



General
Assembly 2022

NP0 – L.F. Richardson's 1922 centennial

Turbulent Energy Cascade in the Gulf of Mexico

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1922 Richardson: energy cascade from large to small scales

L. F. Richardson, Cambridge Univ. Press. (1922).

1941 Kolmogorov-Obukhov: $E_k \propto \epsilon^{2/3} k^{-5/3}$

A. N. Kolmogorov, Dokl. Akad. Nauk SSSR 30, 299. (1941)

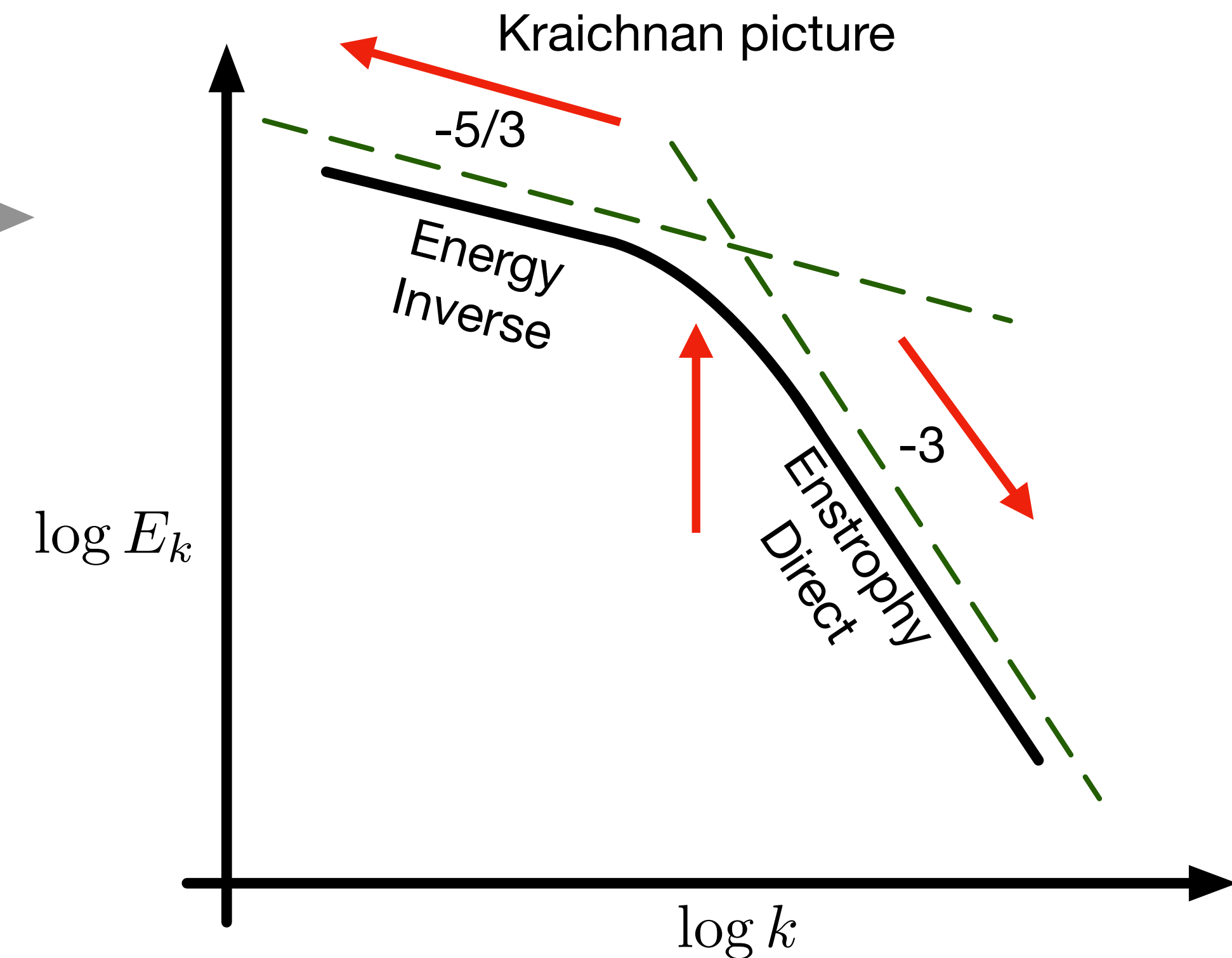
A. M. Obukhov, Dokl. Akad. Nauk SSSR, 5, 453–466. (1941)

