Modelling ice-ocean interactions using Firedrake

Will Scott[1,2], Stephan Kramer[1], Benjamin Yeager[3], Paul Holland[4], Keith Nicholls[4], Martin Siegert[1,2], Matthew Piggott[1,2] and recently Dorothee Vallot (Swedish Met Office/Grenoble/St Andrews)

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Why Finite Elements?

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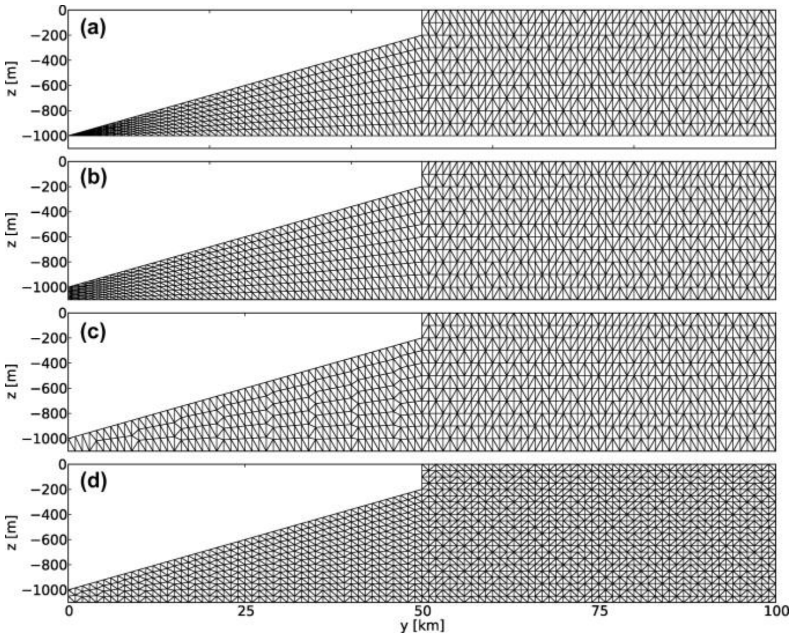
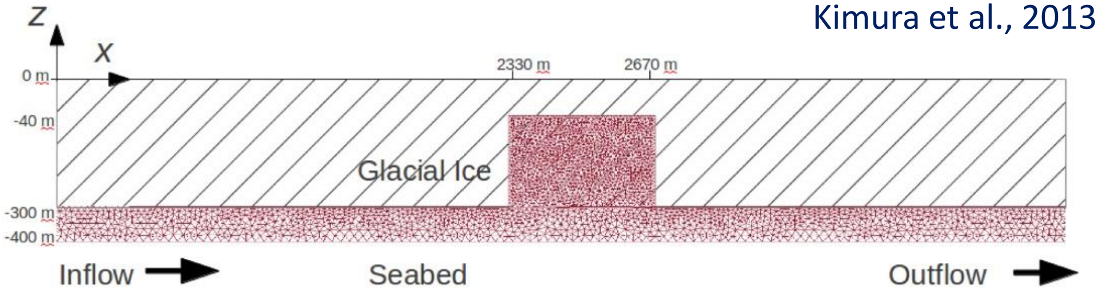
- Finite Elements on unstructured meshes
  - Good at fitting complicated geometries
  - Mesh resolution where you need it
- Fluidity project (Kimura et al. 2013, 2015, Jordan et al. 2014,15, Yeager, 2018)
  - Retain full physics so that order one aspect ratio is possible
  - DG Finite Elements
- Firedrake: abstraction is useful
  - Automatic adjoint



