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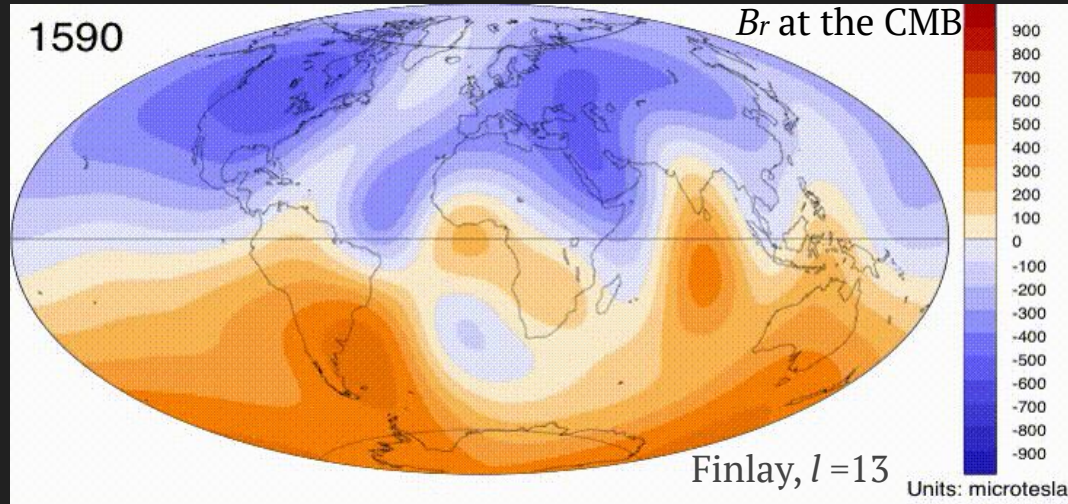
A laboratory study of turbulent magnetoconvection:

Could thermoelectricity induce asymmetry in geomagnetic secular variation?

Yufan Xu¹, Susanne Horn^{1,2}, Jonathan Aurnou¹



The Geodynamo

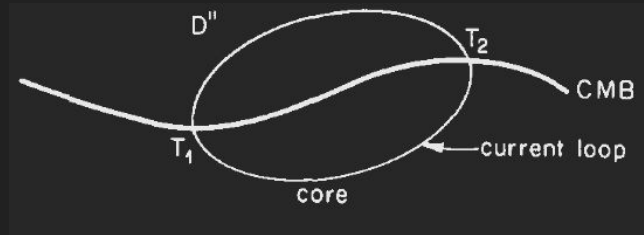


- ❖ Time-dependent secular variation
- ❖ Inclination anomaly from paleomagnetic data
 - Asymmetry during field reversals
 - Hemispheric asymmetry
 - Debates
- ❖ Source of asymmetry
 - Deeper core flows? Or near-CMB dynamics?
 - Thermoelectricity

Conundrum of Thermoelectricity at CMB

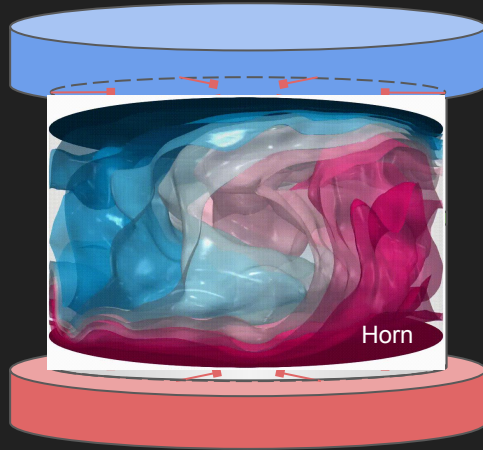
$$\Phi_{TE} \sim \tilde{S} \Delta T_p$$

- ❖ All focused on **toroidal field generation**. Too small to explain polarity asymmetry.
 - Elsasser (1939): Thermoelectricity (TE) → geomagnetism
 - Runcorn (1954), Merrill et al. (1979) (1990): TE from CMB
 - TE generates Mercury's dynamo? (Stevenson 1987, *EPSL*; Giampieri & Balogh 2002, *P&SS*)
- ❖ To characterize the system: laboratory experiment!



Merrill et al. 1990

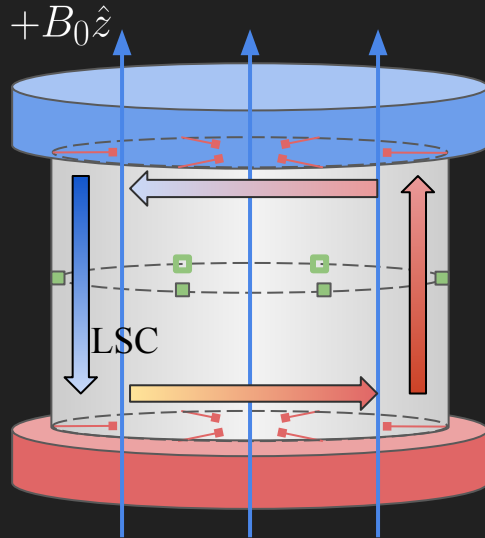
Experiment Setup



Working fluid: liquid gallium ($Pr = 0.027$, $Rm < 0.015$)



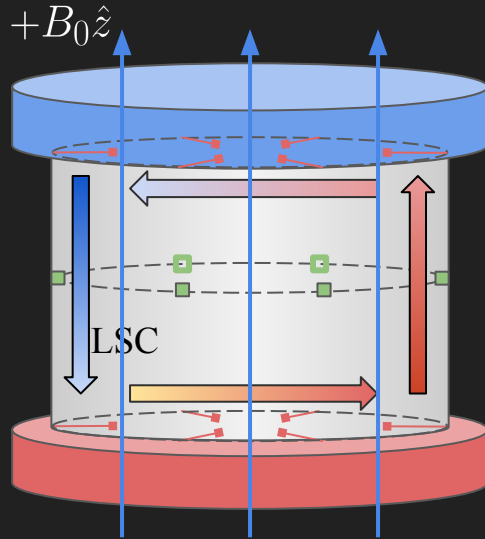
Experiment Setup



Working fluid: liquid gallium ($Pr = 0.027$, $Rm < 0.015$)

- ❖ Thermal signals \rightarrow fluid motions in liquid metal.
- ❖ Classical turbulent Rayleigh-Bénard convection setup with a vertical B field
- ❖ Large Scale Circulation (LSC) \rightarrow self-sustaining ∇T at the top/bottom boundaries.
- ❖ B field should inhibits convective fluid motions (low Rm).

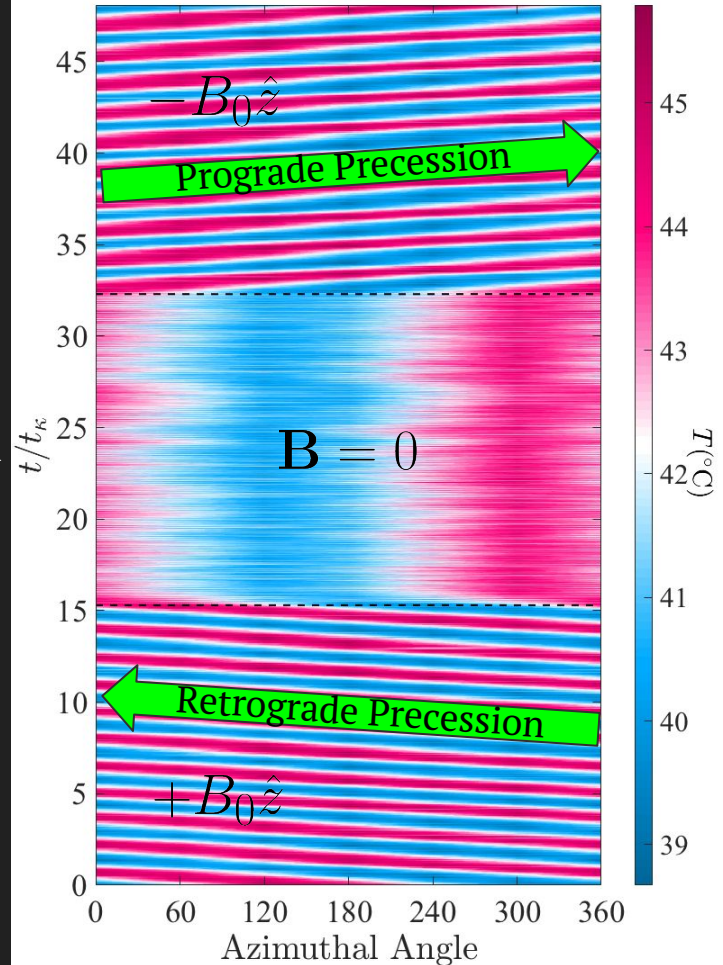
Experiment Results



- ❖ Slow **magneto-precession (MP)** mode
- ❖ Sensitive to the polarity of the magnetic field
- ❖ No significant change in heat transfer

