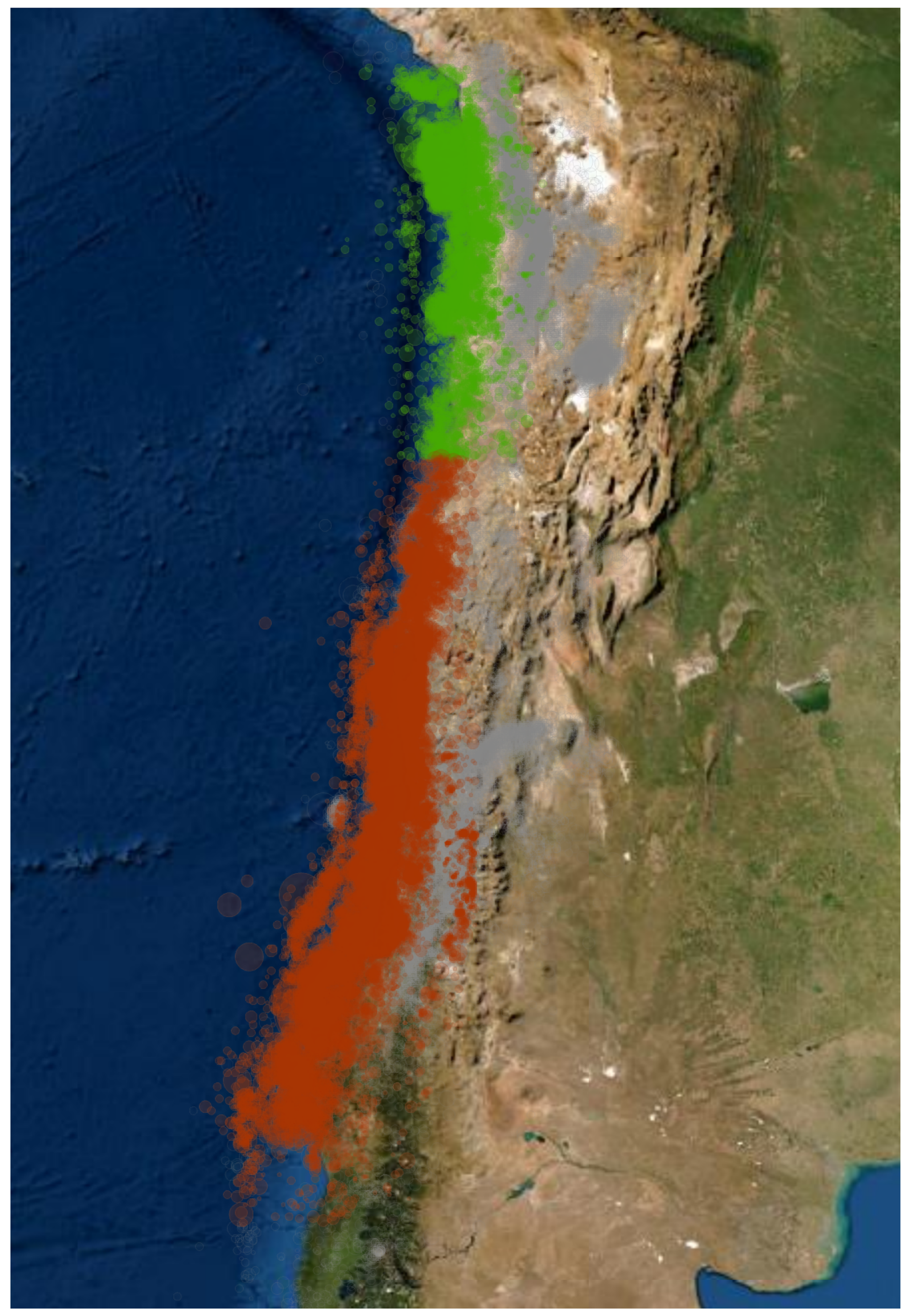
Predictive properties of an anisotropic ETAS Space-time model applied to Chilean seismicity

Marcello Chiodi[1], Orietta Nicolis[2] , Giada Adelfio[1], Nicoletta DÁngelo[1], A. Gonzàlez[2] [1]Univ. of Palermo, Italy. [2]Univ. Andres Bello, Chile.

Introduction



- Nazca plate is being subducted beneath South America at a rate of 90 mm/yr in the north and approximately 40mm/yr in the center-south of Chile; in this paper we will present brief results only on north area;
- a value of magnitude $m_0 = 2.8$ has been taken as a magnitude threshold;
- only events from 2007 have been taken and until 2020: before 2007 catalogue accuracy is probably different;
- earthquakes deeper than 70 km have been excluded because they occur within the continental plate in the crust.

With this selection, we used 7,232 events for northern area. Catalogues contained information also on covariates, like GPS positioning, distance from Nazca plate, and so on, but they will not be used in this presentation.