Kimura et al., 2013

fluidity → Firedrake



Jordan et al., 2014

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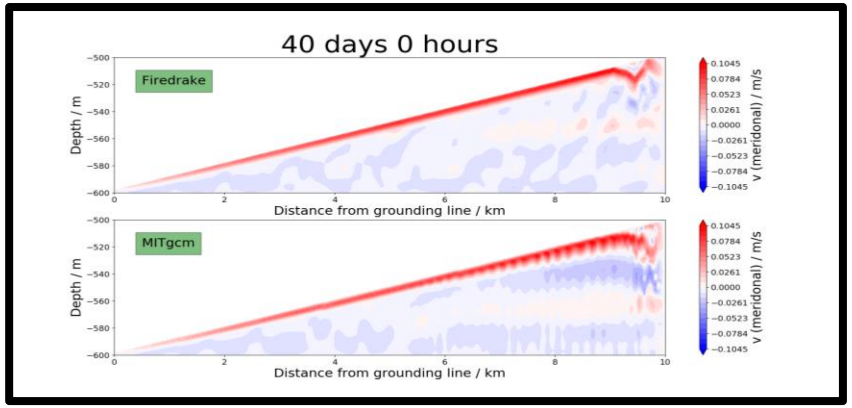
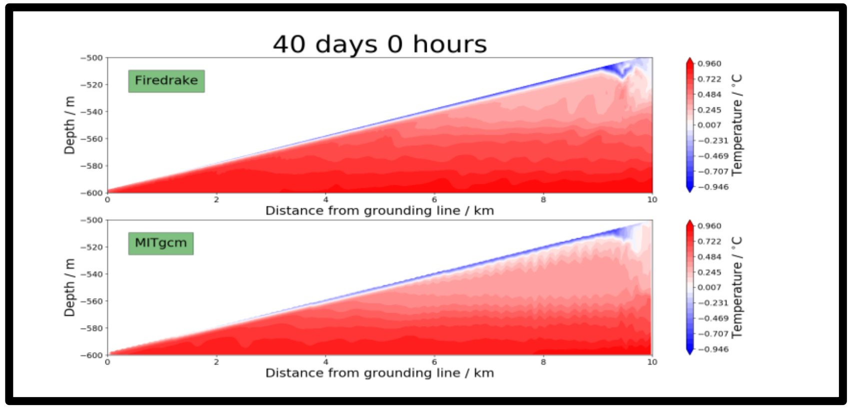
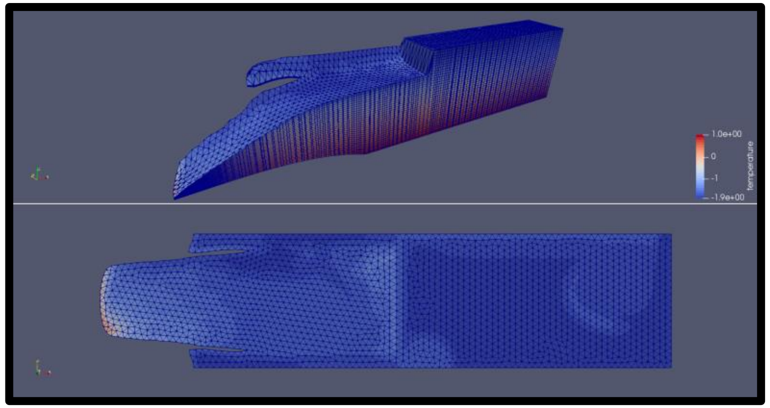
Selected highlights so far…

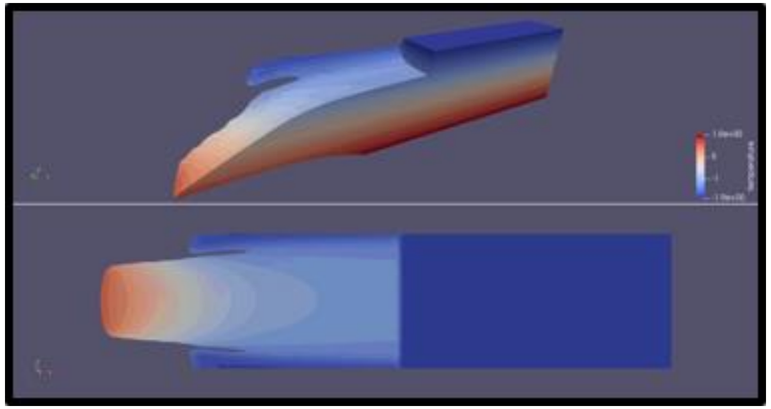
2d!