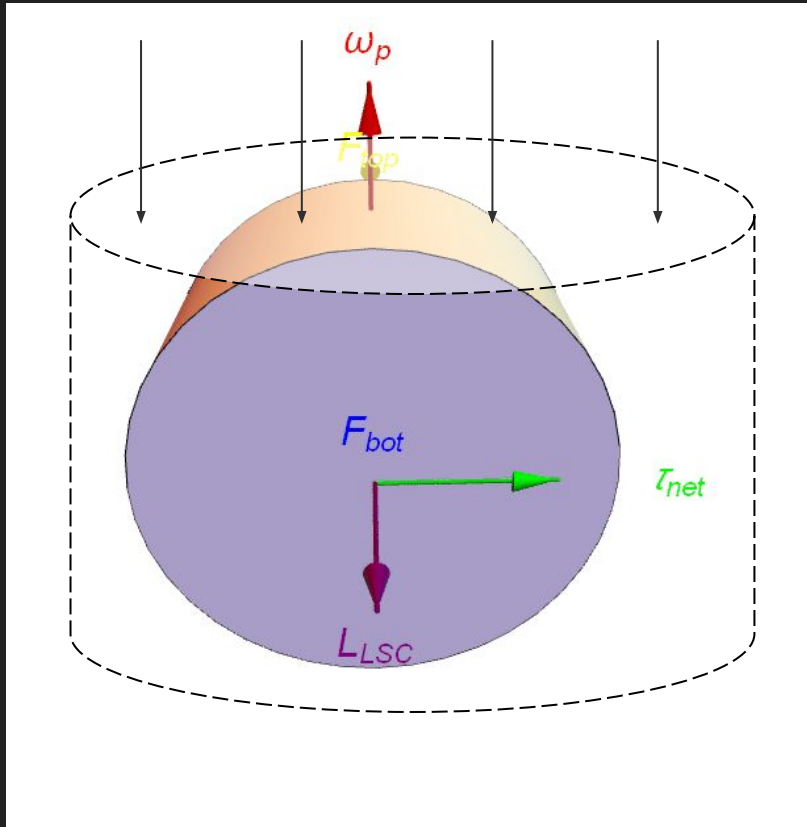
Thermal
Diffusion
Times

Midplane Sidewall Temperature Contour



Xu et al. 2022, JFM

Analytical Model for TEMC



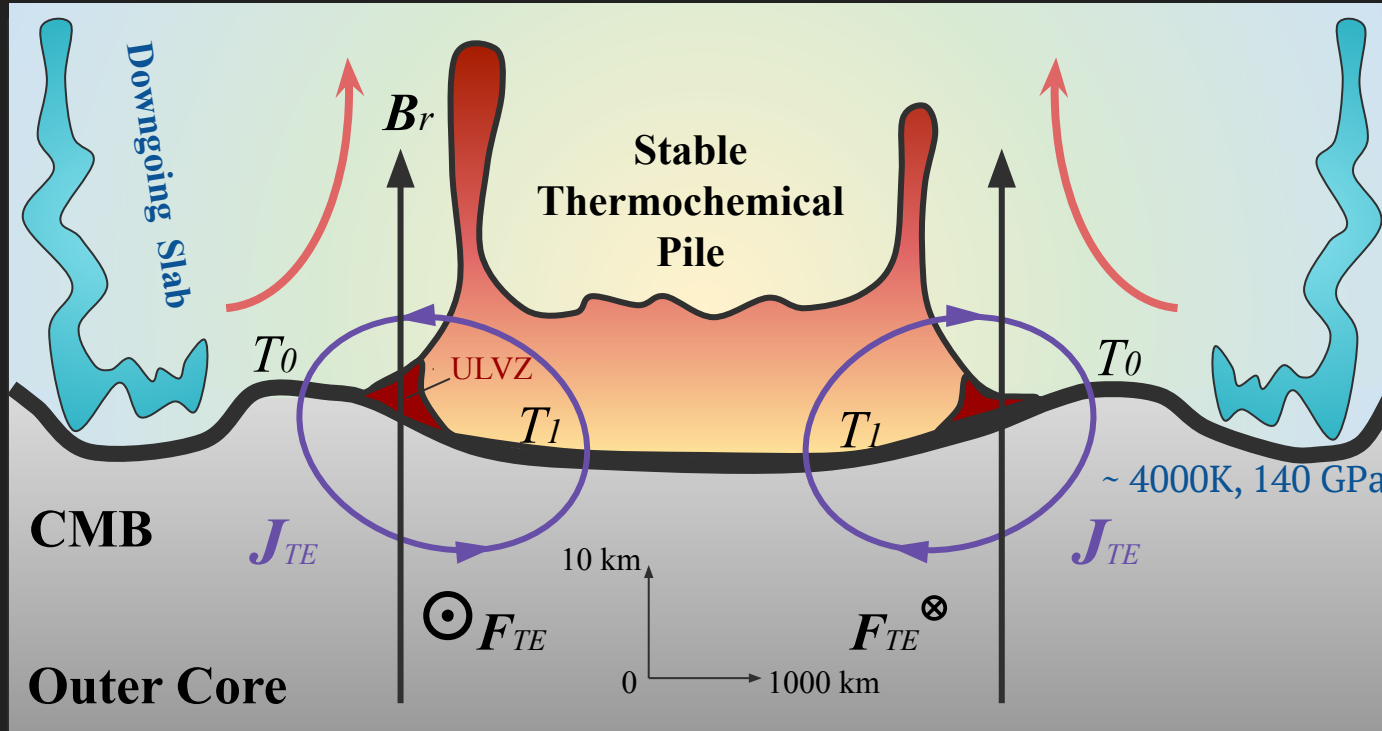
- ❖ Thermoelectric precession angular frequency:

$$\omega_{MP} = \frac{\tau_{net}}{L_{LSC}} (-\hat{e}_b)$$

$$\omega_{MP} = \frac{\pi^{1/2} \sigma_0 X_0 \boxed{B} T}{\Gamma^{3/2} 2\rho U_{ff} H} (-\hat{e}_b)$$

- ❖ Precession reverses its direction if B is flipped
- ❖ Agree with experimental data <16%

Thermoelectricity at CMB?



- ❖ TE currents with B field can exert torques on core fluid near CMB mega-ULVZs
- ❖ Potential new dynamic mechanism for core-mantle coupling.
- ❖ Induction equation: changes in velocity field that can cause polarity asymmetry in geomagnetic fields.
- ❖ Current uncertainty: CMB properties

Xu et al. 2022, *JFM*. Adapted from Garnero, McNamara & Shim (2016) and Deschamps, Rogister & Tackley (2018)

Looking for Geophysical Evidence and Collaborations

❖ **Mantle dynamicists:**

- Thermal gradient along CMB?

❖ **Mineral Physicists:**

- Seebeck coefficients of CMB materials? Possibly can be studied by first-principle simulations.

❖ **Theorists/experimentalists:**

- Controlled ∇T TEMC experiments including effects of rotation & stratification.

❖ **Geophysicists:**

- Observations of polarity asymmetry in paleomagnetic secular variations near mega-ULVZs.

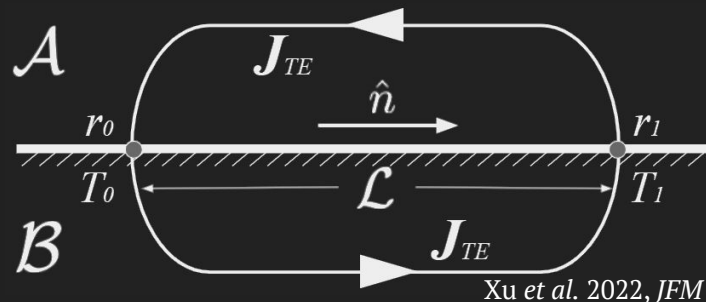
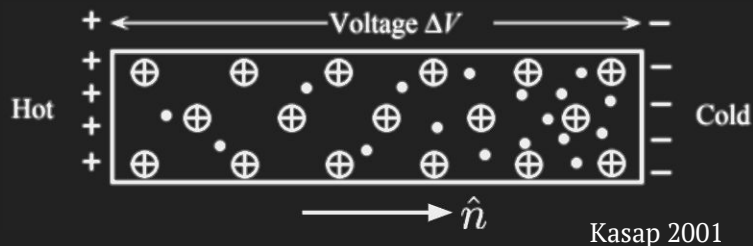


Xu et al. 2022, JFM

Backup

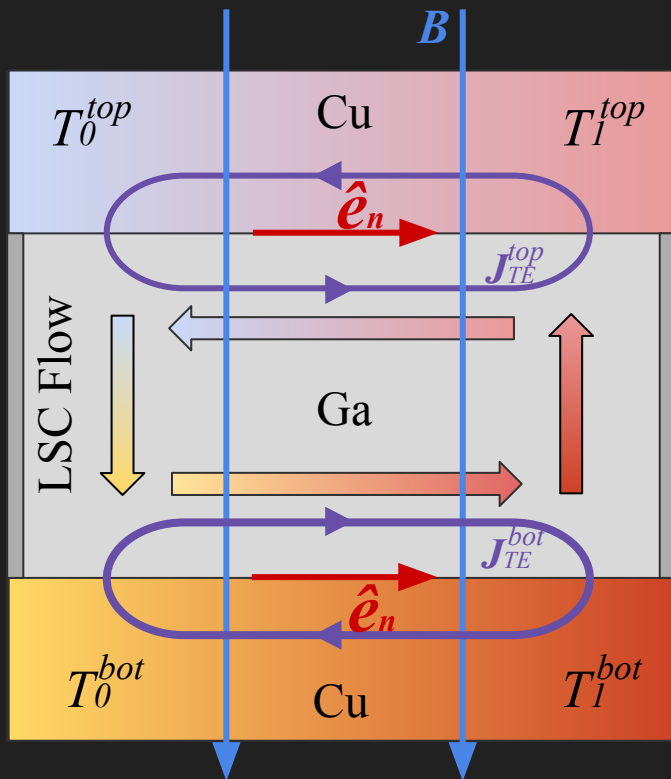
Thermoelectric Effects

- ❖ The Seebeck Effect : Temperature gradient \longrightarrow Electric potential $\Phi_{TE} \sim \tilde{S} \Delta T_p$



- ❖ TE currents generated from an imposed temperature gradient with a perpendicular B field \longrightarrow Lorentz forces F_L on liquid metal (Shercliff 1979).