1967 Kraichnan: $E_k \propto \begin{cases} k^{-5/3} \\ k^{-3} \end{cases}$

R. H. Kraichnan, *Phys. Fluids*, 10(7), 1417–1423. (1967)

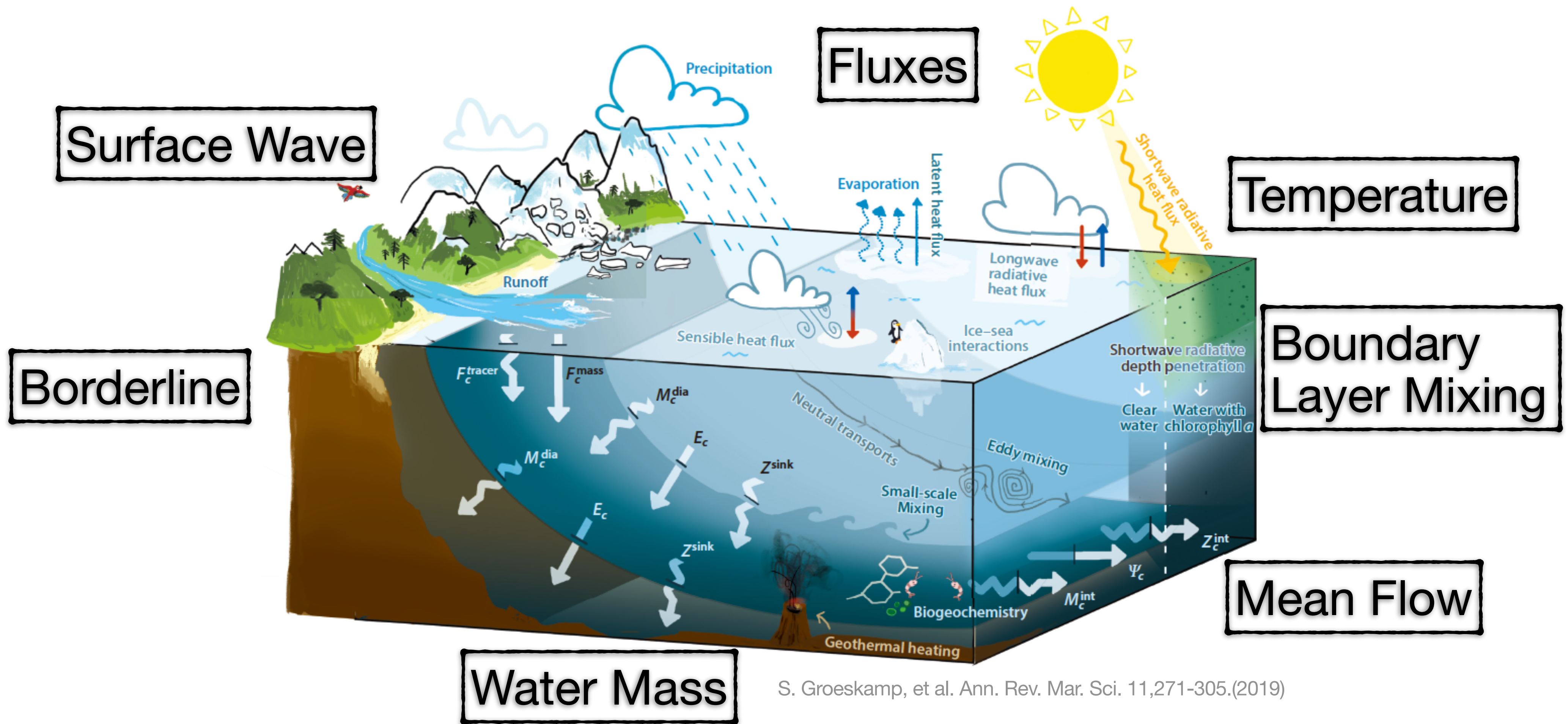
1971 Charney: Geostrophic Turbulence

J. G. Charney, *J. Atmos. Sci.*, 28(6), 1087–1095. (1971)



Background

— complexity of oceanic flow



S. Groeskamp, et al. Ann. Rev. Mar. Sci. 11,271-305.(2019)

– Under different hypotheses

Kolmogorov's 4/5th law of 3D, HIT

$$\langle \Delta u_L^3(r) \rangle = -\frac{4}{5}\varepsilon r$$

Extension to d-dimension

$$\langle \Delta u_L^3(r) \rangle = -\frac{12}{d(d+2)}\varepsilon r$$

(Falkovich et al. 2001)

$$\langle \Delta u_L^3(r) \rangle + 2\langle \Delta u_L(r)\Delta u_T^2(r) \rangle = -\frac{4}{3}\varepsilon r$$

(Lindborg 1996; Antonia et al. 1997)

typical geophysical scaling

$$\langle \Delta u_L^3(r) \rangle + \langle \Delta u_L(r)\Delta u_T^2(r) \rangle = -2\varepsilon r$$

(Lindborg et al. 2001)

Batchelor–Kraichnan theory

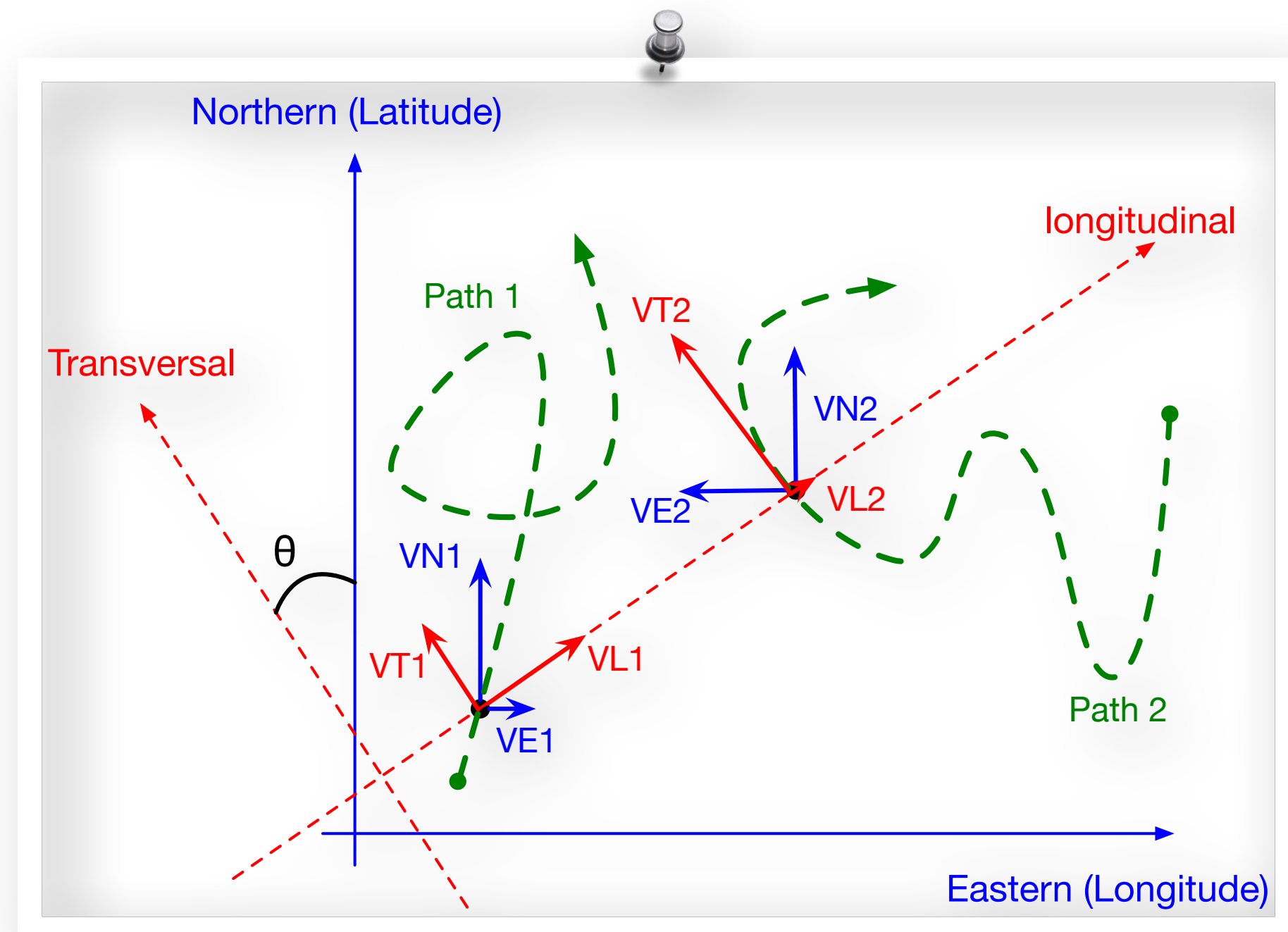
$$\langle \Delta u_L^3(r) \rangle = \begin{cases} \frac{1}{8}\eta r^3 & \text{for } r \ll l_f \\ \frac{3}{2}\varepsilon r & \text{for } l_f \ll r \end{cases}$$

Boffetta and Musacchio, Phys. Rev. E, 82, 016307, (2010)

neglect viscous term, local isotropy...

$$\langle \Delta u_L^3(r) \rangle + \langle \Delta u_L(r)\Delta u_T^2(r) \rangle = \frac{1}{4}\varepsilon_\omega r^3$$

(Lindborg 1999)