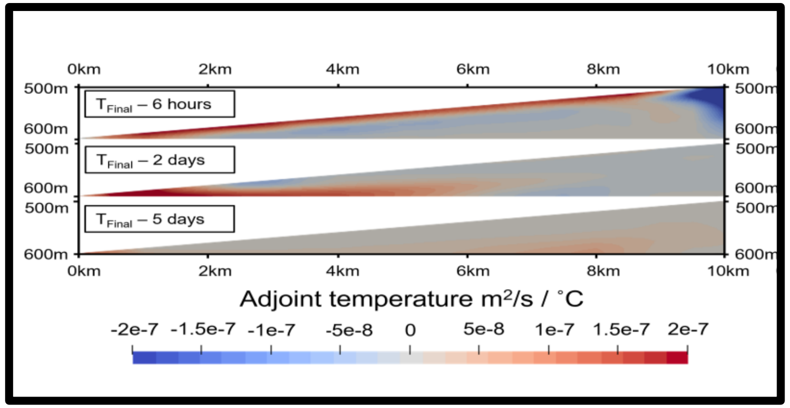


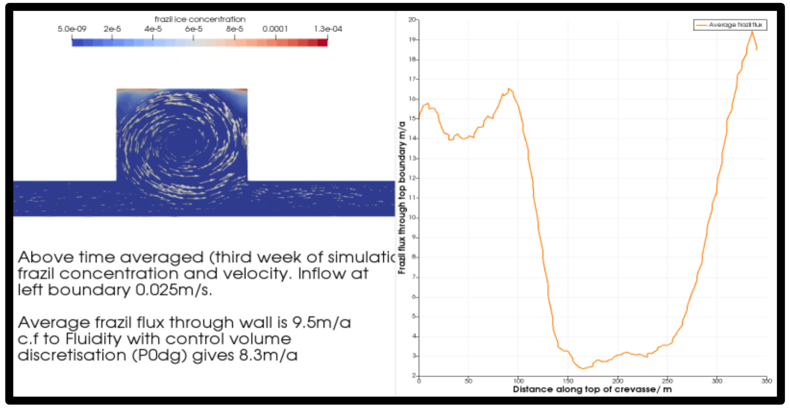
3d!





Adjoints and crevasses!