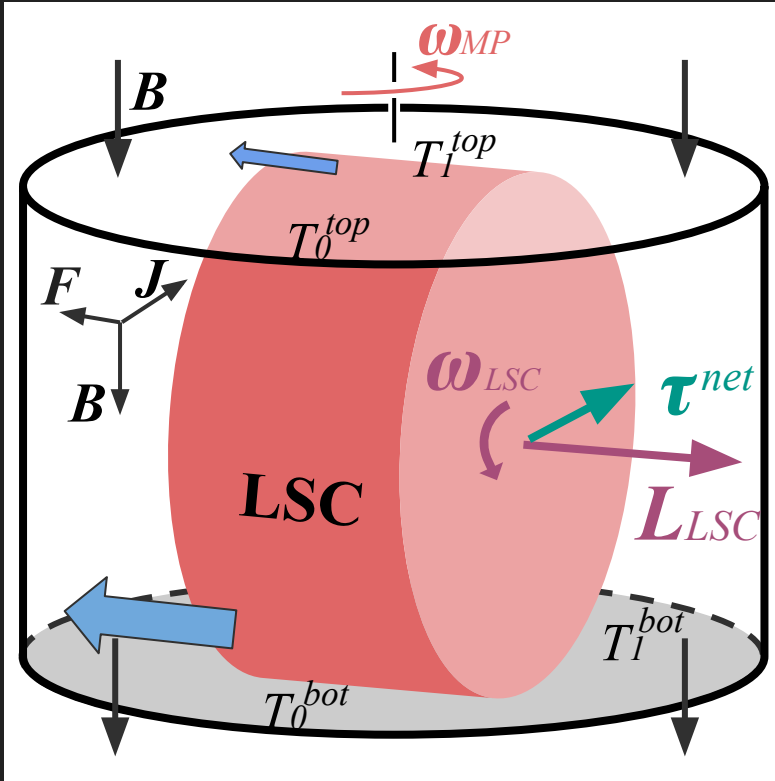
Analytical Model for TEMC



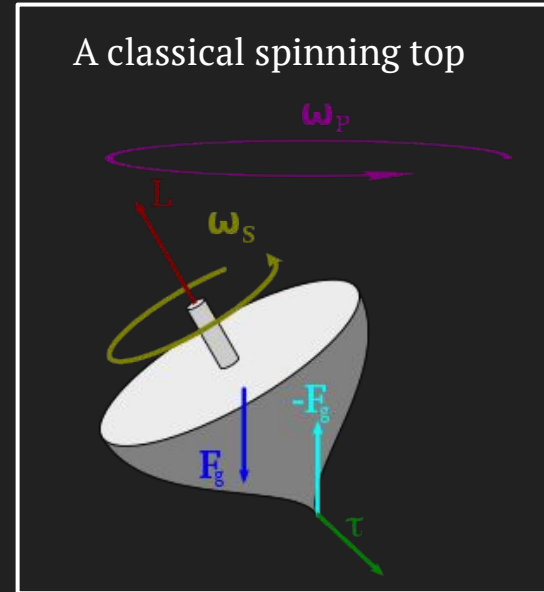
Xu et al. 2022, JFM

- ❖ LSC creates δT_{\perp} at top and bottom boundaries
- ❖ δT_{\perp} generates TE currents J_{TE} at the interfaces
- ❖ TE currents J_{TE} and the vertical B field generate Lorentz Forces $\mathbf{F}_L = \mathbf{J}_{TE} \times \mathbf{B}$
- ❖ The net \mathbf{F}_L generates torques perpendicular to the moment of inertia of the LSC.
- ❖ TE magneto-precessional mode!

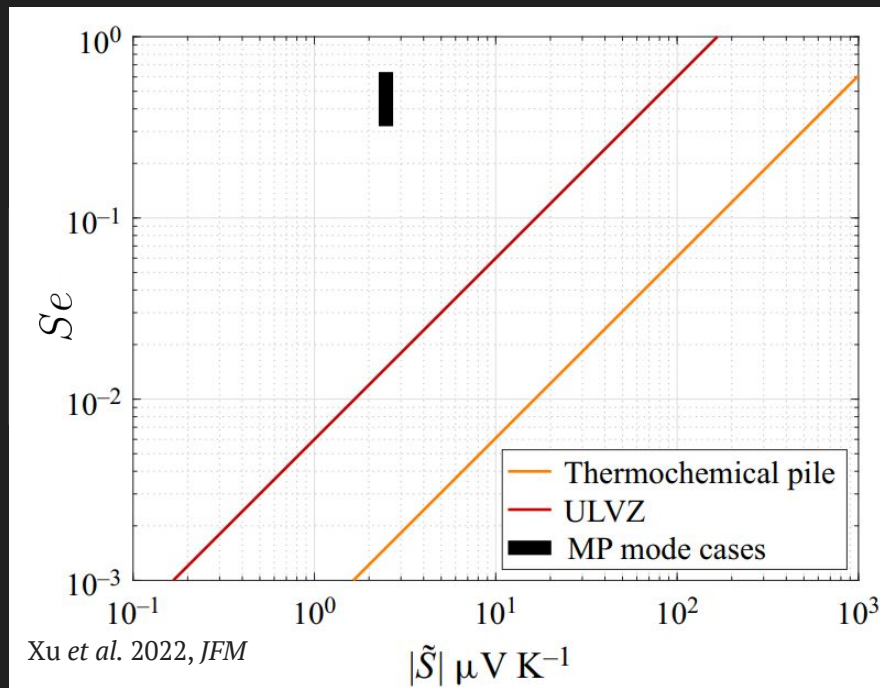
Analytical Model for TEMC



Xu et al. 2022, JFM



Seebeck Number Estimates



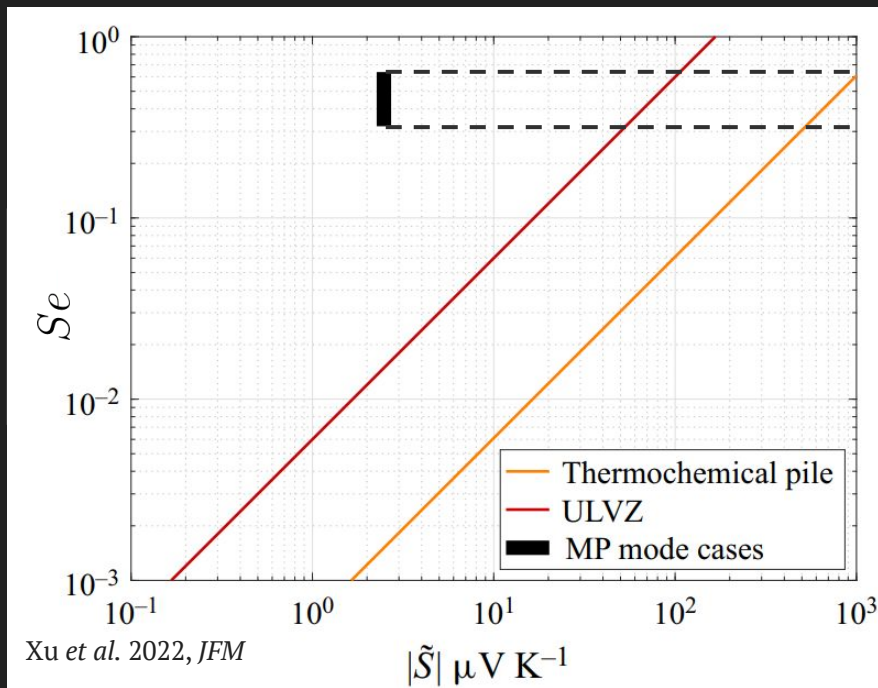
Seebeck Number, Se , measures significance of the TE dynamics:

$$Se = \frac{|\tilde{S}|\Delta T_p}{U_C B_r L} = \frac{\text{TE Potential}}{\text{Motion Induced Potential}}$$

- ❖ ΔT_p : lateral temperature difference $\sim 300\text{K}$
- ❖ U_c : Outer core flow speed near CMB $\sim 0.1\text{mm/s}$
- ❖ B_r : Radial geomagnetic field $\sim 1\text{mT}$
- ❖ Length of thermochemical pile $\sim 5000\text{km}$
- ❖ Length of ULVZs $\sim 500\text{km}$

