# Energy flux quantification in the oceanic internal wavefield

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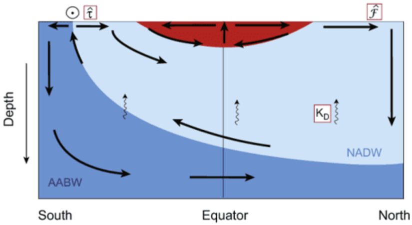
# Oceanic circulation, mixing and internal waves

- **Dense water** sinking to the abyss at high latitudes **must eventually rise**
- Mechanism: **Turbulent diapycnal mixing** of "dense bottom water"/"light water above"
- Modelled as **diffusivity** $K_D$ in advection-diffusion closure (*Munk '66*)
- One of the main drivers of the **Meridional Overturning Circulation (MOC)**



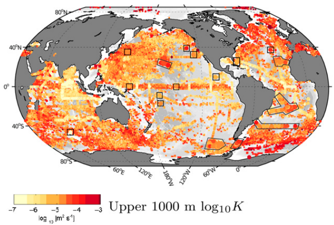

from *Sévellec, Fedorov, Deep Sea Res. '11*
see also *Whalen et al., Nat. Rev. '20*

**Finescale Parameterization:** $\mathcal{P} = \mathcal{P}_0 \frac{f}{f_0} \frac{N^2}{N_0^2} \frac{E^2}{E_0^2}$

**Turbulent production** $\mathcal{P}$ has 2 components:

- **mixing**: $K_D \simeq \frac{R_f}{\rho_0 N^2} \mathcal{P}$
- **Dissipation**: $\epsilon = (1 - R_f)\mathcal{P}$

and is **quantified heuristically** in terms of parameters of **internal wave spectrum**!



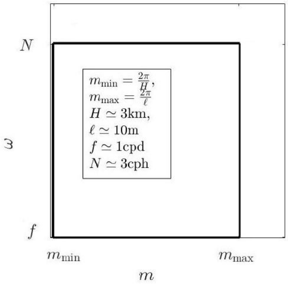

*Waterhouse et al., JPO '14*: finescale estimates of global patterns of mixing

# Internal Wave Turbulence

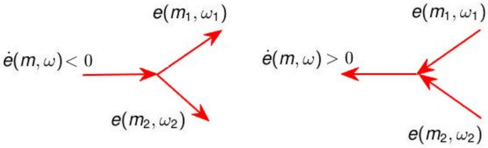
- Boussinesq approximation for ocean with **stable stratification**
- Assume isopycnal coordinates, zero total potential vorticity, hydrostatic balance, spatial homogeneity, horizontal isotropy.
- Assume "Random phase" statistics: energy spectrum $e(m,\omega)$ evolves according to the **Wave Kinetic Equation** (*Lvov & Tabak, Phys. D '04, Olbers '74* )

$$\dot{e}(m,\omega) = \mathcal{I}(e(m,\omega))\,.$$

- $I(e(m,\omega))$: **Collision Integral** quadratic in $e(m,\omega)$



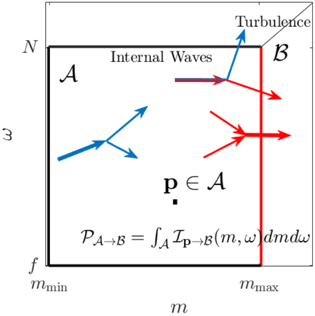

The Collision Integral quantifies the **inter-scale energy exchanges** via **3-wave resonances**



# Calculation of inter-scale energy fluxes

Method to compute **energy flux between "control volumes" in Fourier space** from Collision Integral, *Dematteis and Lvov, JFM '21*

- $\mathcal{I}_{\mathbf{p}}$: energy budget of mode **p** in relation to all other modes
- $\mathcal{I}_{\mathbf{p}\to\mathcal{B}}$: energy budget of mode **p** such that the outcome of the interaction **p** is in region $\mathcal{B}$
- Consider **control volume** ($\mathcal{A}$) containing internal-wave band and integrate $\omega_{\mathbf{p}}\mathcal{I}_{\mathbf{p}\to\mathcal{B}}$ over $\mathbf{p}\in\mathcal{A}$