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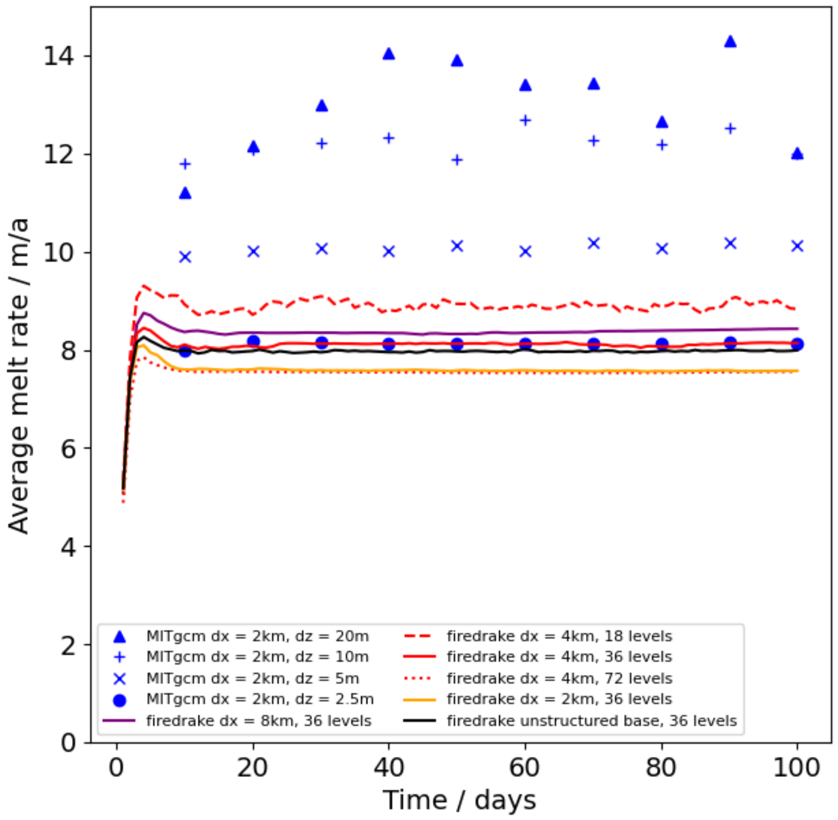
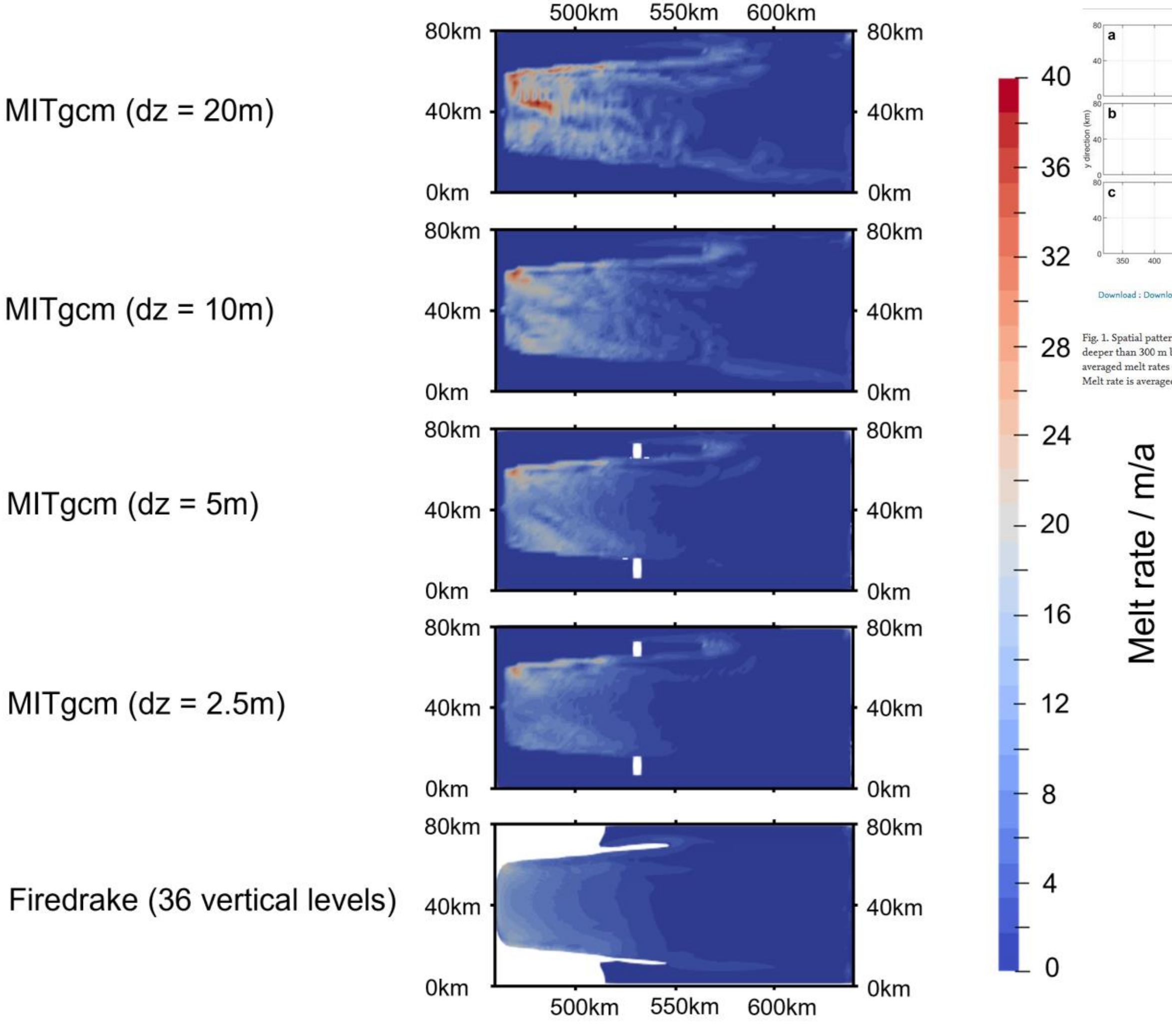
3d ISOMIP+: Melt sensitive to grid resolution





