

Shape of self-gravity wakes in Saturn's rings: Implications for structure, dynamics and origins

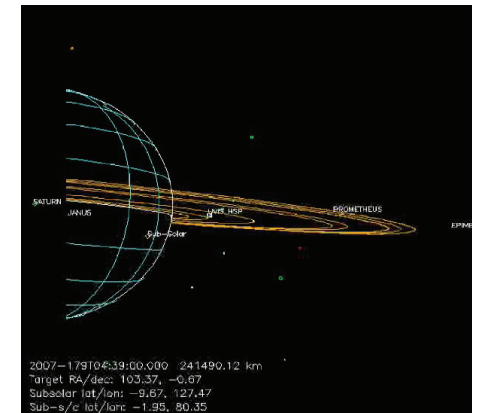
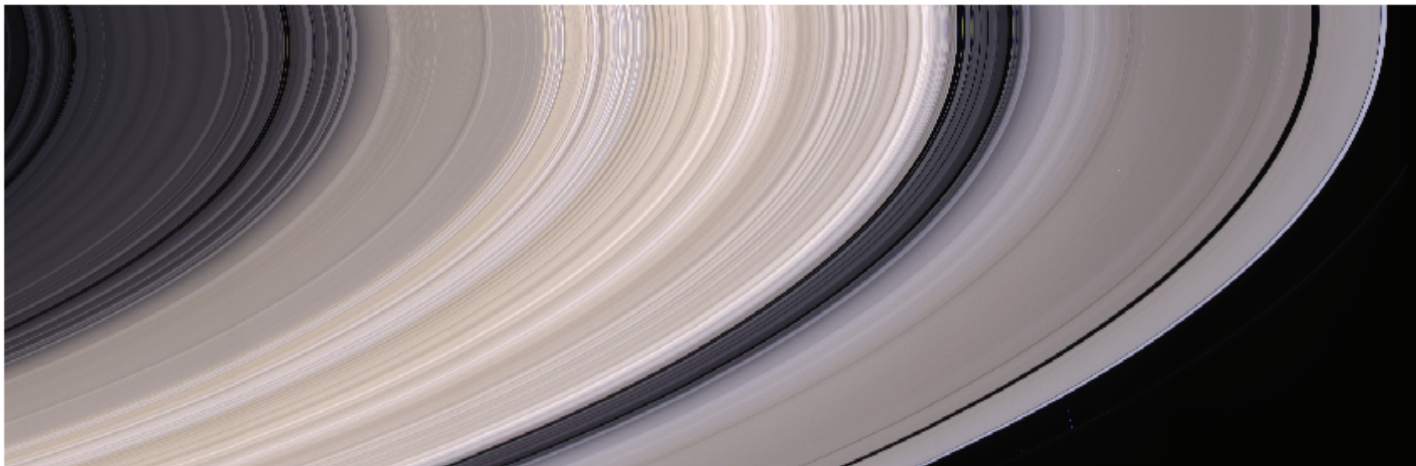
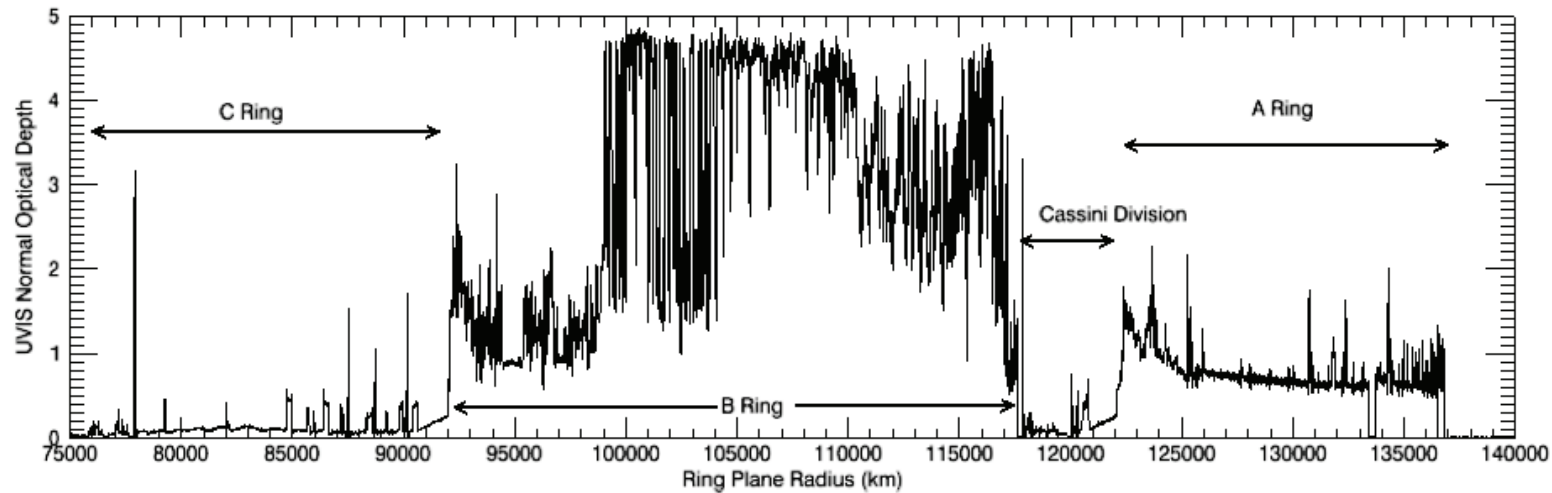
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Saturn's Rings to the Star and to the Eye



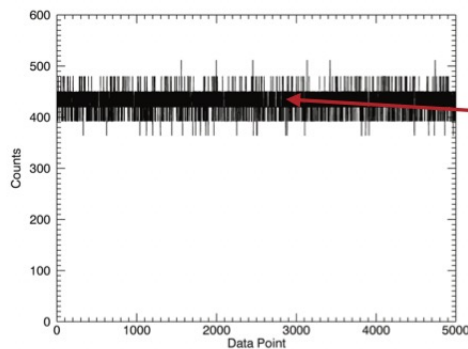
Star occultation view

Ring occultation statistics probe meter-size structures and times scales of hours