Method paper to appear: *Dematteis and Lvov '22*

Wave turbulence analogy to Kraichnan's '59 DIA calculation of turbulent fluxes using control volumes in Fourier space

# Comparison with finescale parameterization

$$\mathcal{P}_{\rm out,h} = \frac{\Gamma C_{\rm h}}{1-\nu}\left[1-\left(\frac{\ell}{2b}\right)^{1-\nu}\right] f^{1+\nu} N E^2,$$

$$\mathcal{P}_{\rm out,v} = \frac{\Gamma C_{\rm v}}{\nu}\left[1-\left(\frac{f}{N}\right)^{\nu}\right] f N^{1+\nu} E^2,$$

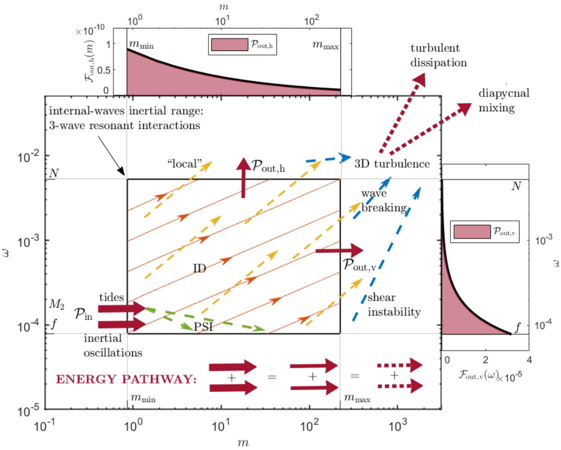
$$\Gamma = \frac{1}{\pi^3}\left(\frac{2\ell}{b}\right)^{\nu}\frac{N_0^{1-\nu} b^3}{c^{1-\nu}\ell}, \qquad \nu = 2a-7 = 0.38, \quad \ell = 10\,\mathrm{m},$$

$$\mathcal{P}_{\rm out,h} \simeq -3.8\times10^{-9}\,\mathrm{W\ kg^{-1}},$$
$$\mathcal{P}_{\rm out,v} \simeq -5.2\times10^{-9}\,\mathrm{W\ kg^{-1}}.$$

This amounts to a total dissipated power

$$\mathcal{P}_{\rm out} = \mathcal{P}_{\rm h} + \mathcal{P}_{\rm v} \simeq -9.0\times10^{-9}\,\mathrm{W\ kg^{-1}}.$$

$$\mathcal{P}_{\rm finescale} \simeq 5.9\times10^{-9}\,\mathrm{W/kg}.$$




For GM scaling ($\nu = 1$), $\mathcal{P}_{\rm out,v} \propto fN^2E^2$ **= scaling of finescale parameterization formula**

We **calculated** the constant $\mathcal{P}_0$ from theory, i.e. the **"Kolmogorov constant" of the problem**: within a factor 1.5 from empirical value!

*Dematteis, Polzin, Lvov, JPO (2022)*

# Conclusions

- Open question on **how to calculate the energy dissipated by internal waves**, with important implications for climate.
- **Finescale parameterization** (synthesis of observation, dimensional analysis and intuition, from 3 decades ago: *Gregg '89, Henyey '91, Polzin et al. '95*) is the **state-of-the-art understanding** of dissipation and mixing:

  $$\mathcal{P} = \mathcal{P}_0 f N^2 E^2$$

  with $\mathcal{P}_0$ **empirically determined**.
- We have **explained how to obtain the formula from the first principles**, with no adjustable parameters.
- **Computation** of $\mathcal{P}_0$ (analogue of "**Kolmogorov constant**" of turbulence) **in agreement with empirical value** up to factor 1.5.
- Future plan is to extend the result to any internal wave spectrum and to test the results directly against oceanic field data and high-resolution numerical models.

# Thank you for your attention!

**The END**