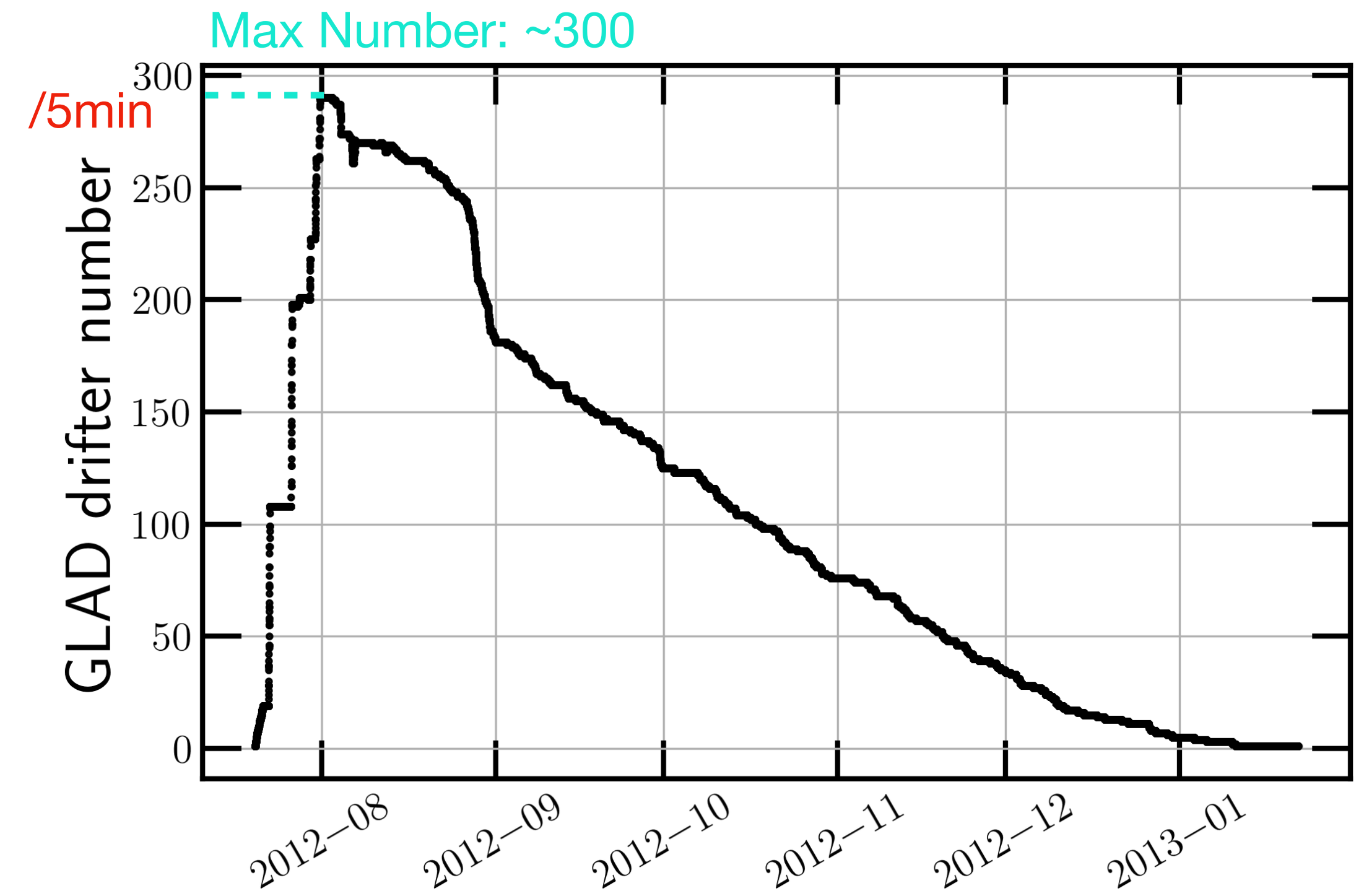
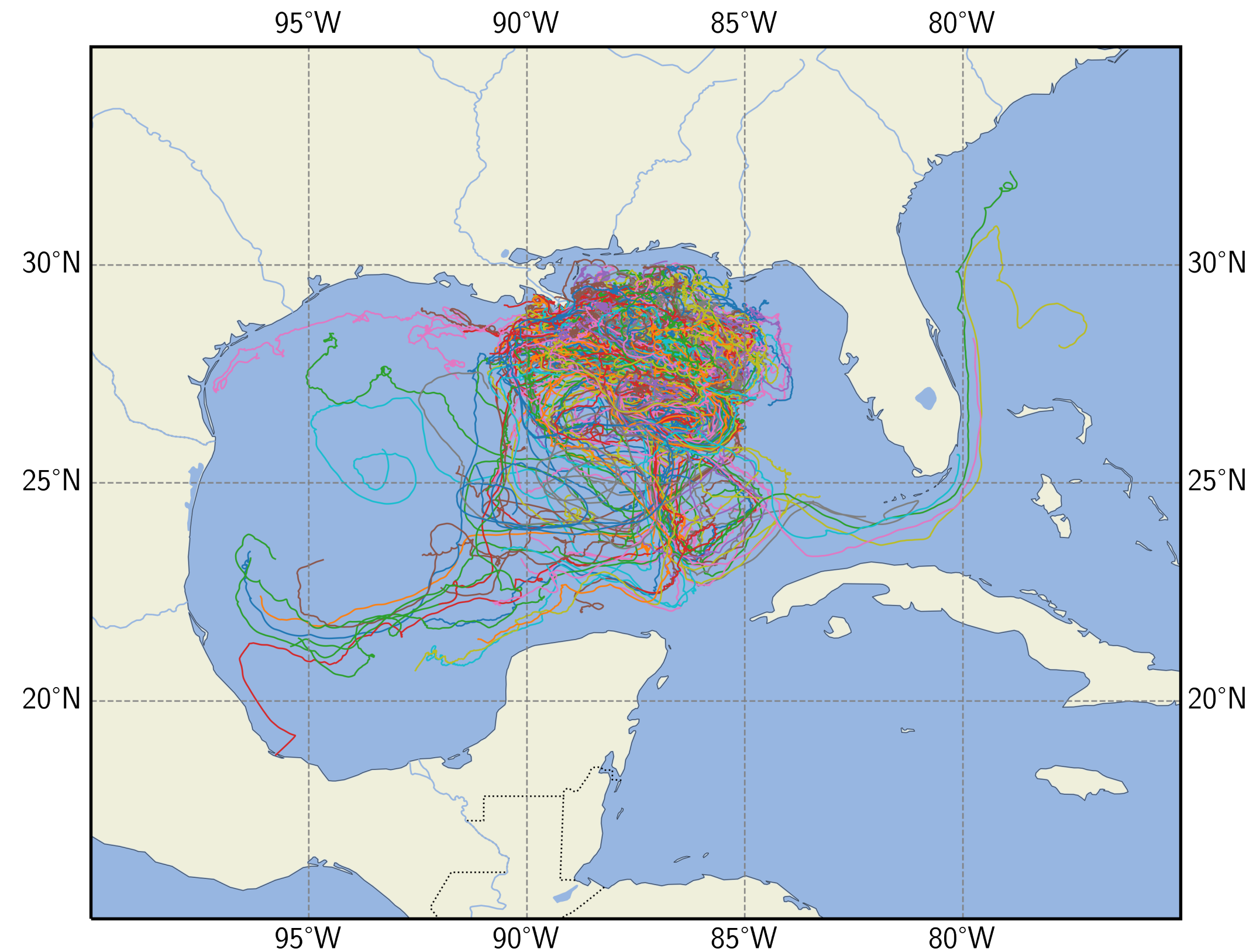


sketch of longitudinal structure-function method

— Grand LAgrangian Deployment (GLAD)