Another Window to Particle and Clump Sizes from UVIS Stellar Occultation Data

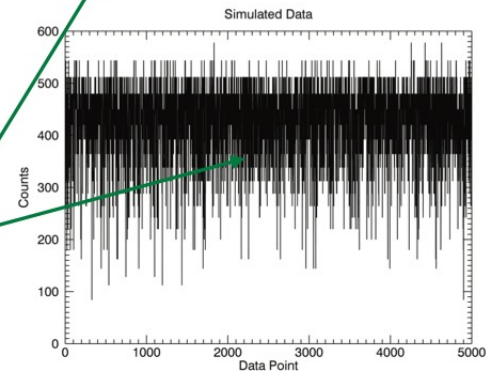
One solution relating variance (E) to particle size (R) and optical depth (τ):

$$E \approx \tau \frac{\pi R^2}{\mu A} e^{-2\tau}$$



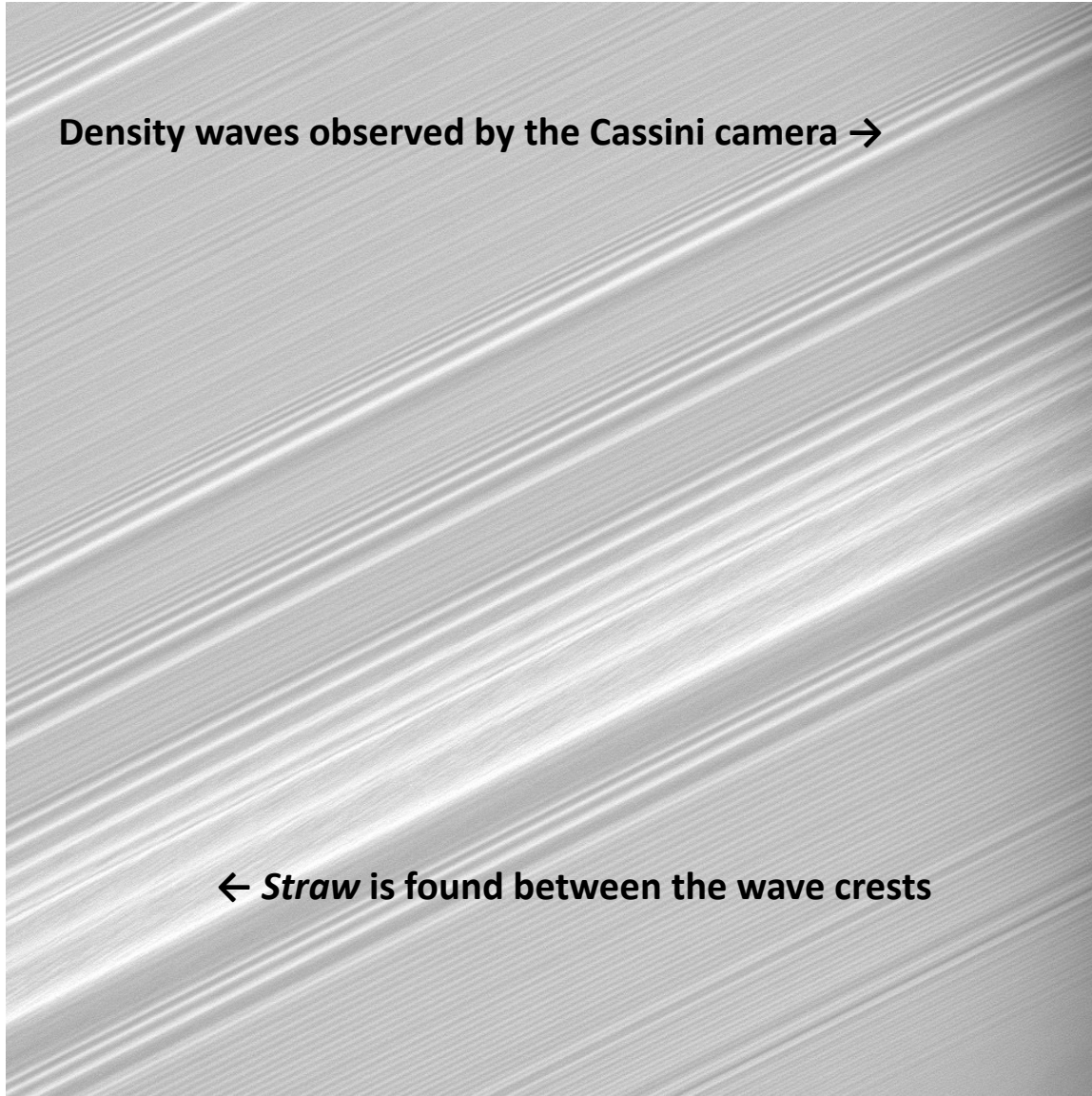
Small particles lead to small variance.

Large particles lead to large variance.

