BIRKELAND CENTRE FOR SPACE SCIENCE

Verification of the Streamer Parameter Model by comparing to Hydrodynamic Simulations

Nikolai G. Lehtinen[1] and Robert Marskar[2]

[1]Birkeland Center for Space Science, University of Bergen, Norway
[2]SINTEF Energy Research, Trondheim, Norway

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European Research Council

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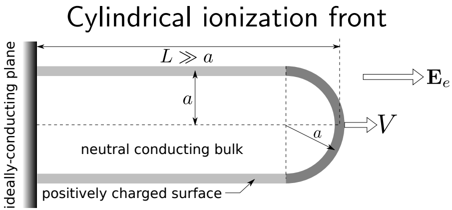
Outline

Streamer Parameter Model (SPM)

Comparisons with HDS

Conclusions

Streamer Parameter Model (SPM) [Lehtinen, 2020, 2021]



- **SPM1** Relationship between E fields, from electrostatic distribution of surface charge.
- **SPM2** Continuity of total (conductivity + displacement) current through the streamer front.
- **SPM3** Propagation stability criterion (Maxwellian relaxation rate ≈ ionization rate).
- **SPM4** Velocity (V)–radius (a) relation, from electron density balance during photoionization/impact ionization.

- No unique solution, but a family of valid solutions (streamer modes).
- Dispersion function $V(a)$ which has a maximum.
- This maximum gives the "physical" solution.
- SPM agrees with experimental values of V, a and negative streamer threshold field $E_{-t} \approx 0.75$–1.25 MV/m [Lehtinen, 2021] and HDS values of V, a, free electron number and maximum electric field [Lehtinen and Marskar, 2021].

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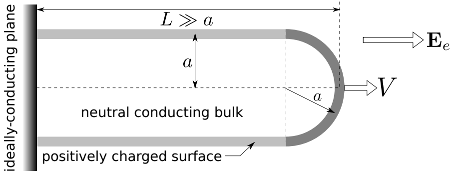

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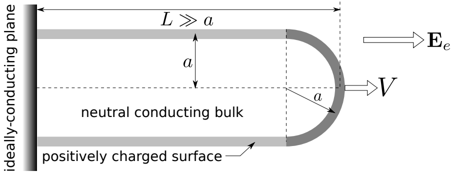
Cylindrical ionization front



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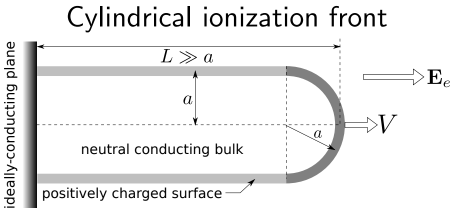

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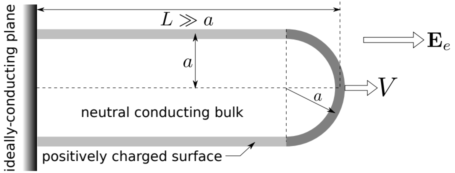

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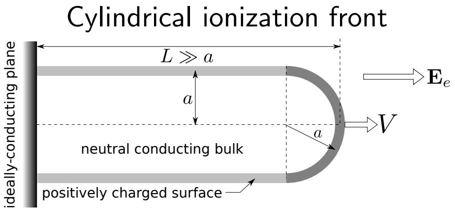

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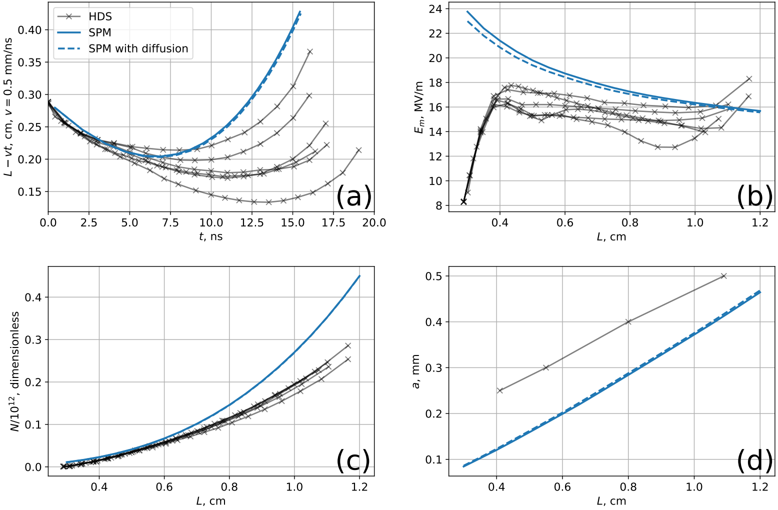
Comparison with hydrodynamic simulations (HDS): Case 1

HDS by Bagheri et al. [2018]: $E_e = 1.5$ MV/m, 1.25×1.25-cm cylindrical box

- No photoionization
- Background electron number density $n_e = 10^{13}$ m^{-3}.

