

# How to calibrate a climate model with neural network based physics?

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# Calibration of parameterizations

- Physical parameterizations are both tuned *offline* and *online* (e.g., to calibrate the mean climate).
- Current neural network (NN) based parameterizations can only be tuned *offline* : there is no guarantee that they will be accurate once they have been plugged into the model.

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## How to calibrate a dynamical system with NN-based physics?

### Calibration of NN parameterizations

- Step 1 : generate a new learning sample  $((x, \theta), y)$ .
- Step 2 : train an NN  $f$  to approximate  $y$  as a function of  $(x, \theta)$ .
- Step 3 : implement the NN parameterization into the model & calibrate the value of  $\theta$ , noted  $\theta^*$ . Use  $\theta^*$  in further simulations.

## The Lorenz'63 model

The Lorenz'63 (hereafter, L63, Lorenz, 1963) is defined with a set of 3 equations.

$$\dot{x}_1 = \sigma(x_2 - x_1), \quad (1)$$

$$\dot{x}_2 = x_1(\rho - x_3) - x_2, \quad (2)$$

$$\dot{x}_3 = x_1 x_2 - \beta x_3. \quad (3)$$

The NN  $\hat{f}$  learns to approximate Eq.3 :

$$\hat{\dot{x}}_3 = \hat{f}(x_1, x_2, x_3; \beta) \quad (4)$$

The learning sample is obtained by generating several L63 orbits, with  $\beta \in [1.6, 3]$ . Finally,  $\dot{x}_3$  is replaced with the NN model.

**Choice of the metric  $m$ .** In our case, the metric of interest is long-term climate (mean value) and standard deviation.

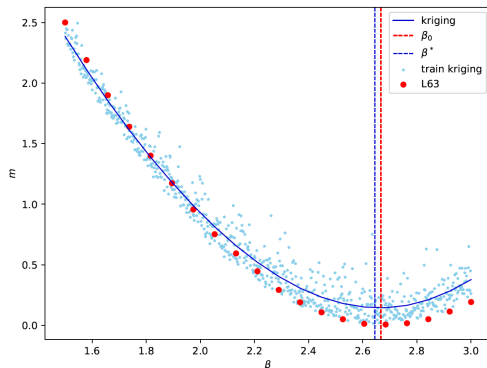
Let  $[\mathbf{x}]^{\hat{f}}(\beta)$  be a long orbit generated using the NN component, the mean value of which is noted  $(\hat{\mu}_1(\beta), \hat{\mu}_2(\beta), \hat{\mu}_3(\beta))$  and of standard deviation  $(\hat{\sigma}_1(\beta), \hat{\sigma}_2(\beta), \hat{\sigma}_3(\beta))$ . If the reference orbit has a mean state  $(\mu_1^0, \mu_2^0, \mu_3^0)$  and standard deviation  $(\sigma_1^0, \sigma_2^0, \sigma_3^0)$ , the estimate value of the metric  $m$  we have used can be expressed as :

$$\hat{m}(\beta) = \sum_{i=1}^3 (\hat{\mu}_i(\beta) - \mu_i^0) + (\hat{\sigma}_i(\beta) - \sigma_i^0). \quad (5)$$

Please note that  $m$  do not depend on the initial condition of the orbits: they are both considered long enough not to depend on it.

# Proof-of-concept experiment : L63

Calibration of  $\beta$  is achieved by minimizing a metric  $m$ , penalizing long-term model bias. Optimization is done on a kriging metamodel, that smoothes the  $\hat{m}(\beta)$ .



Value of metric  $m$  (MSE on mean climate) as a function of the tunable NN input parameter,  $\beta$ .

- Reference value :  
 $\beta_0 = 2.667$
- Optimal value :  
 $\beta^* = 2.664$ .