ETAS Model with covariates for triggering

A model for earthquake description widely used is the Epidemic Type Aftershocks-Sequences (ETAS) model, in which the intensity in time t and location $\mathbf{s}$ conditional to $\mathcal{H}_t$, the history until t, is:

$$\lambda_{\boldsymbol{\theta}}(t,\mathbf{s}|\mathcal{H}_t) = \underbrace{\mu f(\mathbf{s})}_{\text{background}} + \underbrace{\sum_{t_j<t} \frac{\kappa_0 \exp(\eta_j)}{(t-t_j+c)^p}\left[(\mathbf{s}-\mathbf{s}_j)^2+d\right]^{-q}}_{\text{triggered seismicity}}$$

where $(t_j, \mathbf{s}_j)$ is the time and location of individual occurrence of events before time t;
$\eta_j = \boldsymbol{\beta}^\mathsf{T}\mathbf{Z}_j$ is a linear predictor, with $\mathbf{Z}_j$ the external known covariate vector, in this paper only with one classical covariate $z_{j1} = m_j - m_0$ where m_j is the magnitude of the triggering event, and m_0 is a threshold, acting in a multiplicative fashion on the base risk;
$\boldsymbol{\theta} = (\mu, \kappa_0, c, p, d, q, \boldsymbol{\beta})^\mathsf{T}$, with $\boldsymbol{\beta}$ a k-component vector, to be estimated together with the other 6 components of $\boldsymbol{\theta}$; in this presentation we will use only β_1 for the magnitude.
$f(\mathbf{s})$ is estimated with a non-parametric technique based on the FLP method (Forward Likelihood Predictive) proposed by Adelfio&Chiodi for an optimal choice of bandwidth and a variable adaptive kernel.

The `R` package `etasFLP` with variable and adaptive kernel

Computations are implemented in **our open source `R` package `etasFLP`.** The package has many options for the estimation of an ETAS model and for results (`summary`, `plot` with diagnostic, `profile` likelihood,...). It uses a two-stage technique: Maximum Likelihood for parameters estimation and a FLP (Forward Likelihood Predictive) technique for non parametric estimation.
For this presentation we used an experimental version with a variable kernel estimation of background intensities.
Given n observed events in the spatial locations $\boldsymbol{s}_1, \boldsymbol{s}_2 \ldots, \boldsymbol{s}_n$, to estimate $f(\mathbf{s})$:

$$\hat{f}_{\boldsymbol{\Sigma}}(\mathbf{s}) = \hat{f}(\mathbf{s}) = \frac{\sum_{i=1}^n K(\mathbf{s}-\mathbf{s}_i, \boldsymbol{\Sigma_i}) \cdot \rho_i}{\sum_{i=1}^n \rho_i}.$$

where $K(\cdot, \boldsymbol{\Sigma_i})$ is the density of a multivariate normal distribution with covariance matrix $\boldsymbol{\Sigma_i}$; ρ_i is a weight given by the estimated probability that the i-th point belongs to the background; $\boldsymbol{\Sigma}_i = (1-\alpha)diag(\mathbf{h}) + \alpha \boldsymbol{Z_i} \quad (0 \le \alpha \le 1)$ with $\mathbf{h}$ a 2-components bandwidth; $\boldsymbol{Z_i}$ is a locally weighted estimation of the covariance matrix; $\alpha = 0 \ \Rightarrow$ fixed kernel; $\alpha = 1 \ \Rightarrow$ full variable kernel.
Three smoothing parameters are estimated: h_x, h_y (the bandwiths) and α by means of the FLP technique, already present in the package for the bandwidths estimation and now extended to estimate also α.

Summary of the `etasFLP` fitting with two kinds of kernel

Two simple models have been fitted to the north catalog: the first one with constant kernel and the second one with variable adaptive kernel. As expected the second AIC value is considerably better than the the first:
Model1: $AIC = 152279.6; h_x = 8.302; h_y = 14.626.$
Model2: $AIC = 148582.6; h_x = 8.750; h_y = 9.339; \alpha = 0.887.$
As expected parameters estimates are different, even if the correlation between final estimated intensities in the two models is quite high, 0.9927.

Estimates for model 1:

	μ	κ	c	p	γ	d	q	β_1
MLE	0.563	0.025	0.019	1.026	0.848	6.228	1.879	0.299
St.Err.	0.014	0.006	0.002	0.008	0.041	0.695	0.059	0.038

Estimates for model 2:

	μ	κ	c	p	γ	d	q	β_1
MLE	0.713	0.031	0.041	1.152	0.876	6.316	1.964	0.345
St.Err.	0.014	0.008	0.005	0.012	0.040	0.717	0.065	0.038

First results for northern area seem satisfactory, and will be extended to models with some covariates to explore the ability of the the model of obtaining satisfactory forecasting.