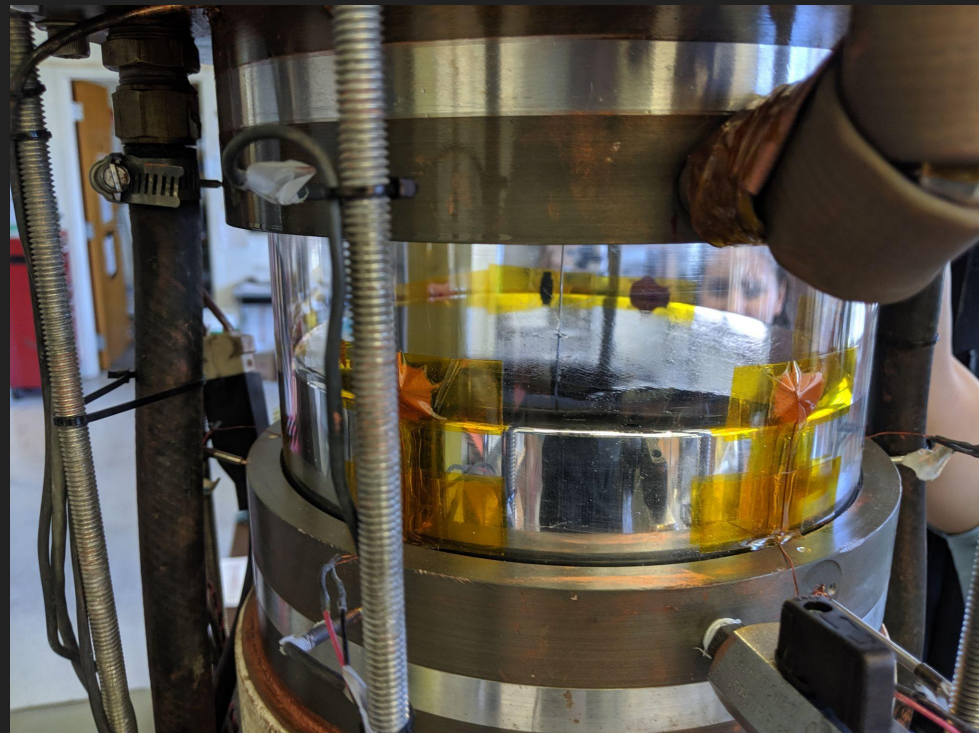
Net Seebeck Coefficient, \tilde{S}

Seebeck Number Estimates



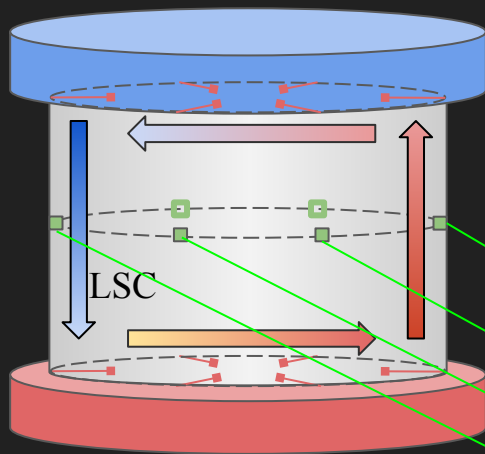
Net Seebeck Coefficient, \tilde{S}

- ❖ TEMC flows are at $Se \gtrsim \mathcal{O}(10^{-1})$
- ❖ Thermochemical pile or ULVZ:
 $|\tilde{S}| \gtrsim \mathcal{O}(10^2) - \mathcal{O}(10^3)$
- ❖ **Mantle dynamicists:**
 - Thermal gradient along CMB?
- ❖ **Mineral Physicists:**
 - Seebeck coefficients of CMB materials? Possibly can be studied by first-principle simulations.
- ❖ **Theorists/experimentalists:**
 - Controlled ∇T TEMC experiments including effects of rotation & stratification.



- Study 1: Electrically-insulated
Boundary: Teflon-coated Aluminum

$$\mathbf{B} = 0$$

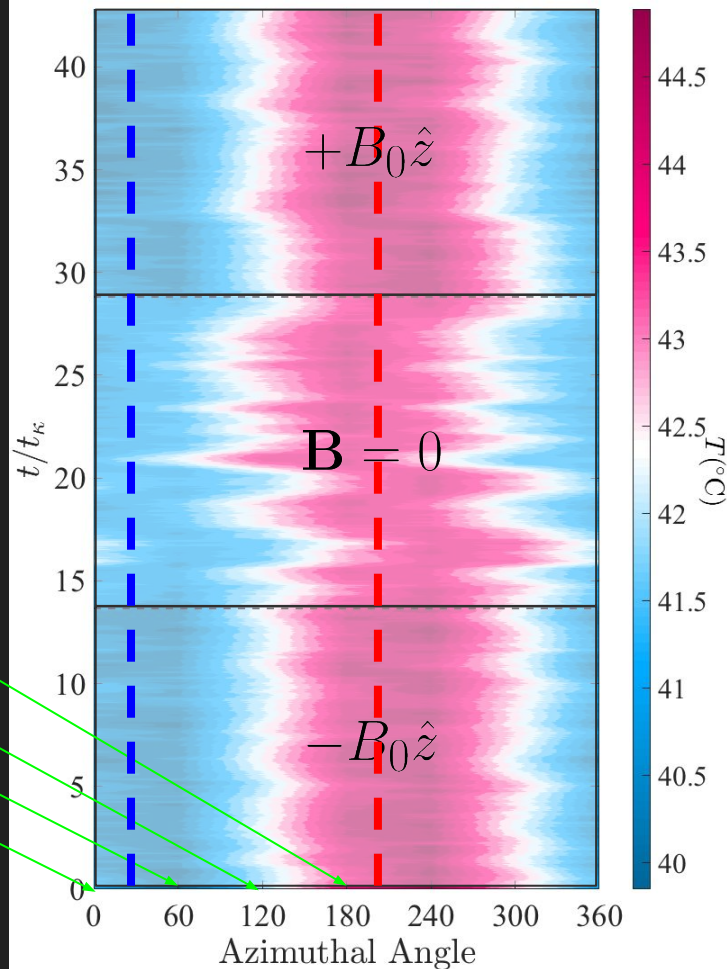


Thermal
Diffusion
Times

$$Ra = \frac{\text{Buoyancy}}{\text{Diffusion}} = 1.62 \times 10^6$$

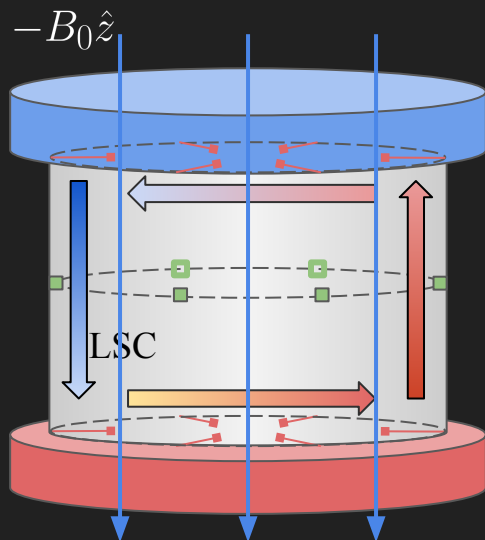
$$N = \frac{\text{Lorentz}}{\text{Inertia}} = 0$$

Midplane Sidewall Temperature Contour



Xu et al. 2022, JFM

- Study 1: Electrically-insulated
Boundary: Teflon-coated Aluminum

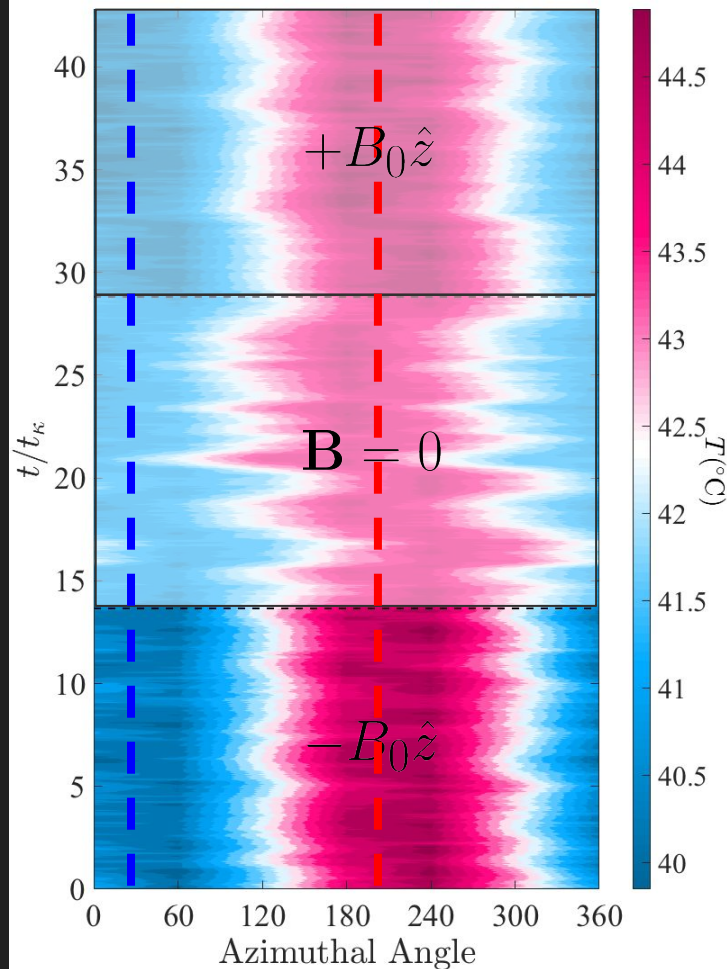


$$Ra = \frac{\text{Buoyancy}}{\text{Diffusion}} = 1.62 \times 10^6$$

$$N = \frac{\text{Lorentz}}{\text{Inertia}} = 0.29$$

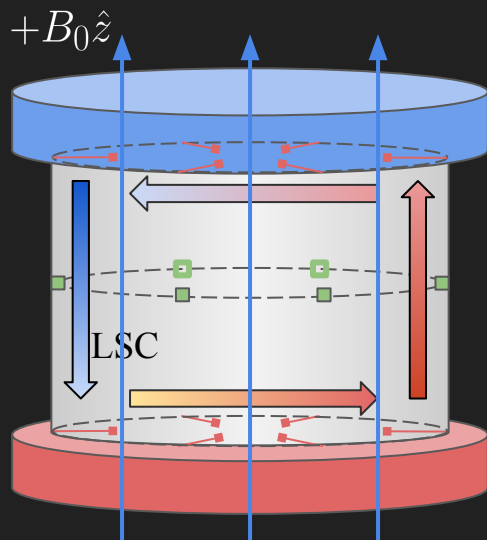
Thermal
Diffusion
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Midplane Sidewall Temperature Contour