Time Span: 2012.7 - 2013.1

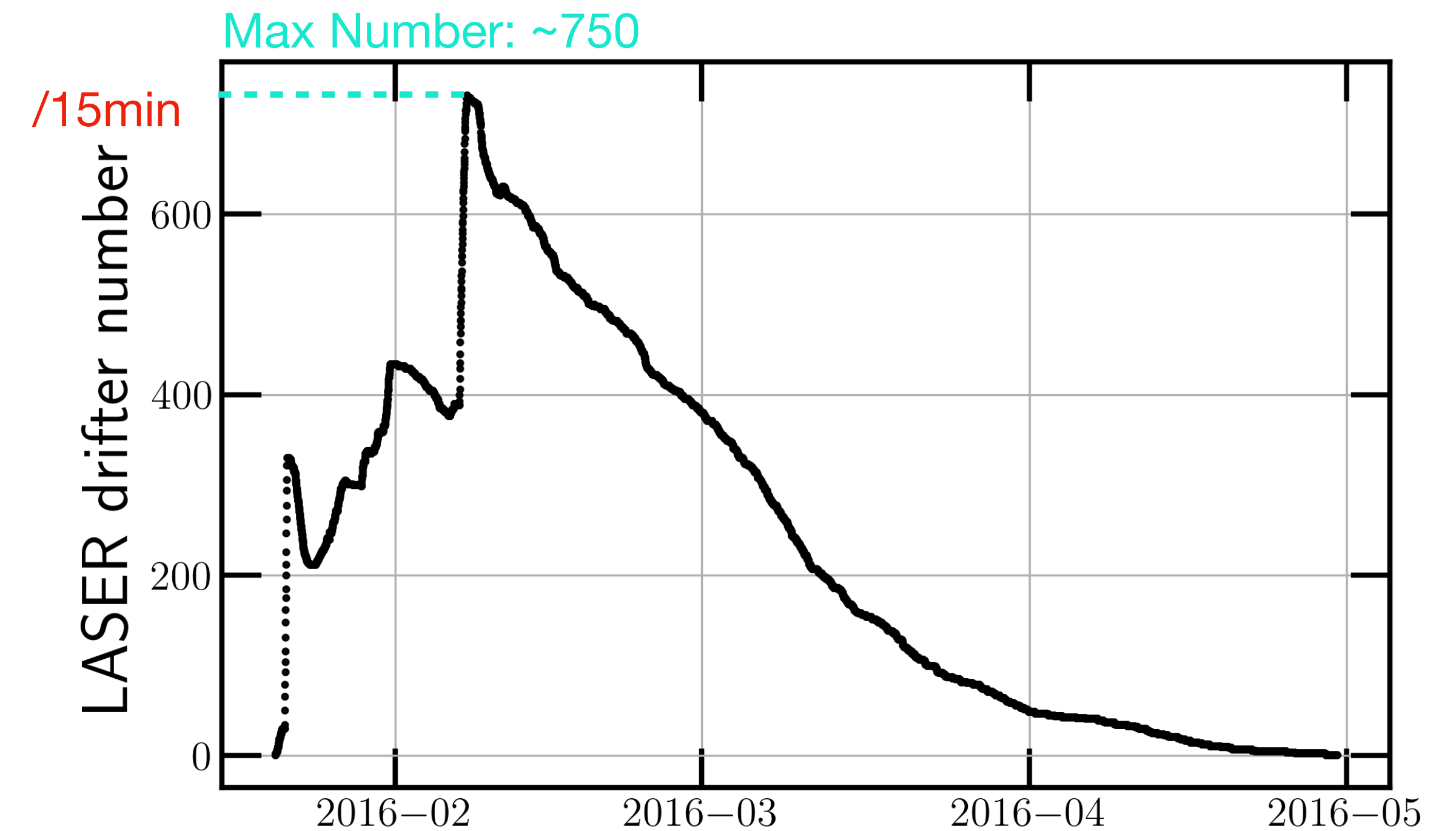
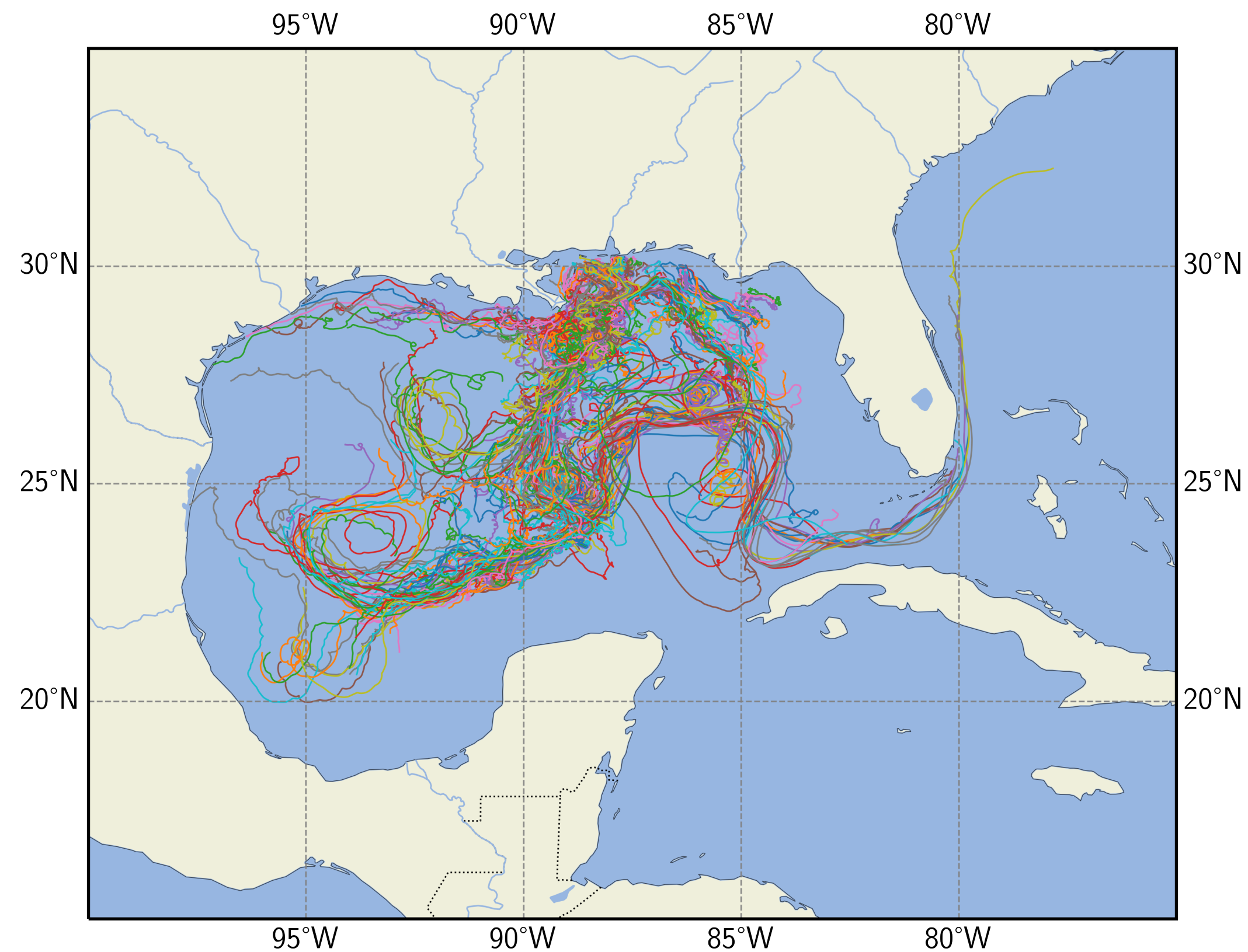
Trajectories



— LAgrangian Submesoscale ExpeRiment (LASER)