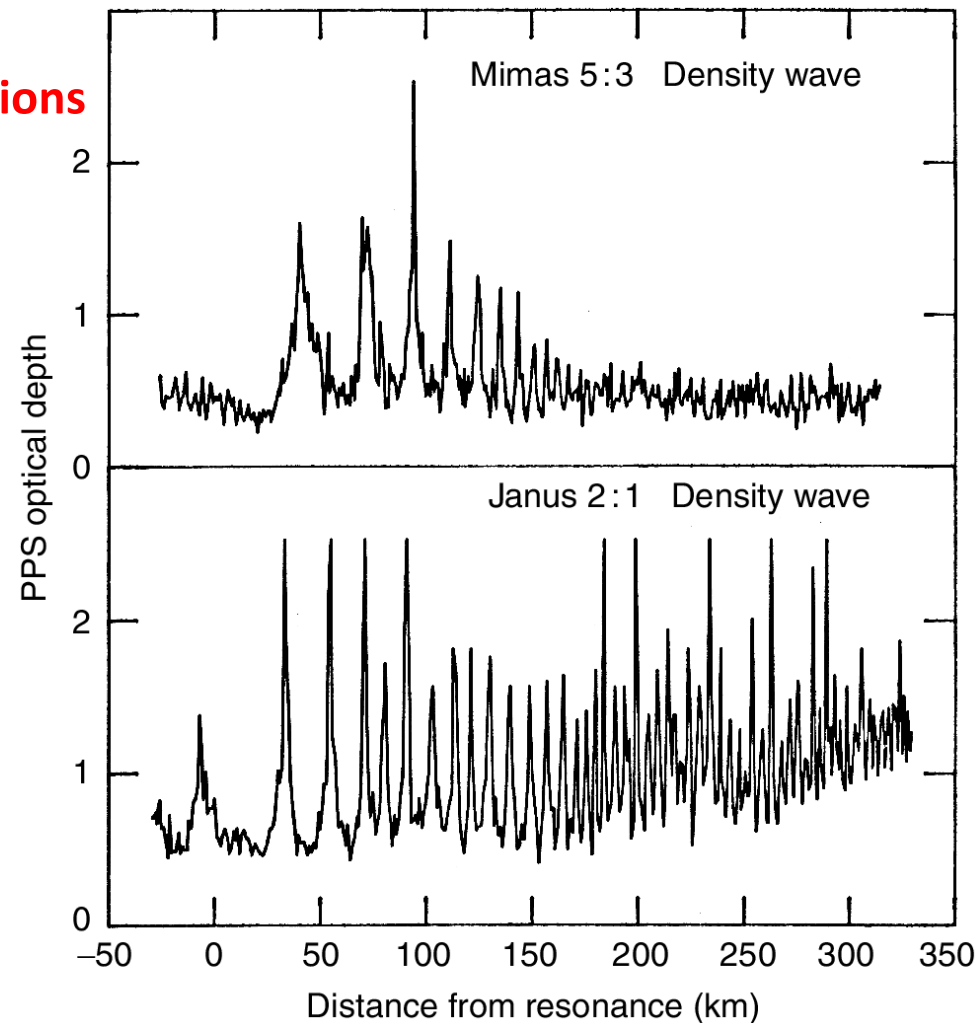


Density waves observed by the Cassini camera →

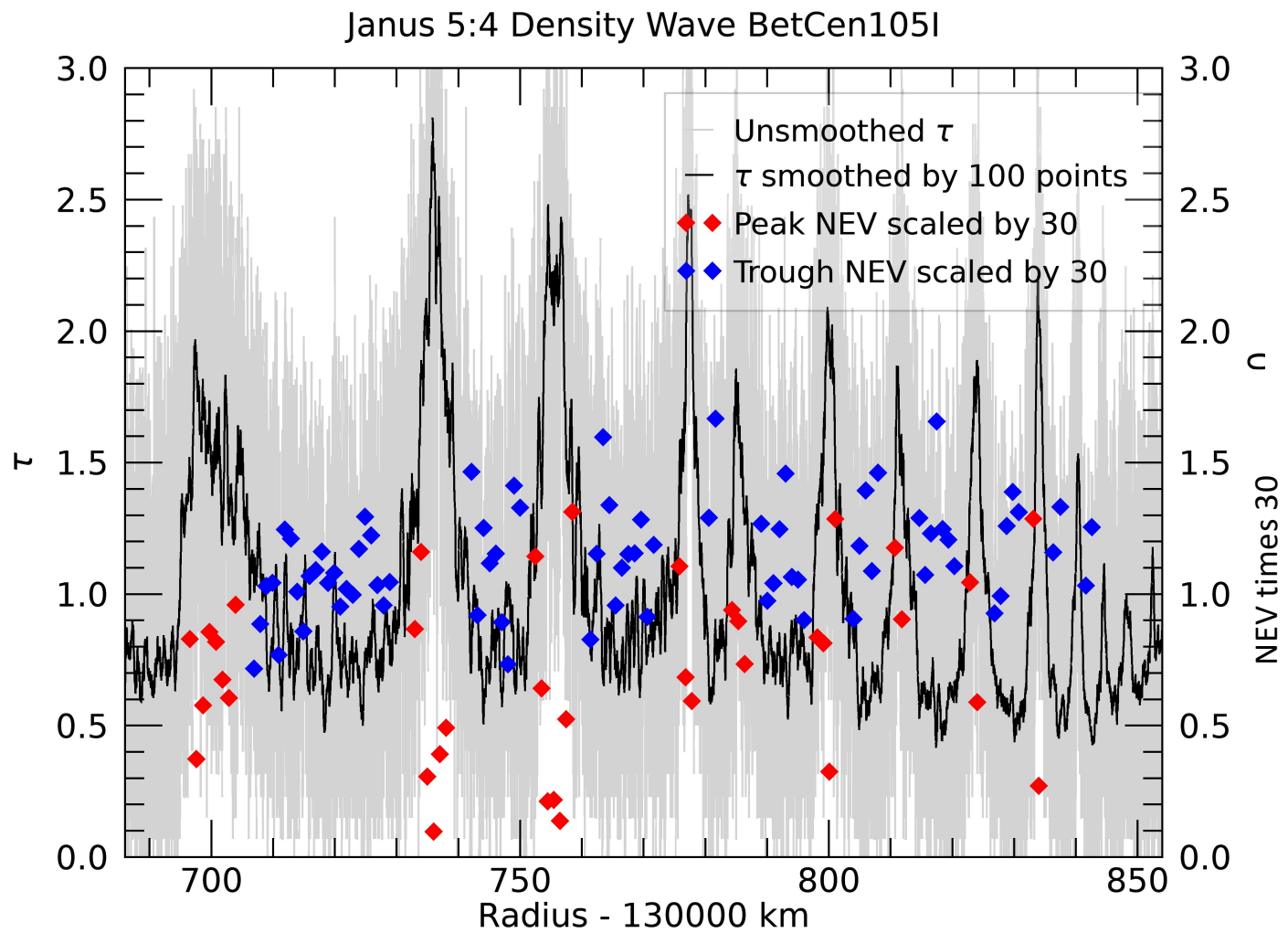
← *Straw* is found between the wave crests



**Voyager occultations
found numerous
density waves
in Saturn's rings**



**Excess variance
(above Poisson statistics)
arises from structures
in Saturn's rings.
Can we use this statistic
to see structural
change *within* a
density wave?**



Between the wave crests, the variance is not symmetric, as expected for constant wakes

A non-linear model for lions (predator) & gazelle (prey)

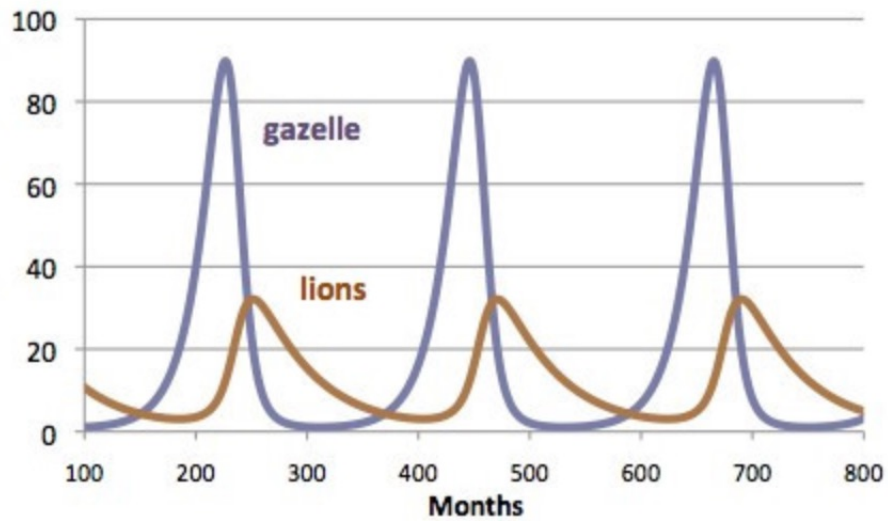


Figure 1: Predator-prey dynamics over 20 years. (1)