Xu et al. 2022, JFM

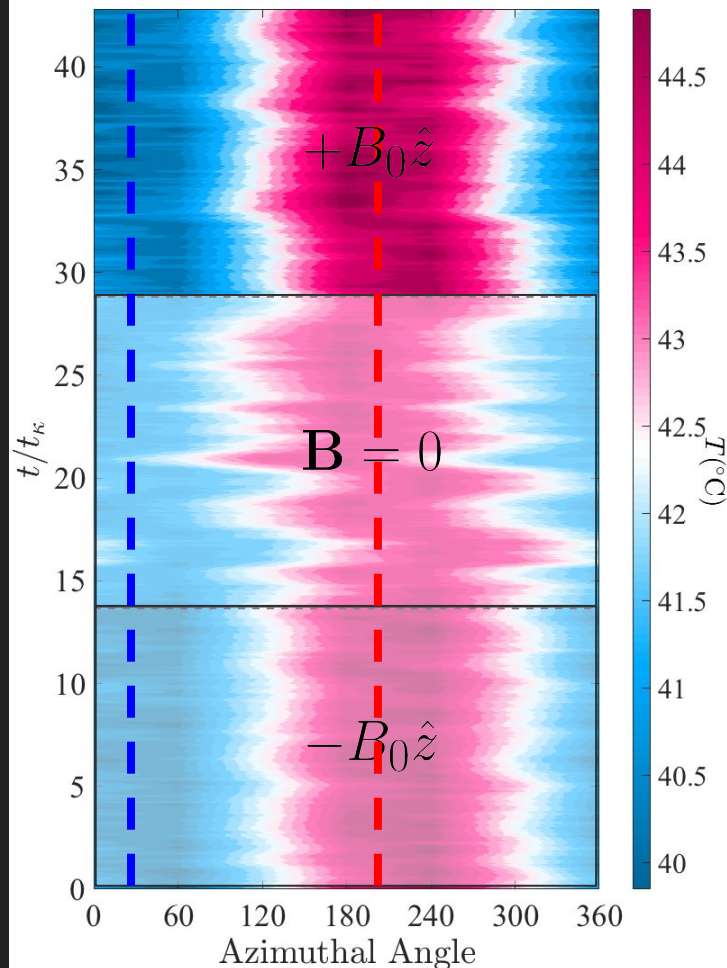
- Study 1: Electrically-insulated
Boundary: Teflon-coated Aluminum



Thermal
Diffusion
Times

- ❖ LSC persists in magnetoconvection
- ❖ Agree with our expectation from the standard Low- Rm approximation (Cioni, 2000; Zürner *et. al*, 2020)
- ❖ MC's characteristic frequency agrees with prediction (Vogt *et. al*, 2018)

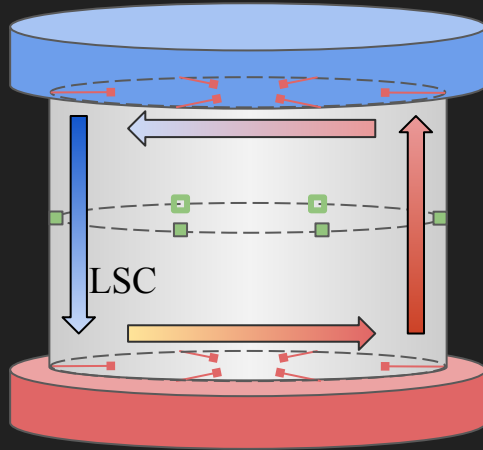
Midplane Sidewall Temperature Contour



Xu *et al.* 2022, *JFM*

- Study 2: Electrically-conducting
Boundary: Copper

$$\mathbf{B} = 0$$

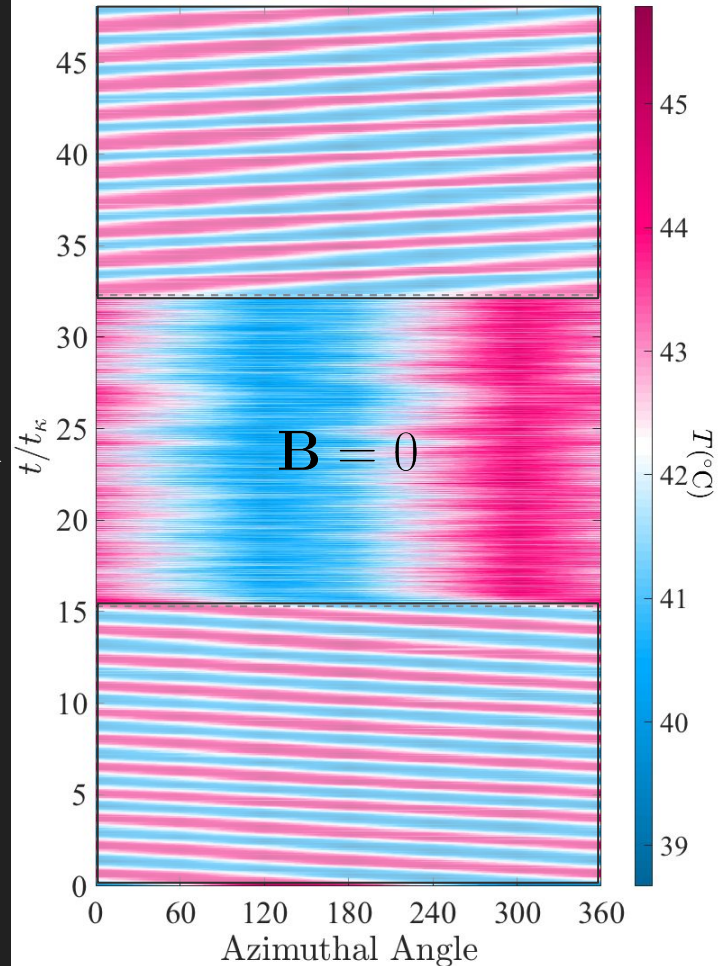


$$Ra = \frac{\text{Buoyancy}}{\text{Diffusion}} = 1.80 \times 10^6$$

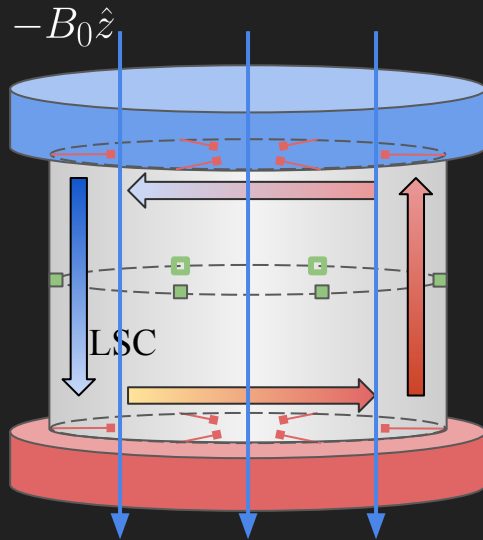
$$N = \frac{\text{Lorentz}}{\text{Inertia}} = 0$$

