A finite element approach to modelling Glacial Isostatic Adjustment on three-dimensional compressible earth models

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Motivation

- Wu (2004) – Iterative Stress Transform (IST) method

a) **Stress transformation** and Abaqus -- solve the equation of motion with the finite element method
b) Green’s function and spherical harmonics – solve the Poisson’s equation for gravitational potential
c) **Suitable for incompressible earth models, both 1D and 3D**

- Wong & Wu (2019) – Iterative Body Force (IBF) approach

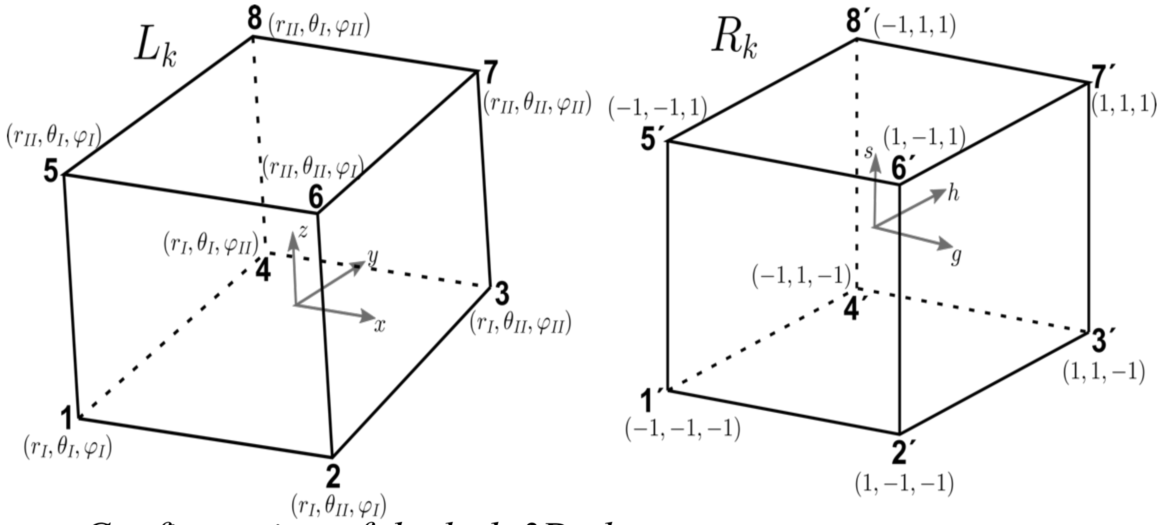
a) **Compressible earth models**
b) **Body forces** and Abaqus
c) Green’s function and spherical harmonics
d) **Suitable for 1D earth models**

- Here, further develop the IBF approach to

a) **Consider 3D earth models**
b) **Avoid spherical harmonics – avoid the leakage of high degree/order load contribution**
c) **Include the degree 1 load – neglected in both Wu (2004) and Wong & Wu(2019)**

The Finite Element Iterative Body and Surface Force Approach (FEMIBSF)

- Classical finite element techniques (e.g., Zienkiewicz et al., 2005)



Configuration of the k-th 3D element.

a) 3D Isoparametric element R_k -- mapping the coordinates, surface area, volume, body and surface force of L_k to those of R_k
b) Bilinear and trilinear interpolations
c) Gauss-Legendre integration rule

- Direct integration instead of spherical harmonics
  ---- calculate load, potential (ϕ), surface and body force, e.g.,

$$\phi = \iint G_{CMB}\sigma_{CMB}dS + \iiint G_V\rho_V dV + \iint G_{SUR}\sigma_{SUR}dS$$
$$= \sum_P G_{CMB}\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)\sigma_{CMB}\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)J_{CMB} +$$
$$\sum_Q G_V(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}})\rho_V(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}})J_V +$$
$$\sum_R G_{SUR}(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}})\sigma_{SUR}(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}})J_{SUR}$$

- Degree 1 deformation

a) View the displacement field in the Earth-load frame with origin at the center of mass
b) Abaqus frame, i.e., the FE frame (e.g., Paulson et al., 2005)
   ---- the initially undeformed spherical solid Earth
c) The FE frame differs from the Earth-load frame by a displacement d_{cm} which consists of the displacement caused by **motion of lithosphere, mantle and liquid core, and the translation due to surface loads** (Klemann & Martinec, 2011)

Validation: case 1, homogeneous compressible model

Material properties for the homogeneous compressible model 1a.

layer	lower radius (km)	upper radius (km)	Density (kg/m³)	Young's modulus (Pa)	Poisson's ratio	Viscosity (Pa·s)
1	0.00	6371.00	5517	3.9325E+11	0.3542	1.00E+22