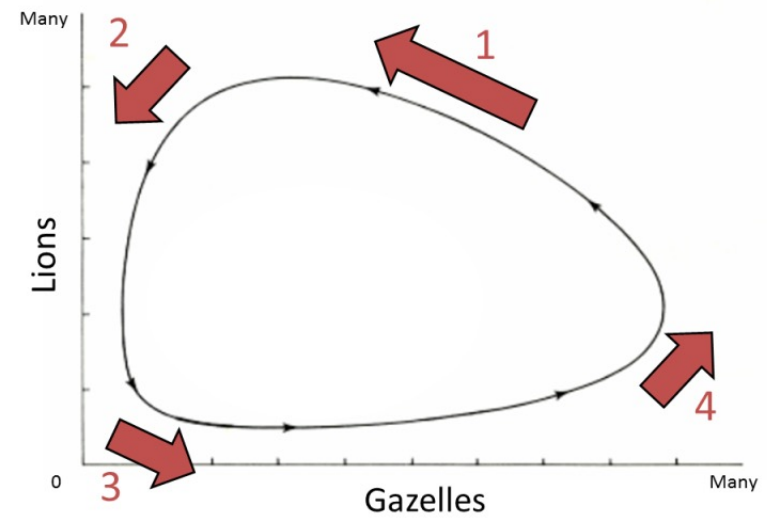
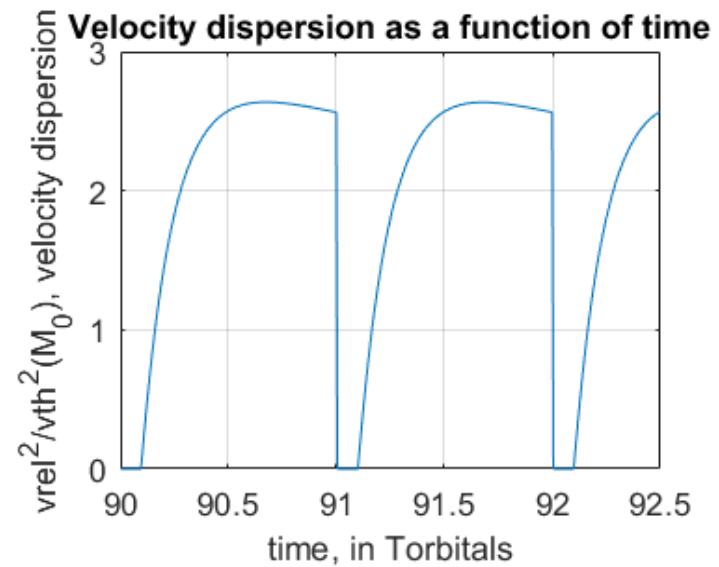
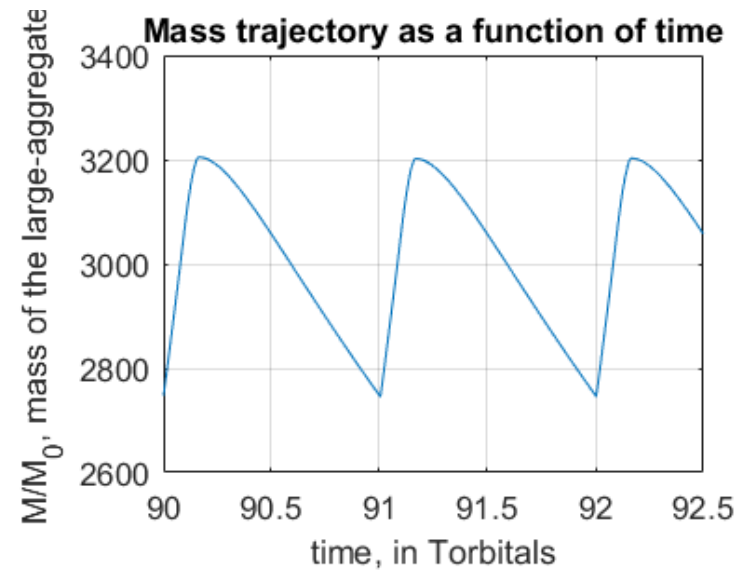
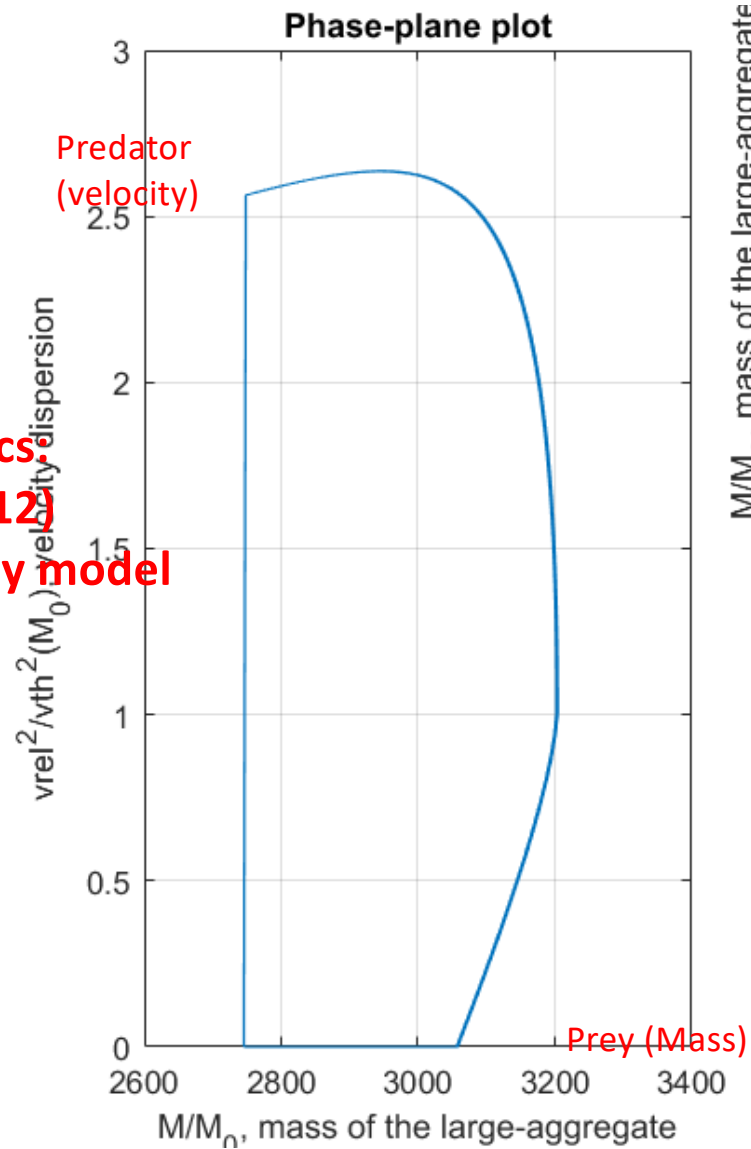