- HDS have $\gtrsim$10% error
- Diffusion does not matter much



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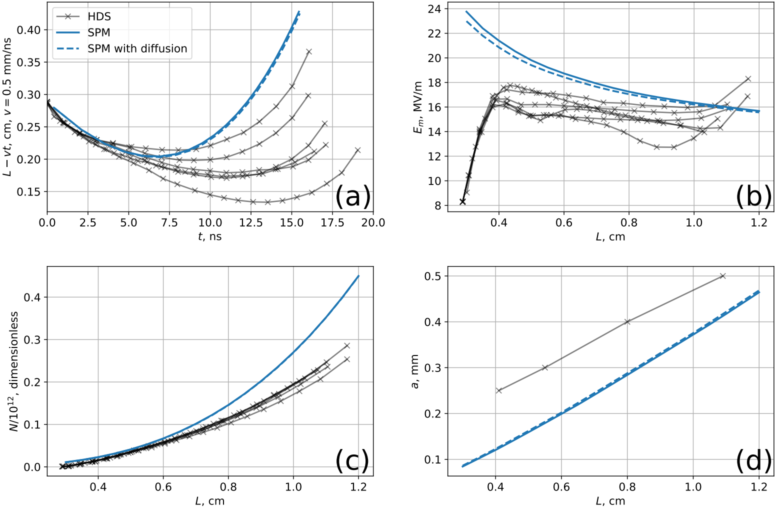

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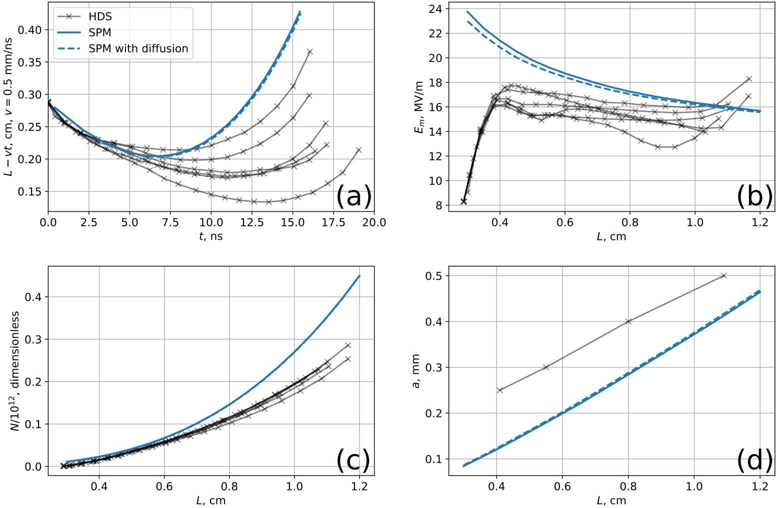

Comparison with hydrodynamic simulations (HDS): Case 2

- No photoionization
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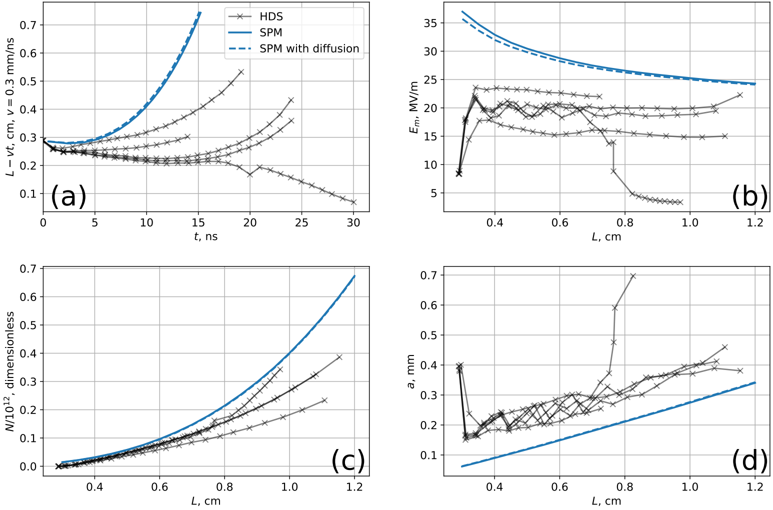
HDS had difficulties for the low number of “seeding” electrons