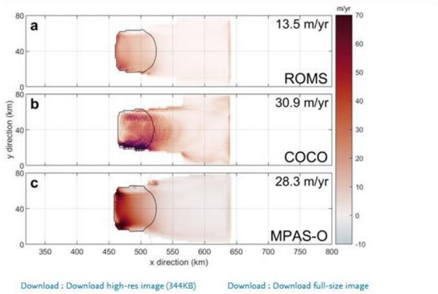


Fig. 1. Spatial pattern of time-averaged melt rate for (a) ROMS, (b) COCO and (c) MPAS-O. Ice deeper than 300 m below sea level is outlined with a black contour in each panel. Spatially averaged melt rates over this region are compared throughout the remainder of the paper. Melt rate is averaged over the last year of integration.

Gwyther et al 2020

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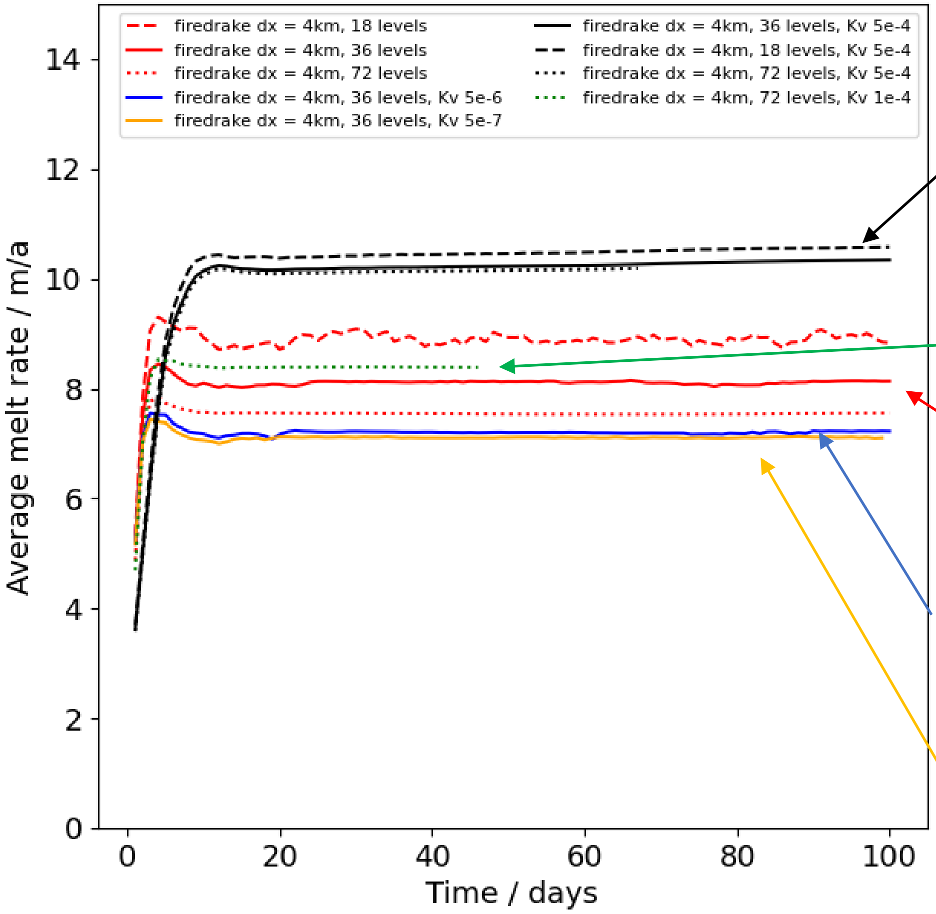
Numerical vs specified diffusivity