Thermal
Diffusion
Times

Midplane Sidewall Temperature Contour



- Study 2: Electrically-conducting Boundary: Copper

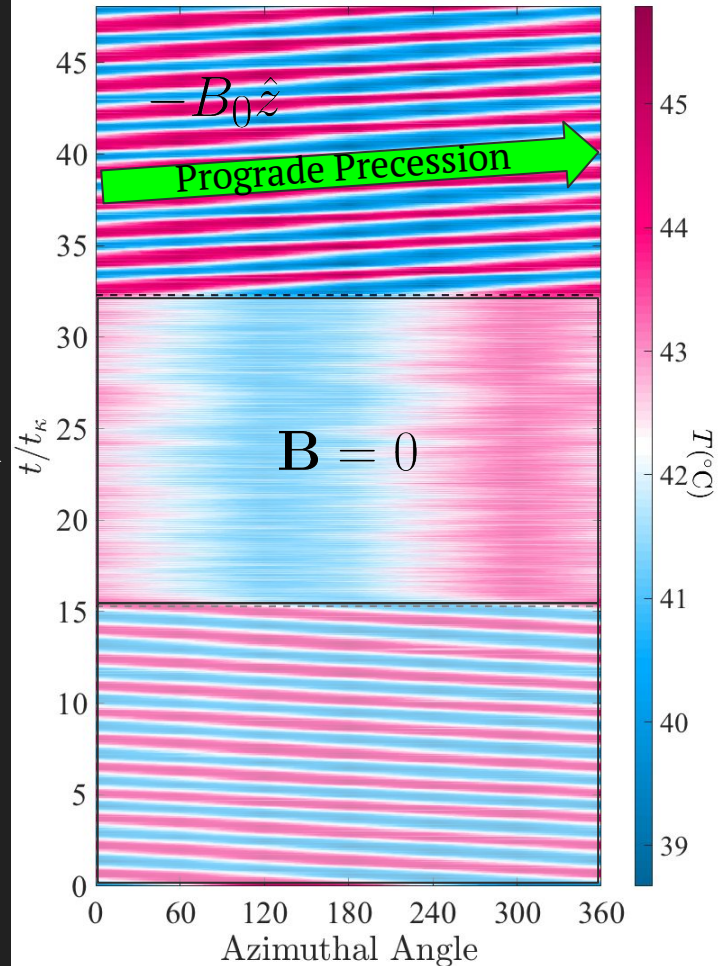


$$Ra = \frac{\text{Buoyancy}}{\text{Diffusion}} = 1.83 \times 10^6$$

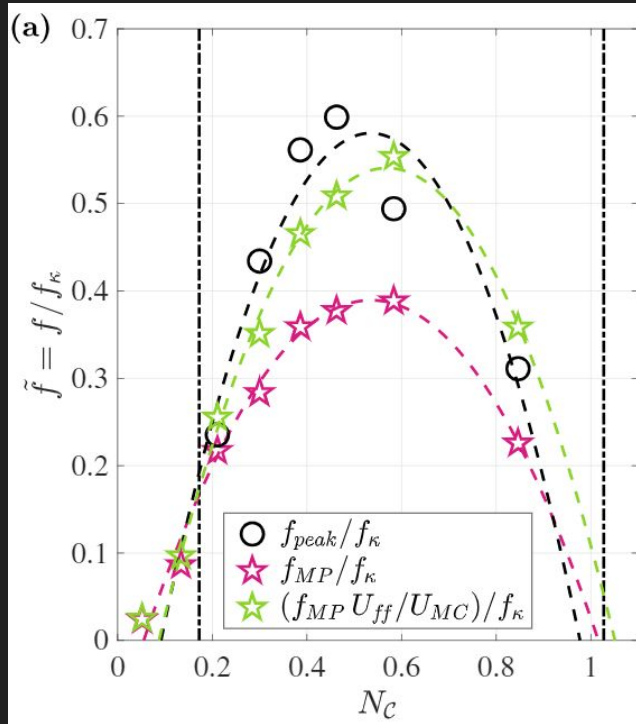
$$N = \frac{\text{Lorentz}}{\text{Inertia}} = 0.31$$

Thermal
Diffusion
Times

Midplane Sidewall Temperature Contour



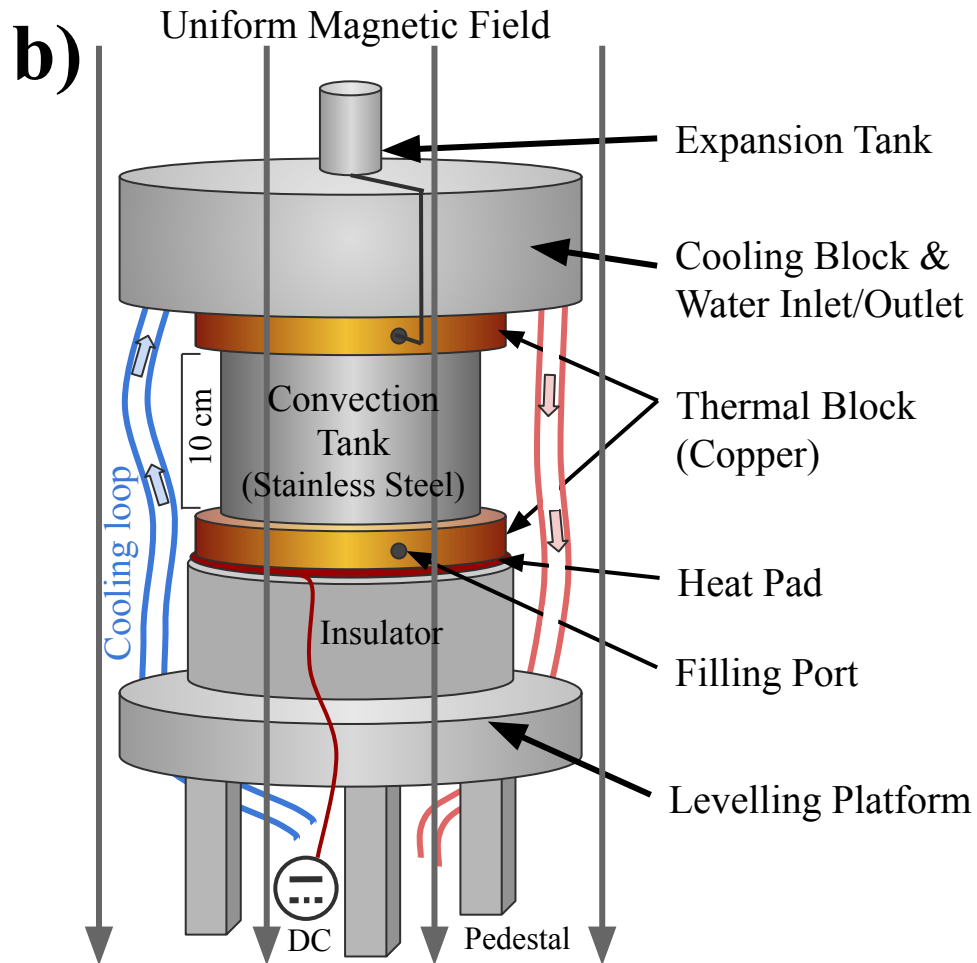
Analytical Model vs. Experimental Data



Lorentz/Inertia

Xu *et al.* 2022, *JFM*

- ❖ Fixed Ra (Buoyancy/ Diffusion), varying N_c (Lorentz/Inertia).
- ❖ <16% difference between the model using U_{MC} (Zürner *et. al*, 2020) and the experiments.
- ❖ When Lorentz force becomes comparable with the inertia ($N_c \gtrsim 1$) \rightarrow multi-cellular flow.



Low- Rm (Quasistatic) Approximation

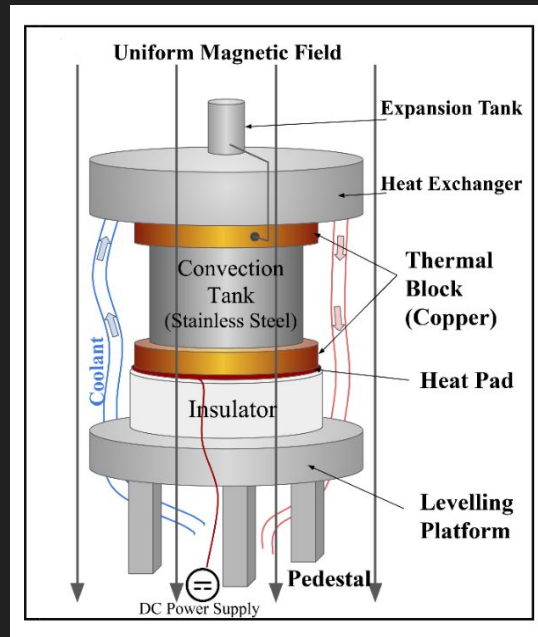
- ❖ In this limit, the induced magnetic field by fluid motion is negligible. The resulting Lorentz force is:

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}_0 \sim -\sigma \mathbf{u}_\perp B_0^2$$

Expectation:

- ❖ The Lorentz forces act as the magnetic drag.
- ❖ No traveling/oscillatory mode.
- ❖ Vertical polarity of the applied field should not matter.

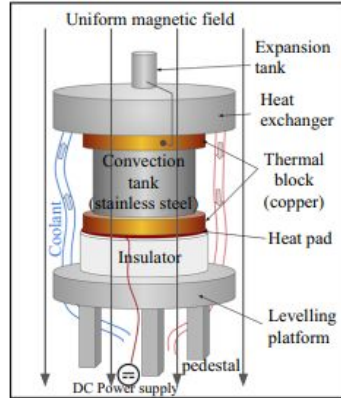
Working fluid: liquid gallium $Rm \lesssim 0.015$



RoMag, UCLA

Number names	Symbol	Definition	Equivalence	Current study
Magnetic Reynolds	Rm	$\frac{U_{ff}H}{\eta}$	$RePm$	$\lesssim 10^{-2}$
Magnetic Prandtl	Pm	$\frac{\nu}{\eta}$	—	1.7×10^{-6}
Prandtl	Pr	$\frac{\nu}{\kappa}$	—	2.7×10^{-2}
Rayleigh	Ra	$\frac{\alpha g \Delta T H^3}{\nu \kappa}$	—	$\sim 2 \times 10^6$
Chandrasekhar	Ch	$\frac{\sigma B^2 H^2}{\rho \nu}$	—	$[0, 8.4 \times 10^4]$
Seebeck	Se	$\frac{ \tilde{S} \Delta T / H}{U_{ff} B}$	—	$\sim [10^{-2}, 1]$
Aspect ratio	Γ	$\frac{D}{H}$	—	2.0
Reynolds	Re	$\frac{U_{ff}H}{\nu}$	$\sqrt{\frac{Ra}{Pr}}$	$\lesssim 8.7 \times 10^3$
Péclet	Pe	$\frac{U_{ff}H}{\kappa}$	\sqrt{RaPr}	$\lesssim 2.2 \times 10^2$
Convective interaction	N_C	$\frac{\sigma B^2 H}{\rho U_{ff}}$	$\sqrt{\frac{Ch^2 Pr}{Ra}} = \frac{Ch}{Re}$	$\lesssim 10$
Thermoelectric interaction	N_{TE}	$\frac{\sigma B \tilde{S} \Delta T}{\rho U_{ff}^2}$	$Se N_C$	$\lesssim 10$