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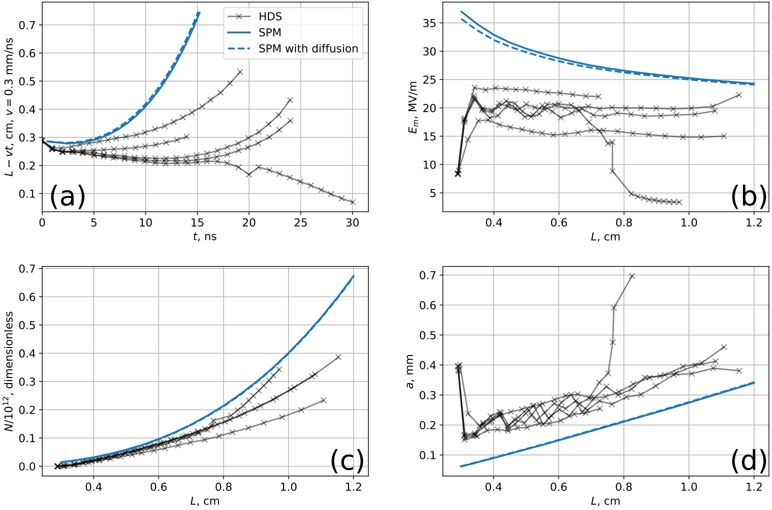

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Comparison with hydrodynamic simulations (HDS): Case 3

- Photoionization
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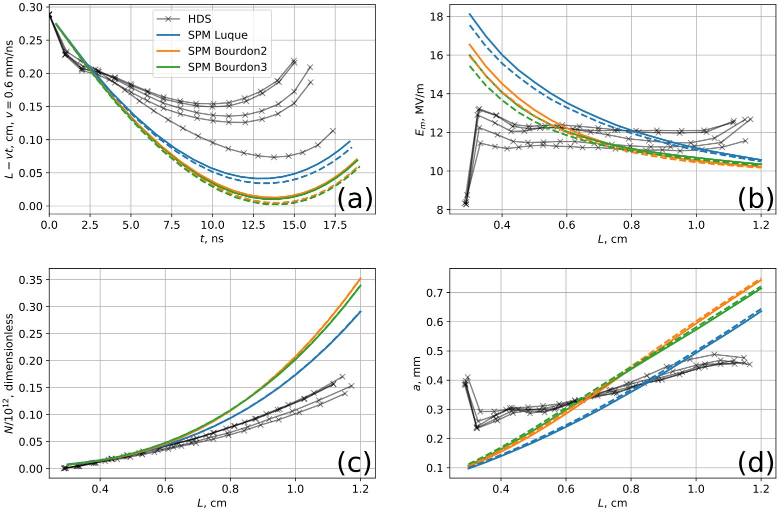

Propagation due mostly to photoionization

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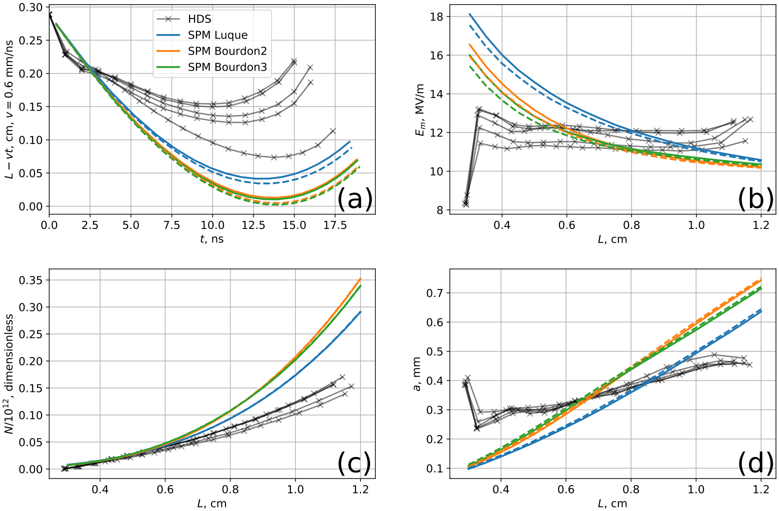

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Acknowledgements

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