Lorenz'96 (Lorenz, 1996)

$K$  large-scale variables  $x_k$  are coupled to  $J$  fine-scale variables  $y_{k,j}$  each.

$$\frac{dx_k}{dt} = -x_{k-1}(x_{k-2} - x_{k+1}) - x_k + F - \frac{hc}{b} \sum_{j=1}^J y_{k,j}, \quad (6)$$

coupling term  $B$

$$\frac{1}{c} \frac{dy_{k,j}}{dt} = -by_{k,j+1}(y_{k,j+2} - y_{k,j-1}) - y_{k,j} + \frac{hc}{b} x_k. \quad (7)$$

NN parameterization  $\hat{f}$ , with  $\theta = c$  :

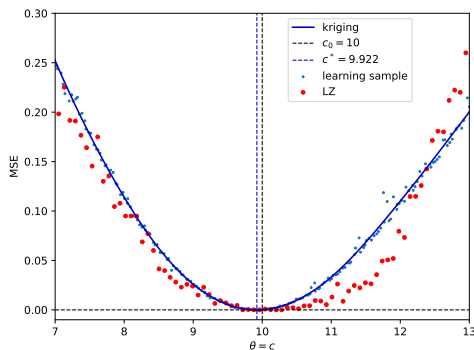
$$\hat{B} = \hat{f}(\mathbf{x}; \mathbf{c})$$

And the resulting dynamical model will be :

$$\frac{d\hat{x}_k}{dt} = -x_{k-1}(x_{k-2} - x_{k+1}) - x_k + F + \hat{B} \quad (8)$$

# L96 : perfect model calibration

In this case, the 'dynamical' part of L96 (Eq.8) is calibrated using the reference value of the parameters, i.e.,  $(h, F, b) = (h_0, F_0, b_0)$  (= perfect model calibration). Calibration of  $c$  is achieved by minimizing a metric  $m$  (analogous to the metric  $m$  used in the L63 case), penalizing long-term model bias.



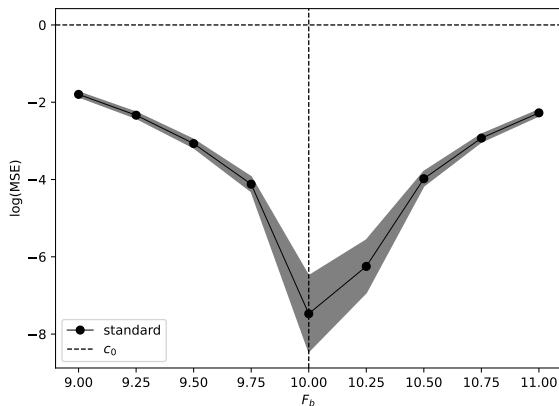
- Reference value :  
 $c_0 = 10.0$
- Optimal value :  
 $c^* = 9.92$ .



## L96 : imperfect model calibration

In the following, we use a biased value  $F_b \neq F_0$  in Eq.8. We suppose that  $F_b$  is not a tunable parameter.

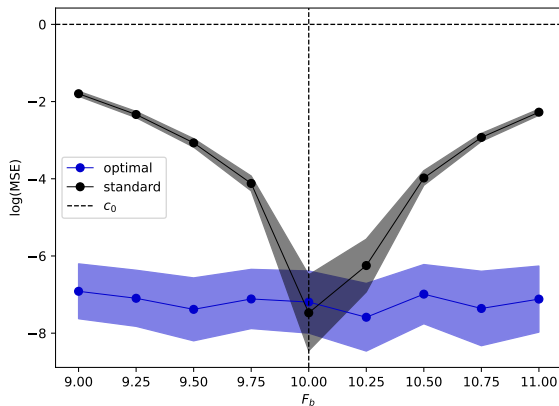
Using  $\hat{B}(x, c_0)$  leads to high model error. The curve below represents  $\log(\hat{m}(c_0))$  for different values of  $F_b$ .



# Imperfect model calibration

Using  $\hat{B}(x, c_0)$  leads to increasing model errors, as the bias on  $F$  increases.

Using  $\hat{B}(x, c^*)$  leads to low model errors, even with large biases on  $F$ .



- It is possible to obtain NN-based physics that can be calibrated online, by adding some parameters  $\theta$  among the input variables of the NN. The value of these additional parameters  $\theta$  can be tuned to ensure low long-term bias of the resulting model.
- We have illustrate our method using the L63 model. We have successfully recovered the reference value of parameter  $\beta$  using our method and  $c$  in the L96 example.
- We have also successfully reduced the value of the metric  $m$  when one of the model parameters that cannot be tuned carried biases in the L96 case study.

**Next step** : implementing our method in a climate model.

You can also check out our paper on Essoar (v1) or in GRL.