Figure 2: Limit cycle corresponding to predator-prey dynamics

Ring dynamics:
Esposito (2012)
Predator-Prey model
Driven by
 $V_{\text{rel}} = 0$ for
0.1 Torb

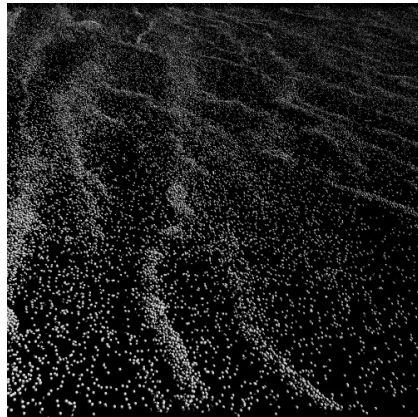


Granola Bar Model for Self-Gravity Wakes

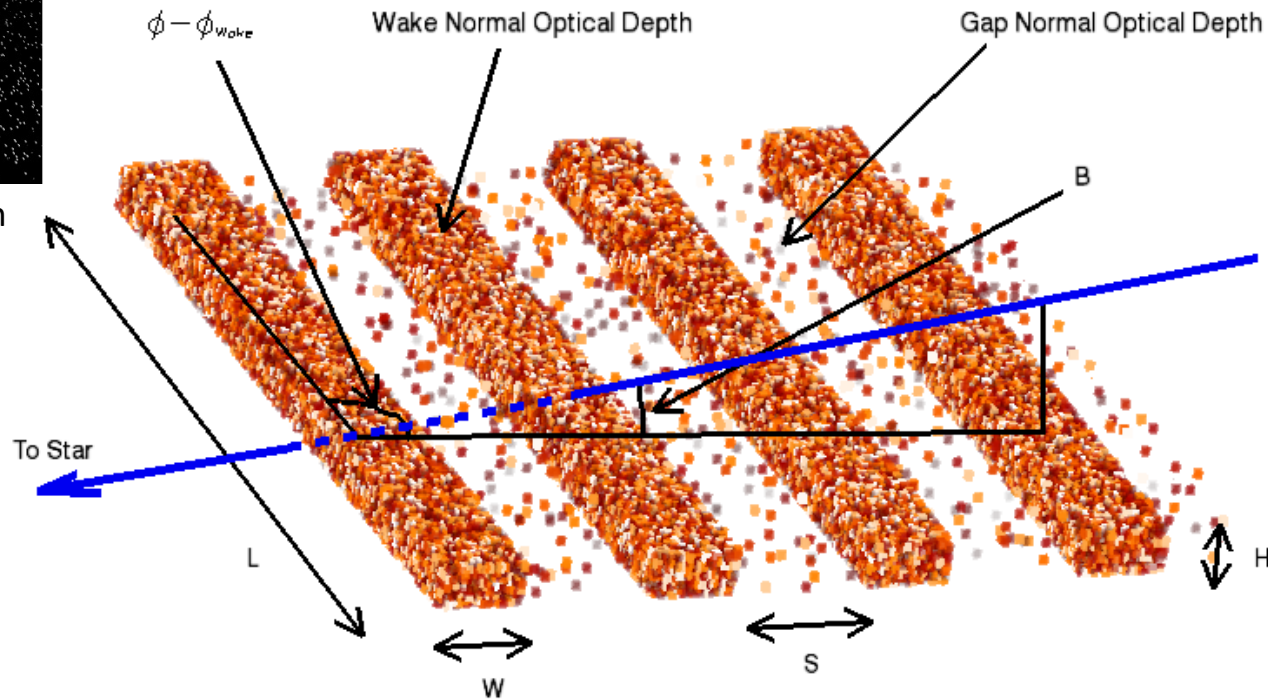
- In Saturn's rings, the ring particles orbit in the Roche zone (surrounding the Roche limit) where accretion competes with disruption
- The ring particles form transient clumps, termed ***self-gravity wakes***, which are tilted and elongated with a typical radial scale of the Toomre wavelength $\lambda_T \sim 60\text{m}$, and a tilt of about 20° .
- We model these with a **Granola Bar Model**: The wakes dominate the statistical moments of the Cassini UVIS star occultations in the A & B rings