Time Span: 2016.1 - 2016.4

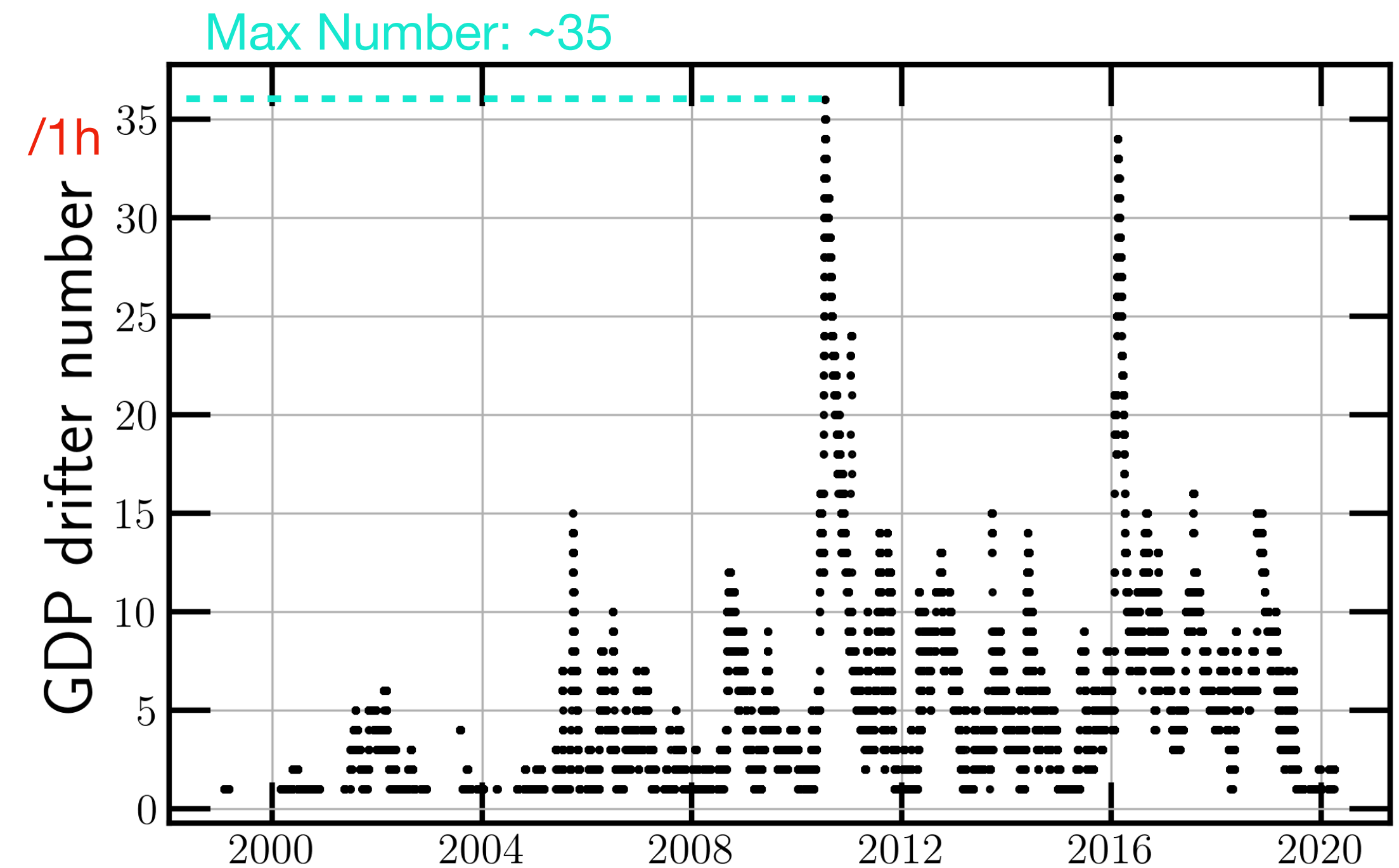
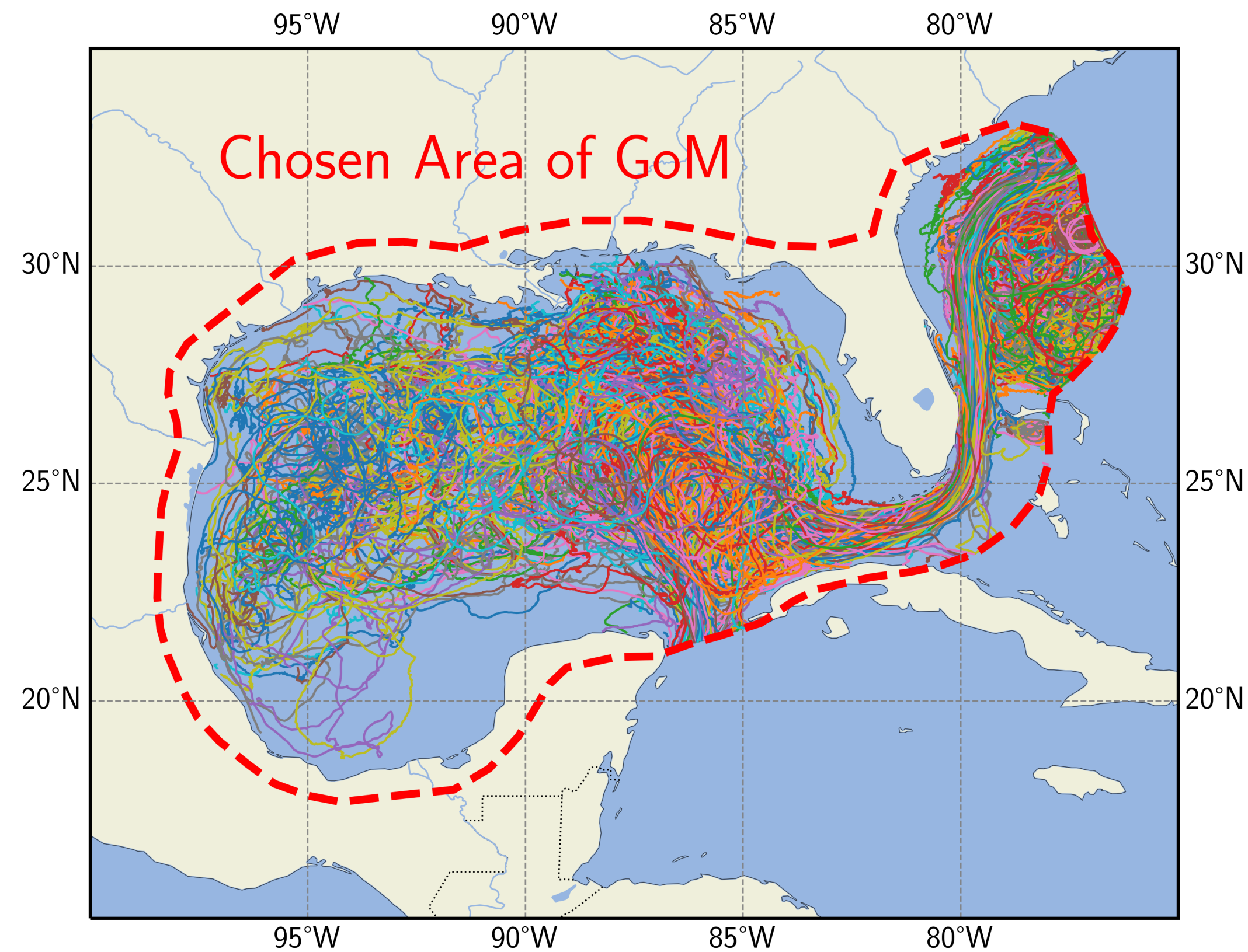
Trajectories



