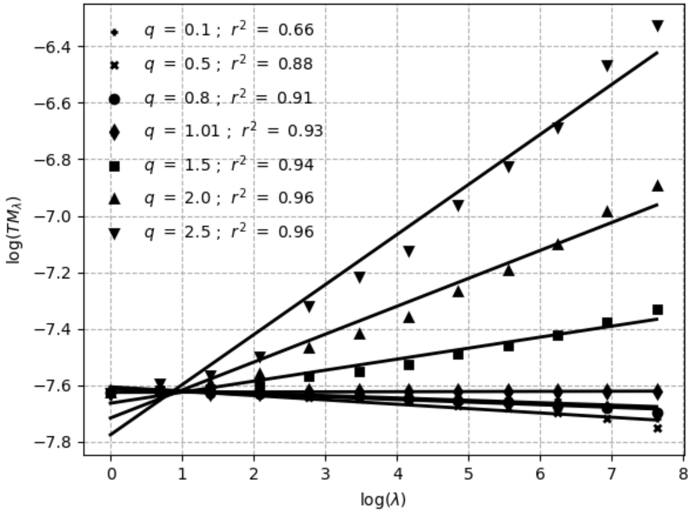

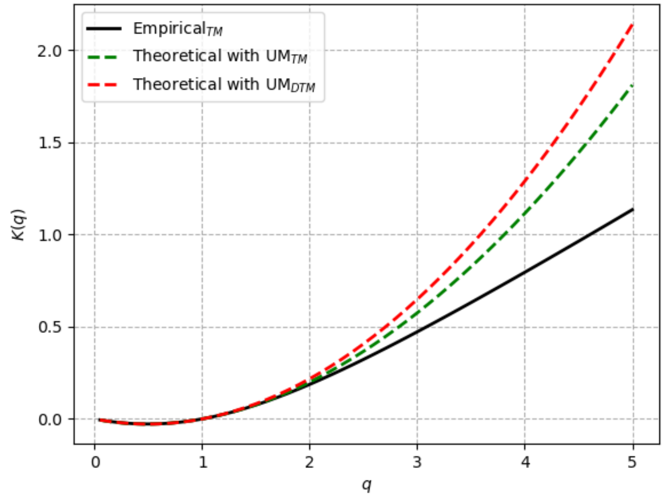

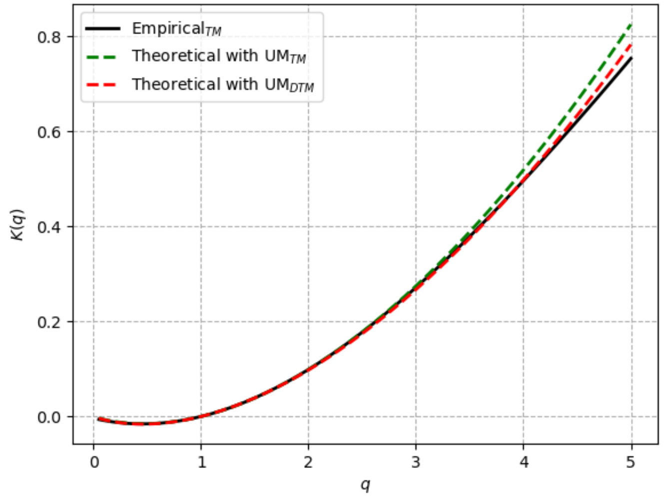

Characterisation of UM parameters

Period	December 2019		January 2020	
Flux/Parameter	α	C_1	α	C_1
Trace Moments				
$C_n{}^2$	1.839	0.106	1.879	0.10
T	1.702	0.055	1.680	0.073
Double Trace Moments				
$C_n{}^2$	1.97	0.11	1.85	0.10
T	1.625	0.056	1.587	0.075

Conclusions & Perspectives

- Scale invariant parameters α and C_1 of ${C_n}^2$ fluctuations during both periods of analysis were higher than the T, which indicates strong multifractality.
- ${C_n}^2$ measurements by the LAS MKI were not only affected by the temperature fluctuations, but additional turbulent processes that have a stronger impact on the heat convection than conventionally expected (e.g., wind speed).

Future work involves:

- Development of a new measurement campaign of heat fluxes, related to the ET from vegetated structures in urban environments, at different spatial scales.
- Analysis of the heat fluxes spatial variability from the new measurement campaign through the UM framework.

References

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