- Melting leads to stratification at ice boundary -> insulates boundary
- Trade off between:
    - Grid type (z ‘steps’ vs sigma ‘layers’)
    - specified diffusivity vs numerical diffusion (function of grid size)
    - Tuning turbulent exchange coefficients in melt parameterisation
- Specified diffusion must dominate numerical diffusion for grid convergence



Black: 10x higher isomip+ diffusivity 5e-4m^2/s (specified diffusion dominates)

Green: 2x higher isomip+ diffusivity 1e-4m^2/s (finer grid)

Red: Specified isomip+ diffusivity 5e-5m^2/s (no grid convergence)

Blue: 10x lower isomip+ diffusivity 5e-6m^2/s

Gold: 100x lower isomip+ diffusivity 5e-7m^2/s (numerical diffusion dominates)

Method of Manufactured Solution testing with Melt

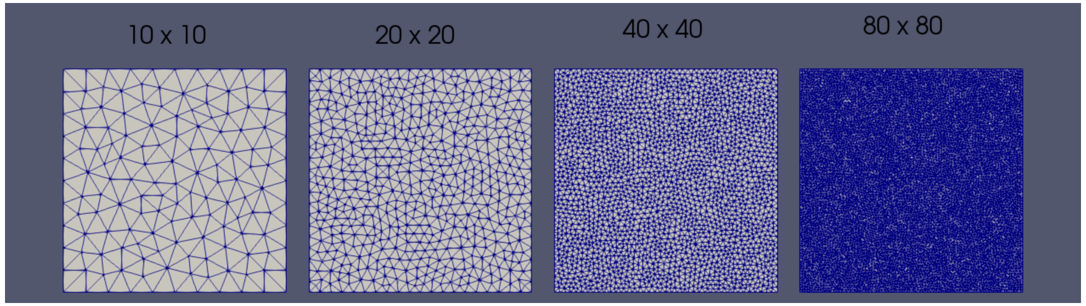
$$u = \frac{x}{L}\cos\left(-\frac{\pi(y+D-H_2)}{H_2}\right)$$

$$v = \frac{H_2}{\pi L}\sin\left(-\frac{\pi(y+D-H_2)}{H_2}\right)$$

$$p = \cos\left(\frac{\pi x}{L}\right)\cos\left(-\frac{\pi(y+D-H_2)}{H_2}\right)$$

$$T = 0.1\sin\left(\frac{4\pi x}{L}\right) + a_T y^2 + b_T y + c_T$$

$$S = 0.345\cos\left(\frac{4\pi x}{L}\right) + a_S y^2 + b_S y + c_S$$



100m x 100m domain with melt on top

	10x10	20x20	40x40	80x80
temperature	6.60e-1 -	1.52e-1 2.12	3.71e-2 2.03	9.43e-3 1.98
salinity	2.19 -	5.95e-1 1.88	1.57e-1 1.92	4.10e-2 1.94
melt	5.31e-7 -	1.21e-7 2.14	2.93e-8 2.04	7.39e-9 1.99
integrated melt	3.71e-7 -	6.23e-8 2.57	1.40e-8 2.16	3.95e-9 1.82
u velocity	2.34e-1 -	5.60e-2 2.06	1.40e-2 2.00	3.52e-3 1.99
v velocity	2.21e-1 -	5.53e-2 2.00	1.41e-2 1.97	3.54e-3 1.99
pressure	3.96e-1 -	1.13e-1 1.80	3.10e-2 1.87	8.36e-3 1.89