— Global Drifter Program (GDP)

Time Span: 1999.2 - 2020.4

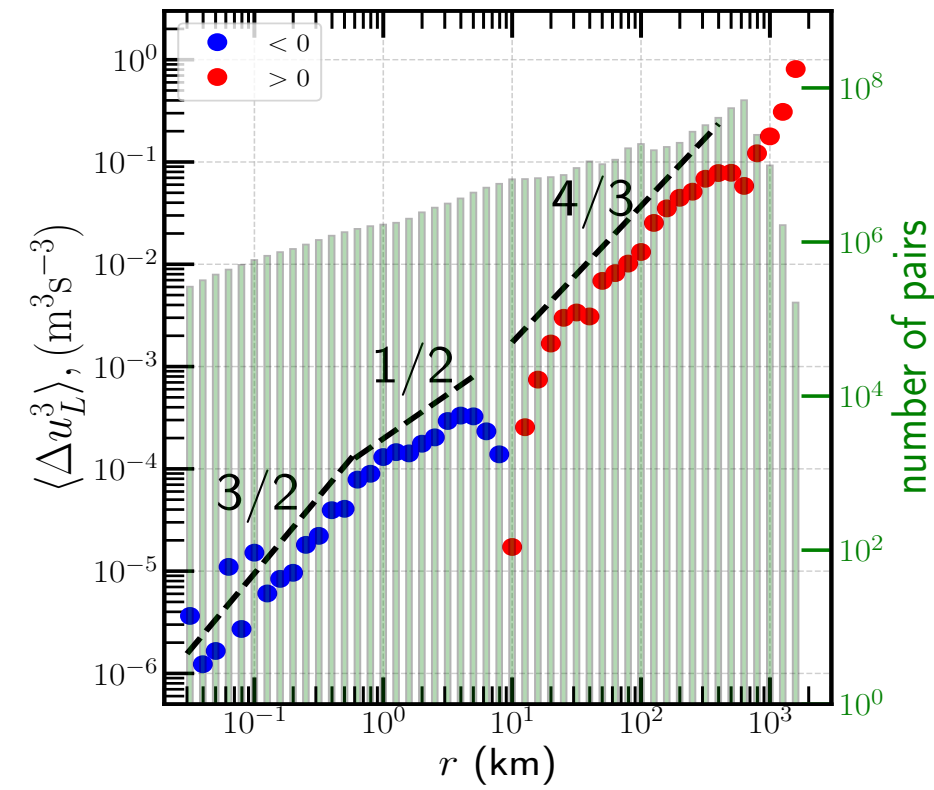
Trajectories



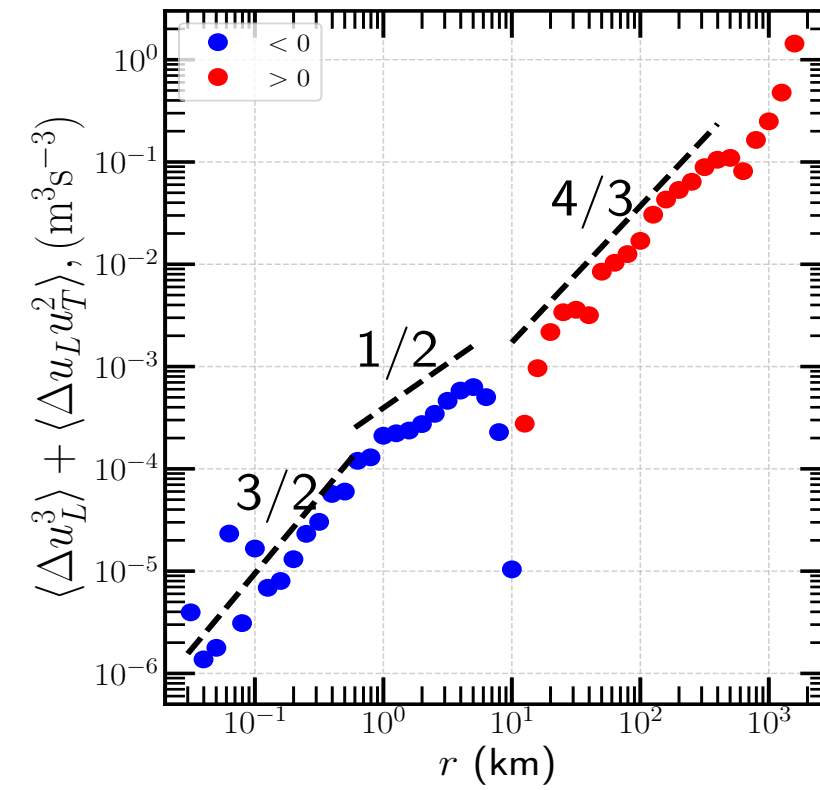
Results

GLAD
Jun. - Jan.
2012

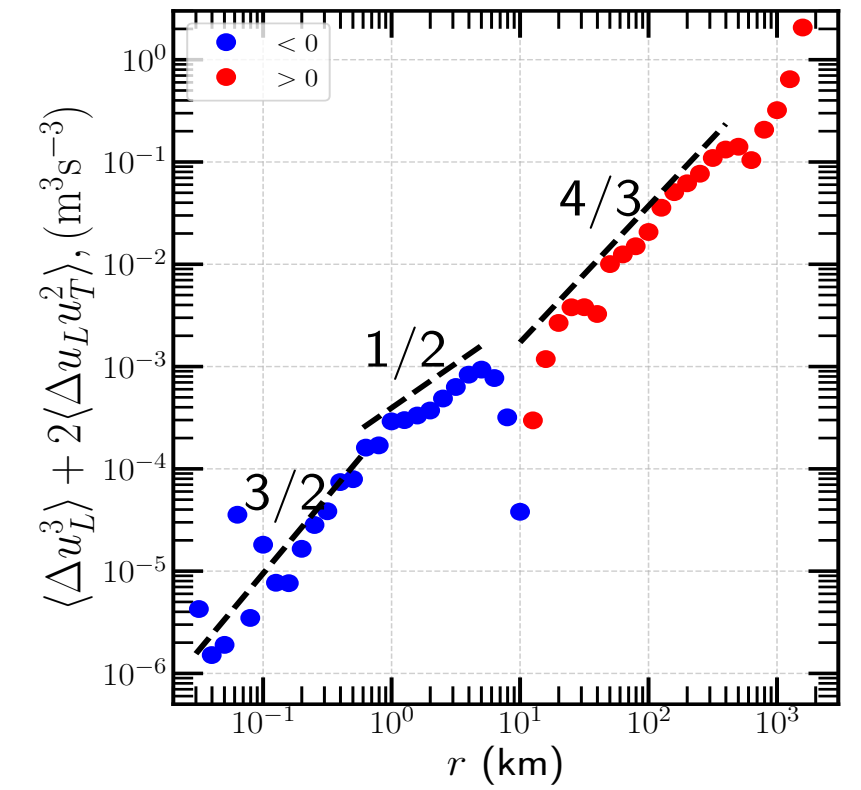
$$\langle \Delta u_L^3(r) \rangle \propto r^?$$