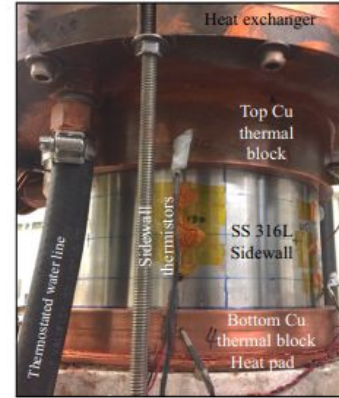
(a)



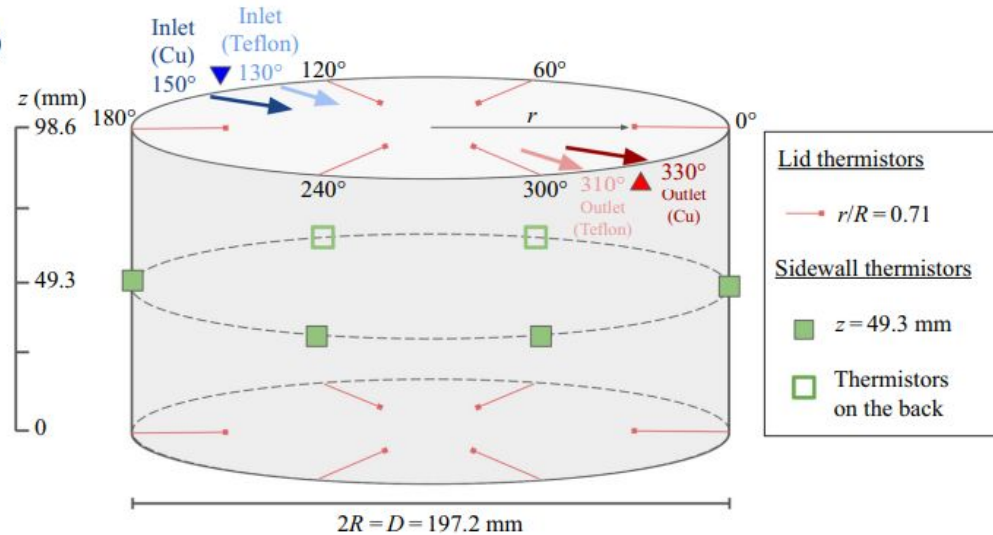
(b)

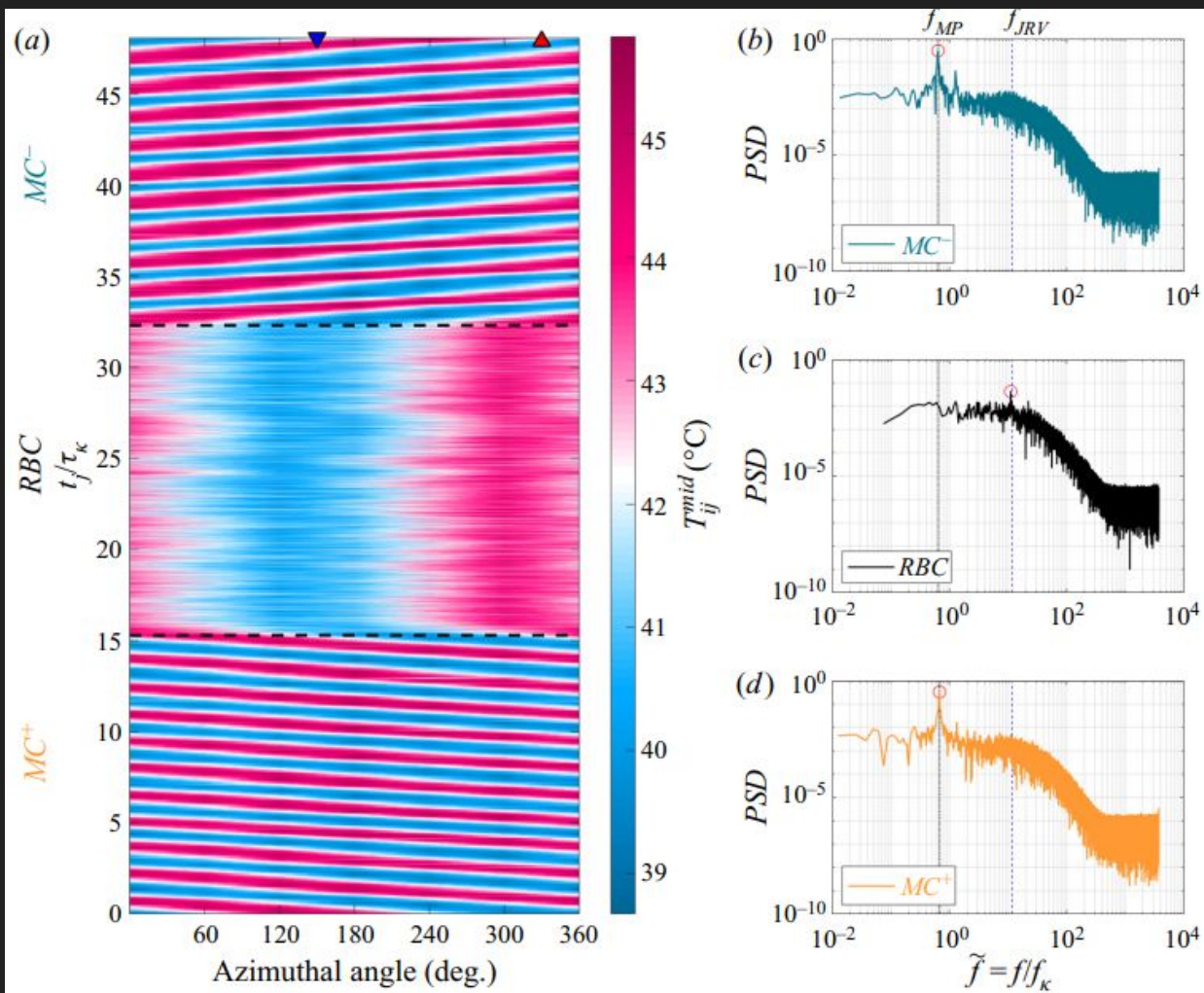


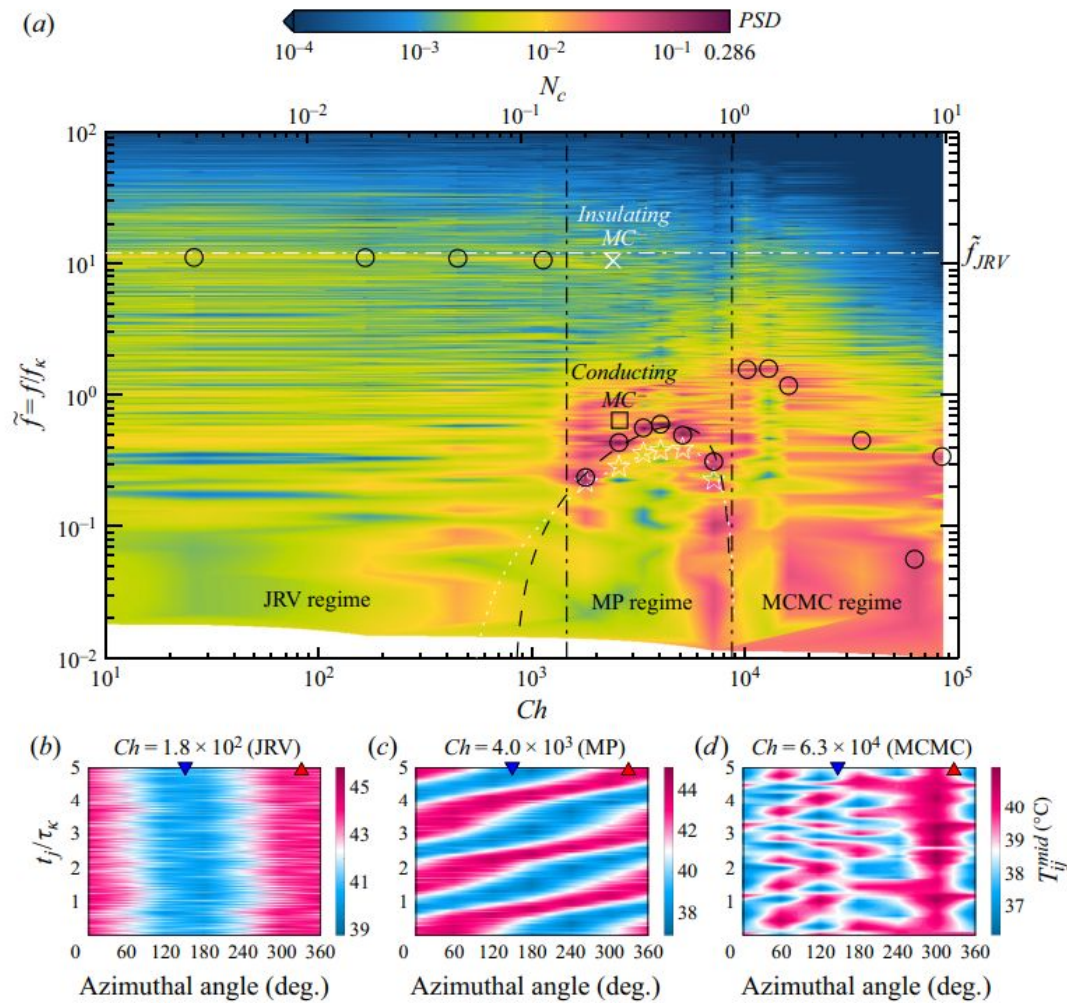
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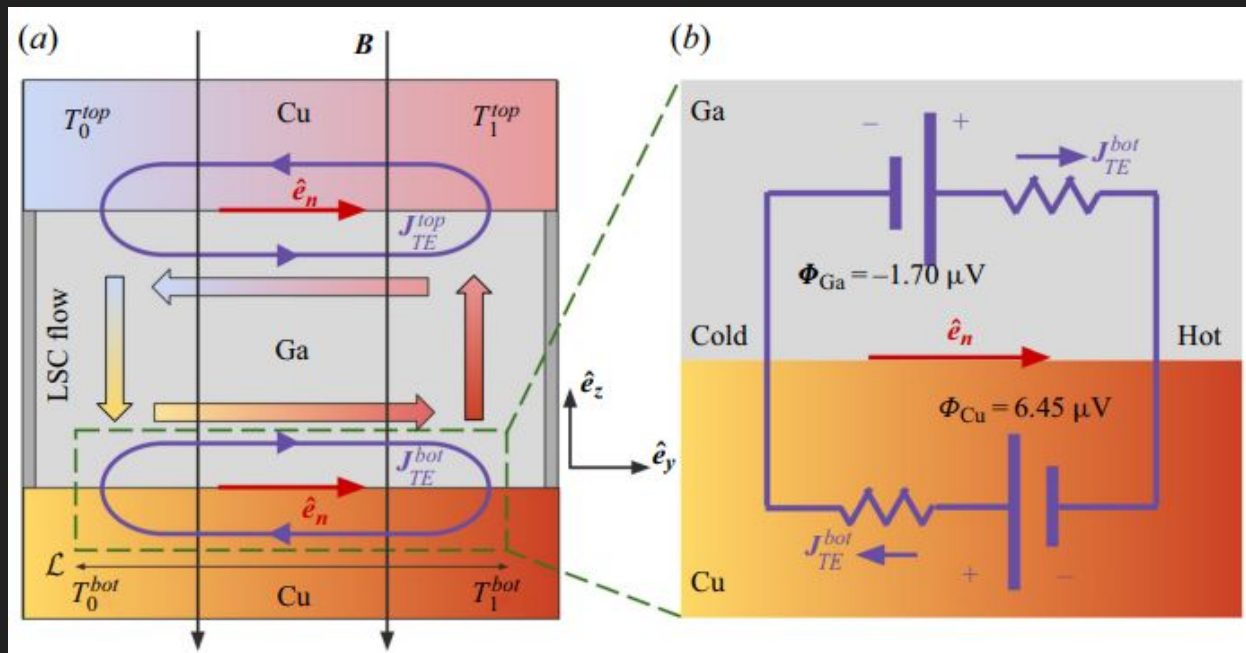