Self-Gravity “Wake” Model

Model Parameters Affecting Measured Optical Depth



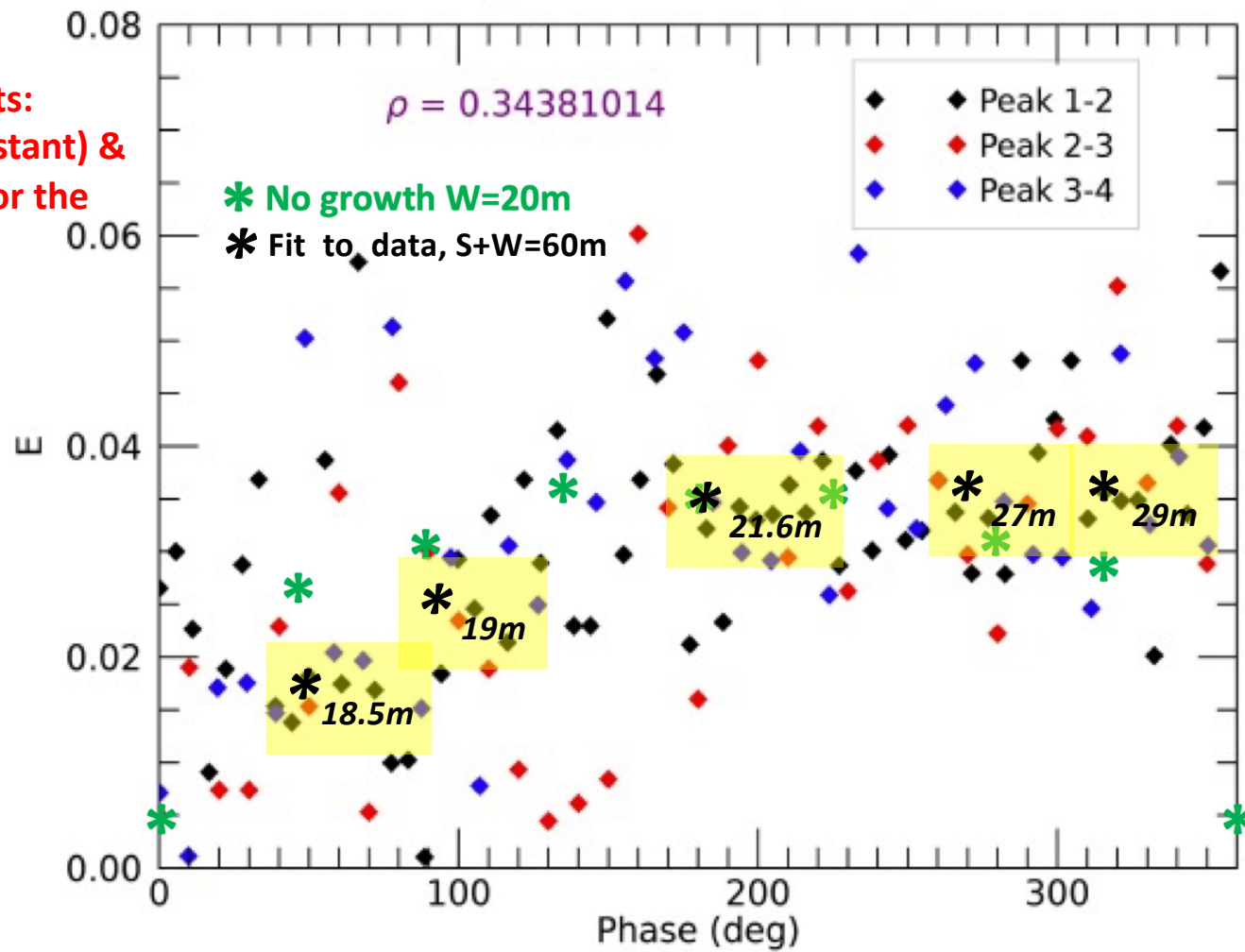
Mark Lewis simulation



Colwell's Granola-Bar-Model (GBM)

BetCen105I, Janus 5:4 Density Wave, E vs Phase

Our results:
Symmetric (constant) &
best-fit values for the
wake width W



Structure, Dynamics, Origins

- Perturbed by passing density waves, **self-gravity wakes** grow and erode on orbital timescales with a full amplitude of **60%**, and a phase lag $\Delta\phi \sim 45^\circ$
- Our analysis shows **W** = 18-29m; **S+W** \sim 60m; **H/W** < 0.12, thus **H** < 4m
- These results are consistent with a simple dynamical model of the rings, analogous to an ecological ***Predator-Prey*** interaction
- Collisions or azimuthal instabilities of these wakes may lead to the ***straw*** features seen in Cassini images
- **Gaps & ghosts** in the rings surround larger fragments remaining from the ring origin, or maybe large aggregates forming recently: These also affect the statistics, and we are investigating this using the **GBM** formalism...

Back-Up Slides

We reproduce *SN90*, but don't assume

- An integer number of particles;
- Cylindrical shadows
- That particle area and optical depth are both small;
- The conditional variance formula;
- Moment Generating Functions or their logarithms (also called cumulants).
- Use no special techniques or approximations
- This formulation naturally accounts for edge effects



Eckert (2020 EPSC) showed that different viewing angle occultations do not follow SN90