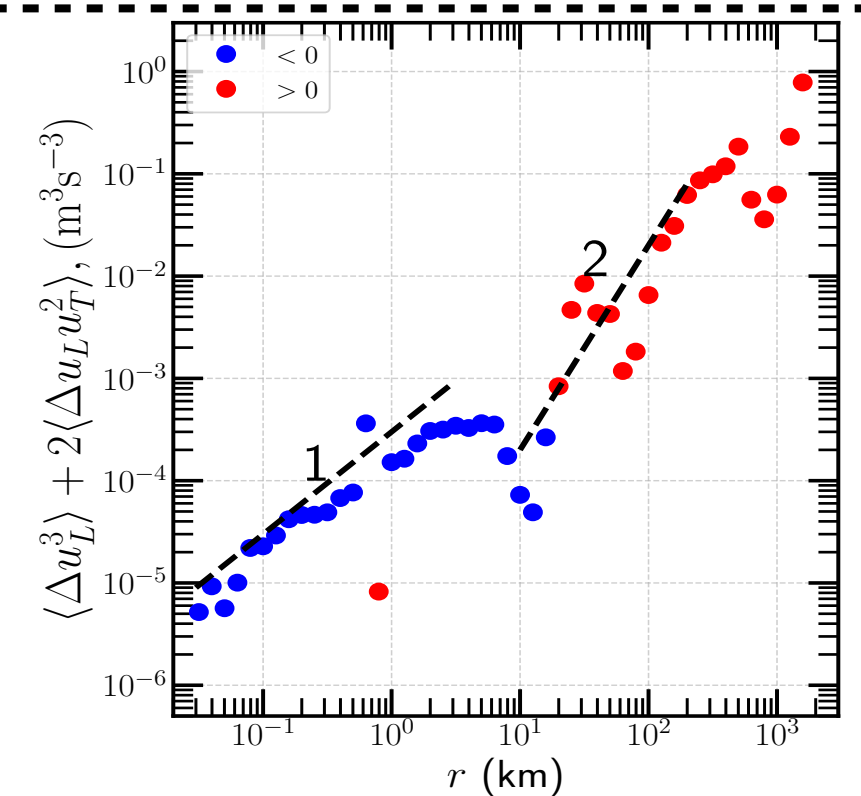
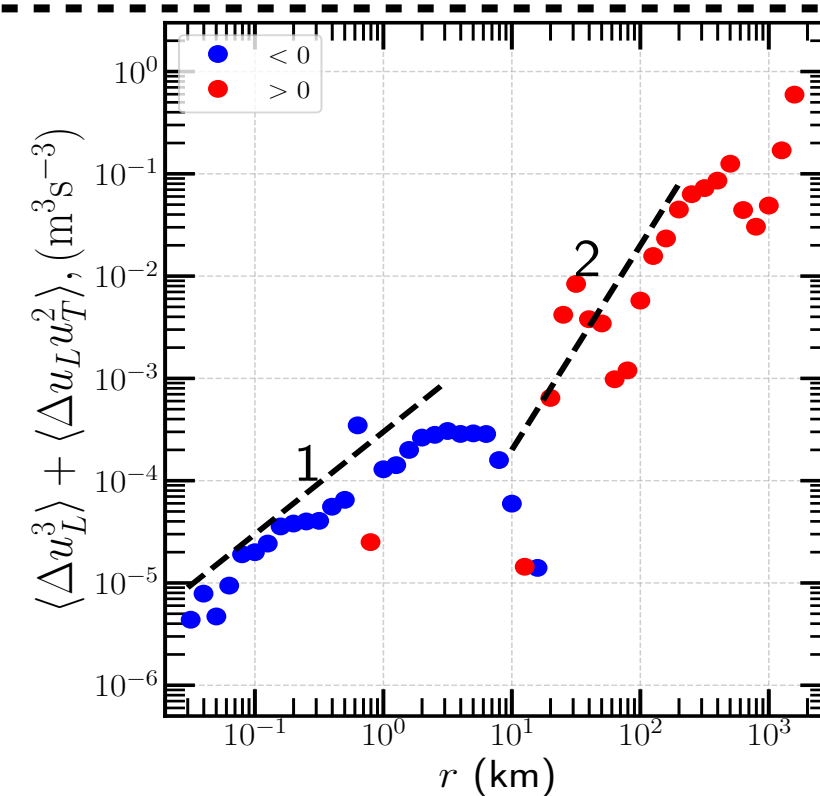
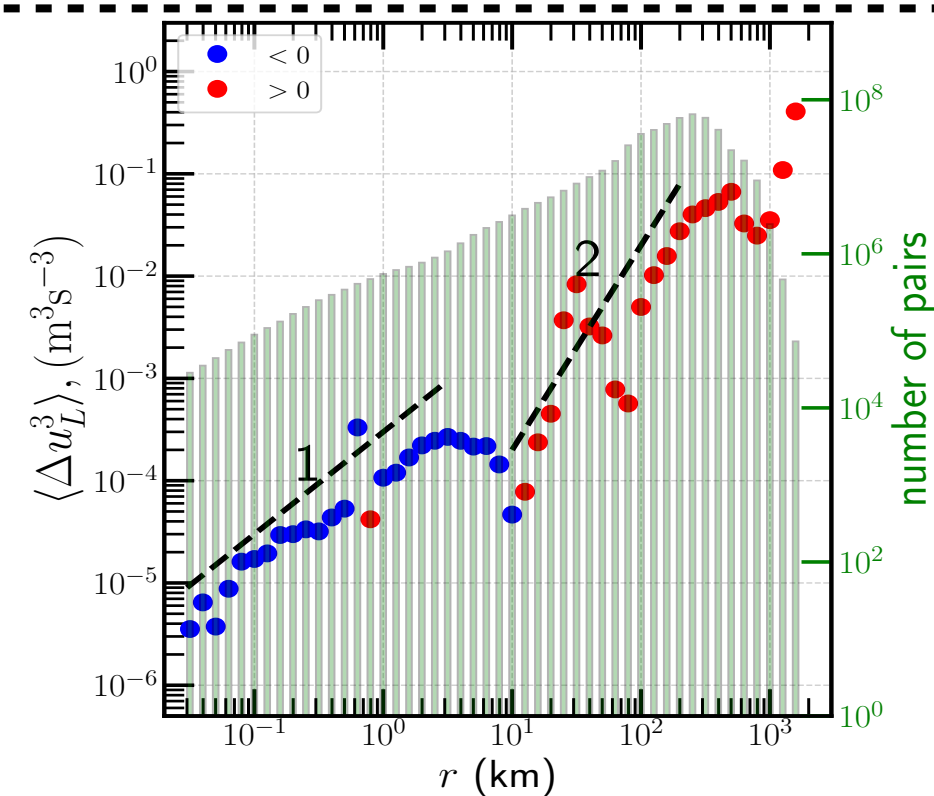
$$\langle \Delta u_L^3(r) \rangle + \langle \Delta u_L(r) \Delta u_T^2(r) \rangle \propto r^?$$