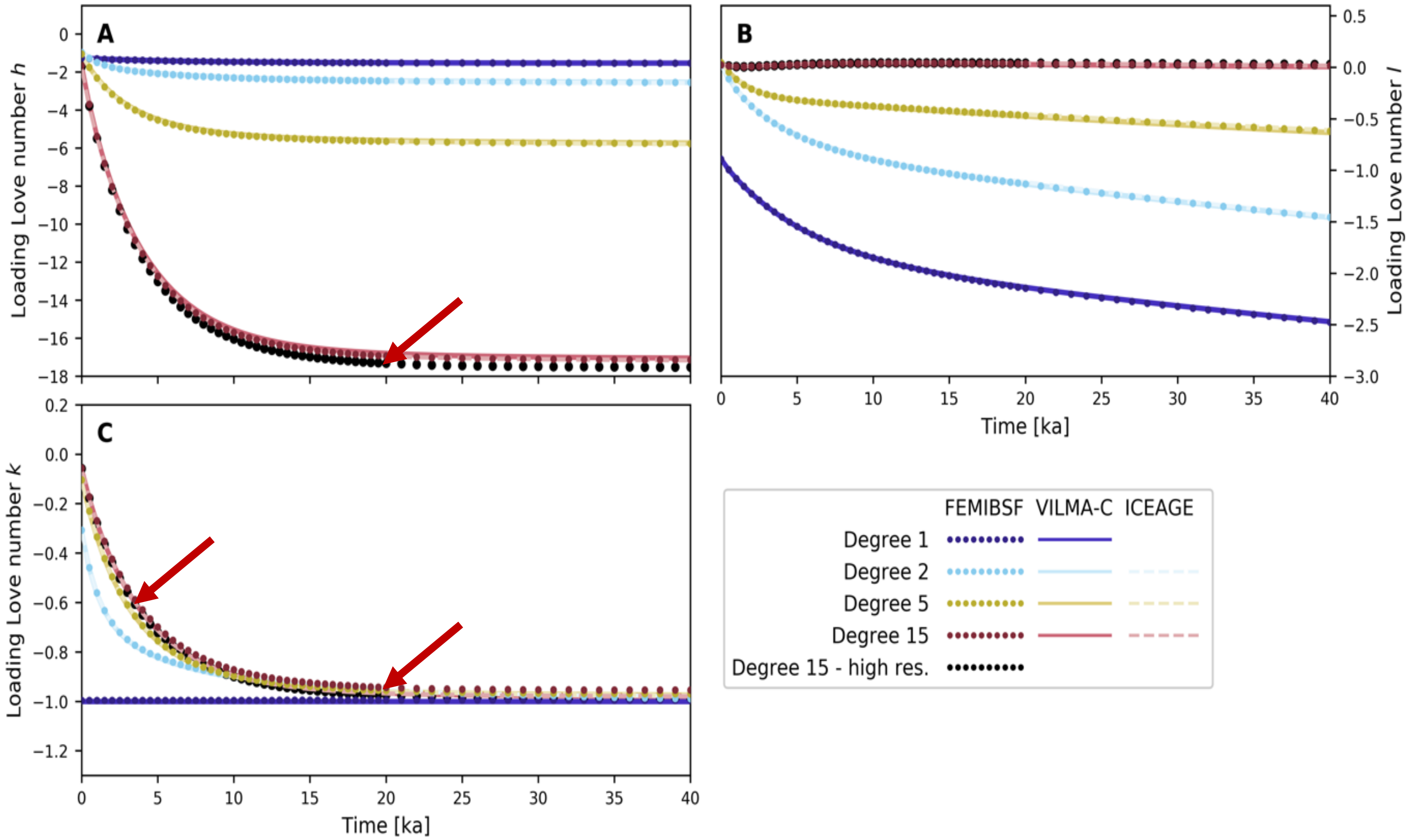
Loading Love numbers h, l and k comparison between finite element (FEMIBSF) and analytical (an) results for the model 1a for degrees (n) 1, 2, 5 and 15 as well as their difference (error) in percent.

n	h_FEMIBSF	h_an	Error (%)	l_FEMIBSF	l_an	Error (%)	k_FEMIBSF	k_an	Error (%)
1a, elastic limit									
1	-1.1828E+00	-1.1827E+00	0.01	-8.5775E-01	-8.5660E-01	0.13	-9.9815E-01	-1.000E+00	0.19
2	-5.8392E-01	-5.8167E-01	0.39	-1.2522E-02	-1.2200E-02	2.65	-2.1996E-01	-2.2009E-01	0.06
5	-7.8259E-01	-7.7766E-01	0.63	1.7091E-02	1.7002E-02	0.52	-1.4698E-01	-1.4761E-01	0.43
15	-9.3041E-01	-9.2332E-01	0.77	1.1810E-02	1.1454E-02	3.10	-6.4592E-02	-6.6333E-02	2.70

Validation: case 2, layered compressible model

Material properties for the layered compressible model 2a.

layer	lower radius (km)	upper radius (km)	Density (kg/m³)	Young's modulus (Pa)	Poisson's ratio	Viscosity (Pa·s)
1	0.00	3485.5	10977	Inviscid core, not modelled but boundary condition at the CMB considered		
2	3485.5	5200.0	5074	6.2410E+11	0.2925	3.20E+21
3	5200.0	5701.0	4527	4.5980E+11	0.2730	1.60E+21
4	5701.0	5971.0	3882	2.8300E+11	0.2920	5.00E+20
5	5971.0	6256.0	3442	1.8870E+11	0.2901	5.00E+20
6	6256.0	6371.0	3192	1.5470E+11	0.2845	1.00E+22



Loading Love numbers comparison between different compressible codes/methods for model 2a. A) h, B) l, and C) k loading Love number. FEMIBSF_high_res. is the same as FEMIBSF but with a higher grid resolution which causes extensive computation time.

Conclusion

- A method called FEMIBSF is presented here which is capable of modelling Glacial Isostatic Adjustment on **3D earth models**, in the meantime **avoiding spherical harmonics usage – away from the leakage of high degree/order load contribution**, and **including the degree 1 load contribution.**

- The validity of FEMIBSF is demonstrated by the **fit of loading Love numbers (LLNs) generated by FEMIBSF to their analytical solutions for the homogeneous model and numerical solutions for the layered model** calculated by the normal-mode approach/code, ICEAGE (Kaufmann, 2004), the iterative body force approach/code, IBF (Wong & Wu, 2019) and the spectral-finite element approach/code, VILMA-C (Martinec, 2000; Tanaka et al., 2011).

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