SN90:
 $E\mu A$ constant
for R constant

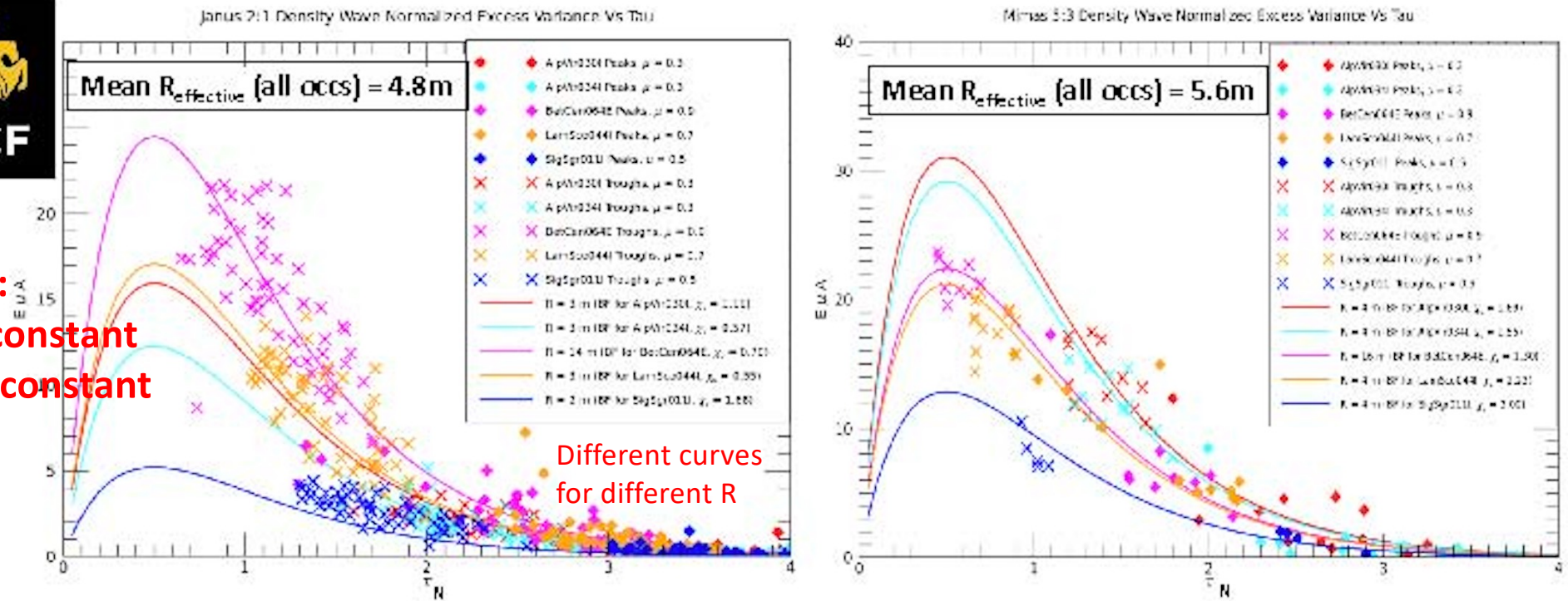
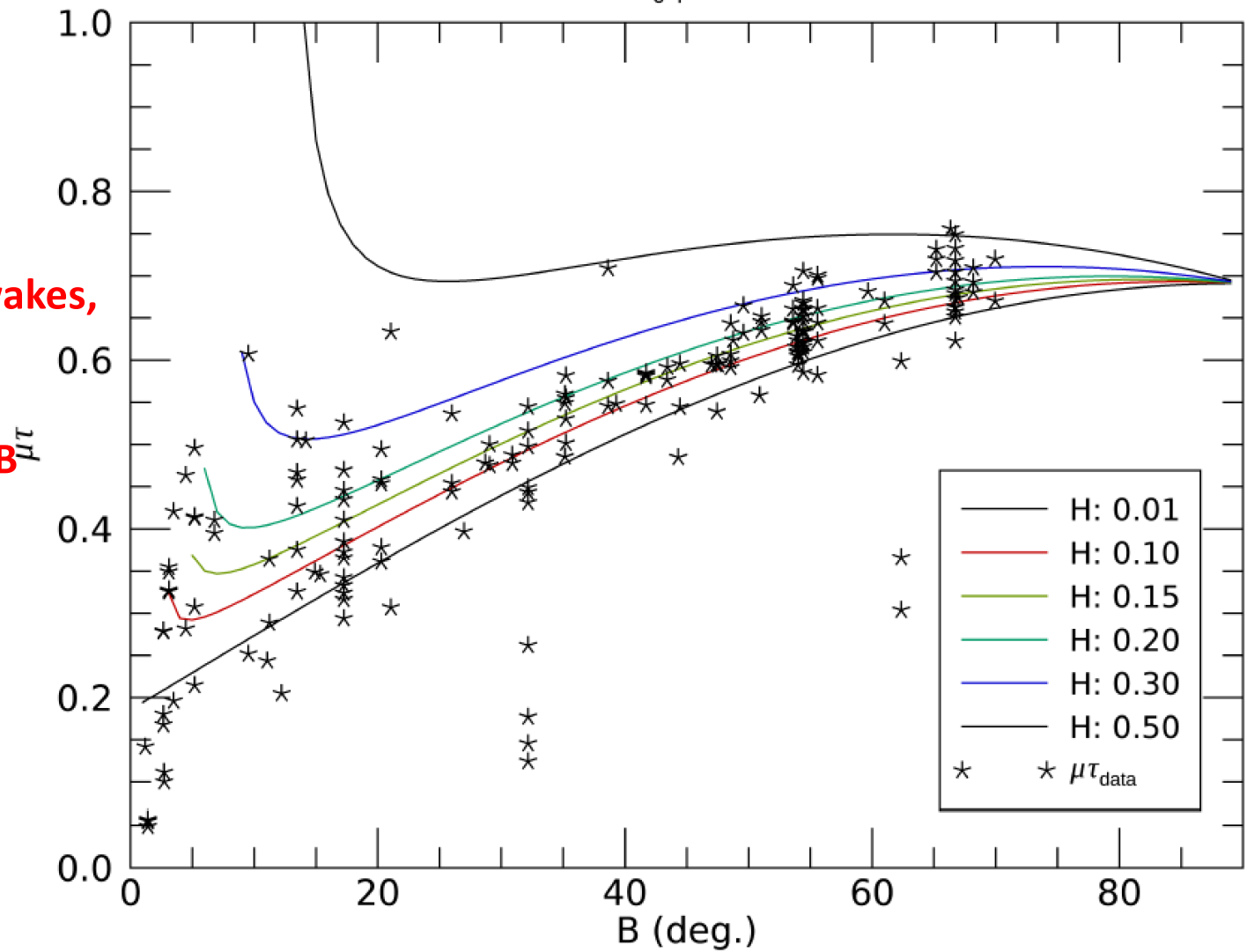


Figure 1. Normalized excess variance times μA to account for variation in $\mu = \sin|B|$ (where B is the elevation angle) and integration area (A) versus τ_N for 5 occs for both waves with troughs (crosses) and peaks (diamonds) distinguished. We test 100 values (1-100 m in increments of 1 m) and determine a best-fit $R_{\text{effective}}$ for each occ by the lowest reduced chi-squared (χ_r) for the trough points **only**. We exclude peaks in our χ_r calculation because at high τ_N , $E(\tau)$ drops off. Note that identical R values do **not** produce identical curves, even for the same star, which indicates that our model does not account for longitudinal structures such as self-gravity wakes. Our results are consistent with the predator-prey model for ring dynamics in which satellite resonances drive the growth of large aggregates that gravitationally accelerate surrounding particles to higher collision velocities which results in the break up of those dumps. Esposito, et al. (2012, Icarus, 217: 103-114)

Radius: 127 Mm, S: 1.50, $\mu\tau_{\text{gap}}$: 0.18, $\phi - \phi_{\text{wake}}$: 45° , Model: GBM

For regularly spaced wakes,
the GBM has
some problems at
small star declination B



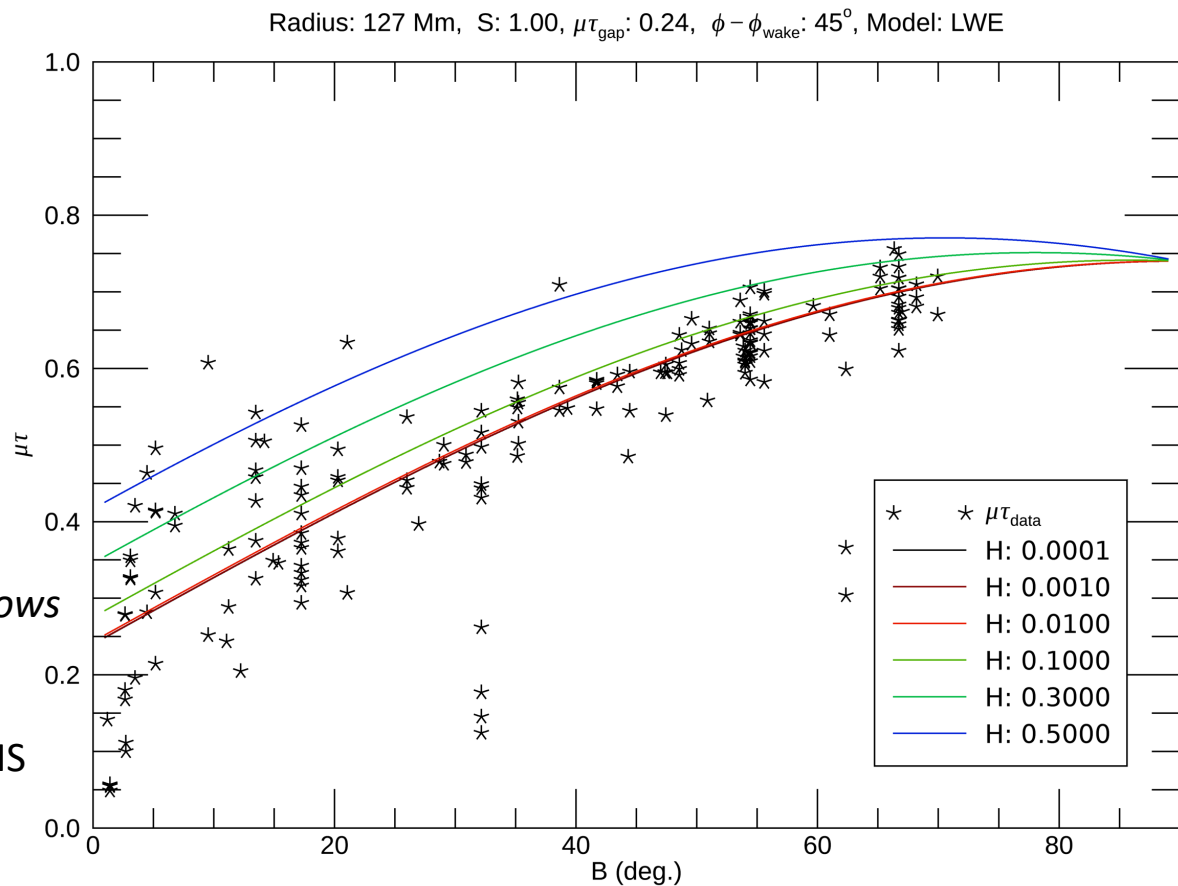
We can remove these issues
by using randomly placed,
possibly overlapping bars