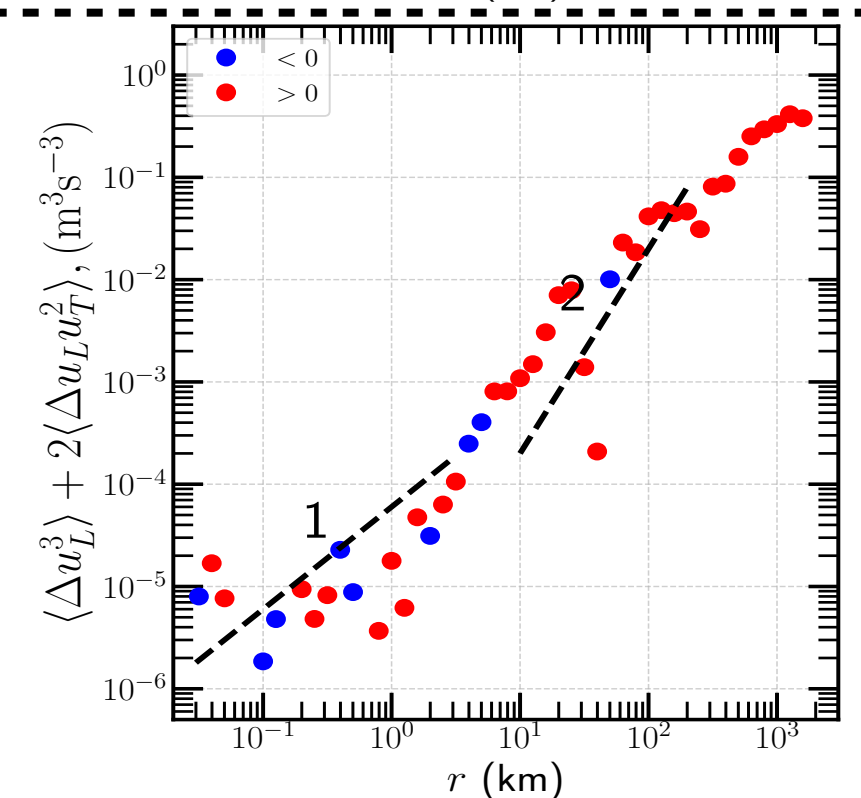
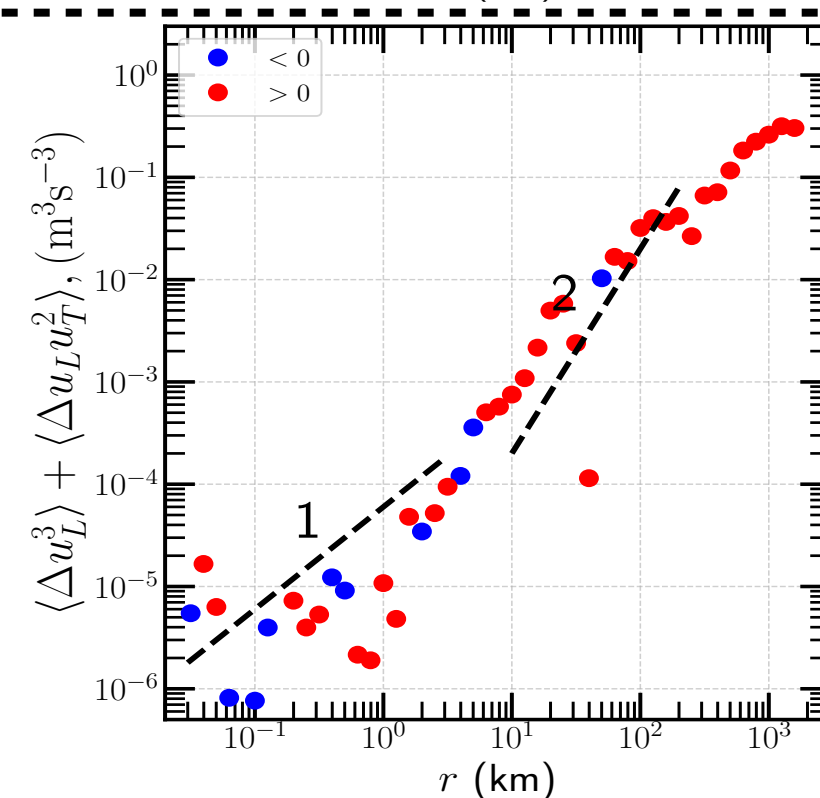
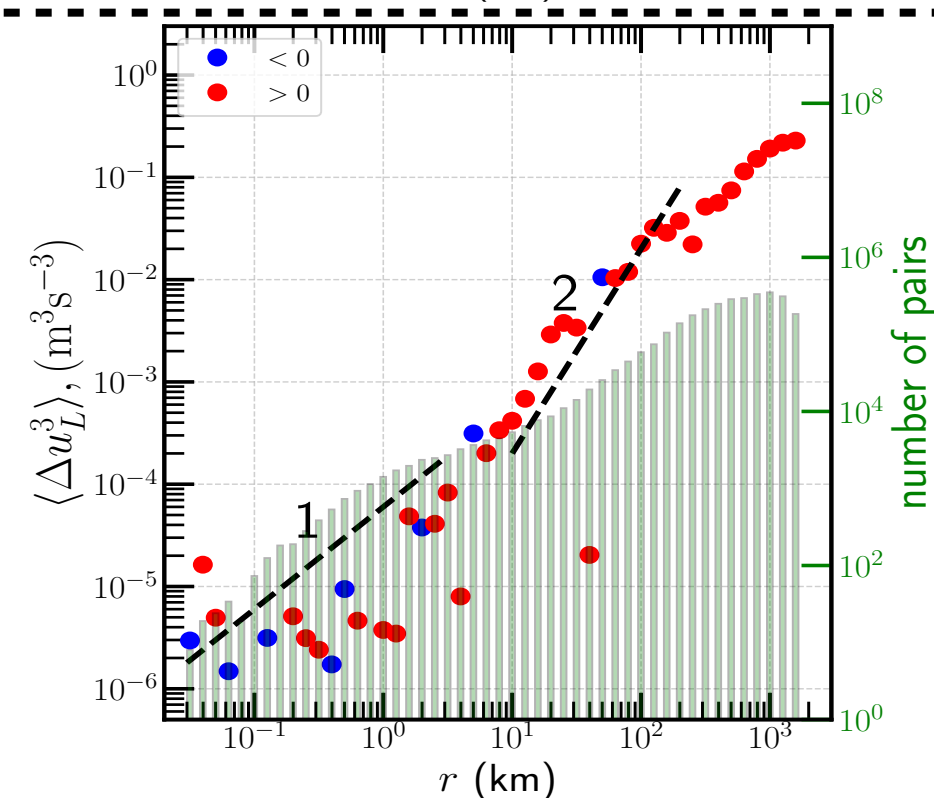
$$\langle \Delta u_L^3(r) \rangle + 2\langle \Delta u_L(r) \Delta u_T^2(r) \rangle \propto r^?$$



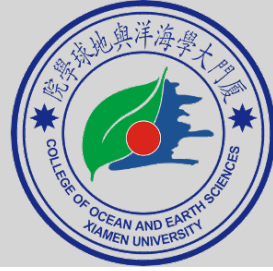
LASER
Jan. - Apr.
2016



GDP
1999-2020



- Sign change of $\langle \Delta u_L^3(r) \rangle$ in the scale of $\sim 10\text{km}$ in Golf of Mexico
- The oceanic cascade is confirmed by r-dependent 3rd SFs.
- Either Kraichnan or Charney theory cannot be excluded.



Thank you !