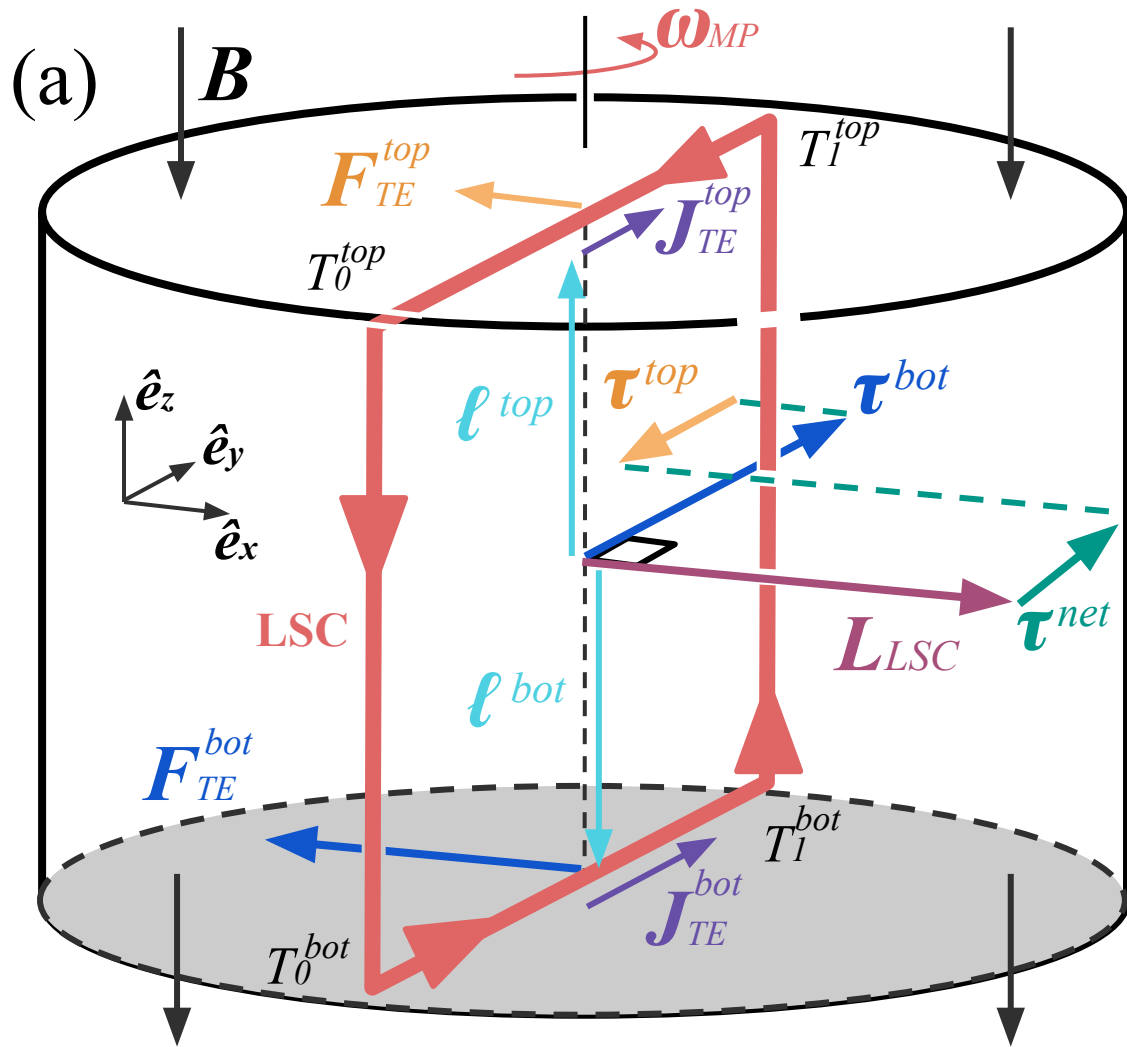
(d)











Symbols	Description	Value
σ_0	Cu–Ga effective electric conductivity, (2.5)	$3.63 \times 10^6 \text{ S m}^{-1}$
B	magnetic field intensity	120 gauss
\mathcal{L}	Horizontal length scale of thermoelectric current loops, $\approx \Gamma H$	197.2 mm
X_0	Cu–Ga Seebeck prefactor, (6.6)	$-7.89 \times 10^{-9} \text{ V K}^{-2}$
ρ	Liquid gallium density	$6.08 \times 10^3 \text{ kg m}^{-3}$
R^*	Effective radius of the LSC, (6.1)	0.08 m
U_{ff}	Free-fall velocity, (2.10)	0.03 m s^{-1}
\bar{T}	Mean fluid temperature, (3.5)	42.50°C
ΔT	Vertical temperature difference across the fluid, (3.3)	7.03 K
T^{bot}	Bottom interface mean temperature	319.23 K
δT^{bot}	Bottom interface mean temperature difference, (5.2)	3.44 K
T^{top}	Top interface mean temperature	312.07 K
δT^{top}	Top interface mean temperature difference, (5.2)	2.24 K
\mathcal{T}	$(T^{bot} \delta T^{bot} - \delta T^{top} \delta T^{top})$	399.11 K^2

❖ Euler's equation:

$$\boldsymbol{\tau}_{net} = \frac{d\mathbf{L}_{LSC}}{dt} = I \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times \mathbf{L}_{LSC}$$

$$\boldsymbol{\omega} = \omega_{LSC} \hat{e}_x + \boldsymbol{\omega}_{MP}$$

❖ Thermoelectric net torque:

$$\boldsymbol{\tau}_{net} = \frac{\sigma_0 V_{LSC} X_0 B \boxed{(T^{bot} \delta T^{bot} - T^{top} \delta T^{top})}}{4\Gamma} (\hat{e}_x \times \hat{e}_b)$$

\mathcal{T}

❖ Thermoelectric precession angular frequency:

$$\boldsymbol{\omega}_{MP} = \frac{\tau_{net}}{L_{LSC}} (-\hat{e}_b)$$

$$\boldsymbol{\omega}_{MP} = \frac{\sigma_0 X_0 B}{2\rho U_{ff} \Gamma R^*} (T^{bot} \delta T^{bot} - T^{top} \delta T^{top}) (-\hat{e}_b) = \frac{\pi^{1/2}}{\Gamma^{3/2}} \frac{\sigma_0 X_0 B \mathcal{T}}{2\rho U_{ff} H} (-\hat{e}_b)$$

Conclusion

- ❖ Thermoelectric effects in turbulent liquid metal magnetoconvection experiments
- ❖ Novel thermoelectric precession mode
- ❖ TE currents with B field can exert torques on core fluid near CMB →
New potential mechanism for core-mantle coupling



Xu et al. 2022, JFM