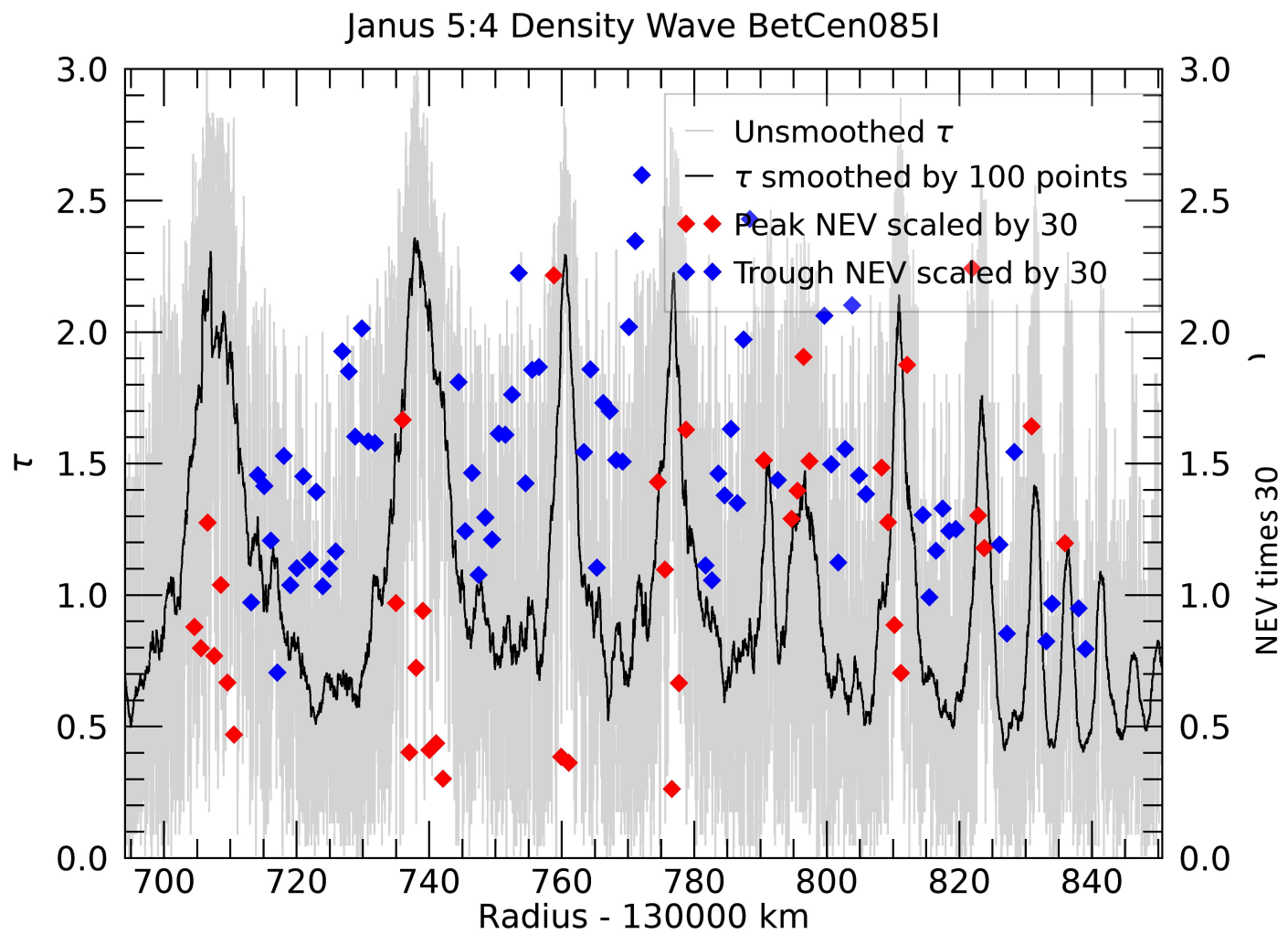
$$T = e^{-\tau_{\text{gap}}/\mu} * e^{-F}$$

where the *filling factor of wake shadows*

$$F = (1 + H/W * \sin\phi * \cot B)/(1+S/W)$$

$S/W=1$ and $H/W<0.12$ can match UVIS
transparency & excess variance





Between the wave crests, the variance is not symmetric, expected for constant wakes

Solve for Compressed Gap Width

- Assuming no wake growth, conservation of particles in the gaps (no accretion or erosion). $W = W_0$ &
- $\tau(t) = \tau_{\text{gap}} * S_0/S(t) + \ln [1 + W_0/S(t)]$ [2]

- Solving:

$$S(t) = [\tau_{\text{gap}} * S_0 + W_0]/\tau \quad [3]$$

- Jerousek 2017, Fig 7: $S/W \sim 2$; $\tau_{\text{gap}} \sim 0.2$ at the Janus 5:4 density wave
- Toomre wavelength in this region $\lambda_T \sim 60\text{m}$; This gives $W_0=20\text{m}$, $S_0=40\text{m}$; $S+W=60\text{m} \sim \lambda_T$
- Note: No free parameters in this No-Growth Wake model