$$\mathbf{S_u} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p' - \nabla\cdot\nu(\nabla\mathbf{u} + (\nabla\mathbf{u}^T)) + \frac{\rho'}{\rho_0}g\hat{\mathbf{z}}$$

$$\mathbf{S}_u = \frac{-\pi\sin(\pi x/L)\cos(\pi(-H_2+D+y)/H_2)}{L} + \frac{x\sin(\pi(-H2+D+y)/H_2)^2}{L^2} + \frac{x\cos(\pi(-H_2+D+y)/H_2)^2}{L^2} + \frac{\pi^2\mu_v\, x\cos(\pi(-H_2+D+y)/H_2)}{H_2^2 L}$$

```
T_flux_bc_sympy = (-(-3.34629218167539e-9*y - 8.83700642033444e-6*(0.143778882407223*(0.000759376404*y + 0.0832)*(0.00101*cos(25*x*y) + 0.0303 - 0.00202*y/sqrt(x)) +\
    (0.000378668071800354*y - 0.499927196057832*sin(25*x*y) - 7.28039421677275e-5*cos(25*x*y) - 0.198293394983272 + y/sqrt(x))**2 + 0.0271555175202523*cos(25*x*y) +\
    0.814665525607568 - 0.0543110350405046*y/sqrt(x))**0.5 + 4.41785984126286e-6*sin(25*x*y) + 5.05643368904362e-7*cos(25*x*y) + 1.69023200045771e-5 -\
    9.84700642033444e-6*y/sqrt(x))/(0.00662632115183246*y + 17.4990226145236*(0.143778882407223*(0.000759376404*y + 0.0832)*(0.00101*cos(25*x*y) + 0.0303 -\
    0.00202*y/sqrt(x)) + (0.000378668071800354*y - 0.499927196057832*sin(25*x*y) - 7.28039421677275e-5*cos(25*x*y) - 0.198293394983272 + y/sqrt(x))**2 +\
    0.0271555175202523*cos(25*x*y) + 0.814665525607568 - 0.0543110350405046*y/sqrt(x))**0.5 - 8.74823730943141*sin(25*x*y) - 0.00127399783041954*cos(25*x*y) -\
    3.46994060312294 + 17.4990226145236*y/sqrt(x)) - 0.0001)*(0.000379688202*y - 1.0026939958122*(0.143778882407223*(0.000759376404*y + 0.0832)*(0.00101*cos(25*x*y)+\
    0.0303 - 0.00202*y/sqrt(x)) + (0.000378668071800354*y - 0.499927196057832*sin(25*x*y) - 7.28039421677275e-5*cos(25*x*y) - 0.198293394983272 + y/sqrt(x))**2 +\
    0.0271555175202523*cos(25*x*y) + 0.814665525607568 - 0.0543110350405046*y/sqrt(x))**0.5 - 0.49872600216958*sin(25*x*y) + 7.30000756830394e-5*cos(25*x*y) +\
    0.282027596558944 + 0.997306004187795*y/sqrt(x))
```

Next steps and longer term aims

- **Crevasse geometry experiment – is rotation important?**
- **Adjoint sensitivity and optimisation**
    - Focus on boundary mixing: parameterised turbulent exchange coefficients and modelled diffusivity
    - Demonstration on an idealised small problem
- *Subglacial discharge / Basal channels*
- *Turbulence*
    - *GLS vs isotropic (vertical)*
    - *LES in horizontal*
- *Free surface*
    - *plus wetting and drying to investigate tidal problem*
- *Ice model coupling*