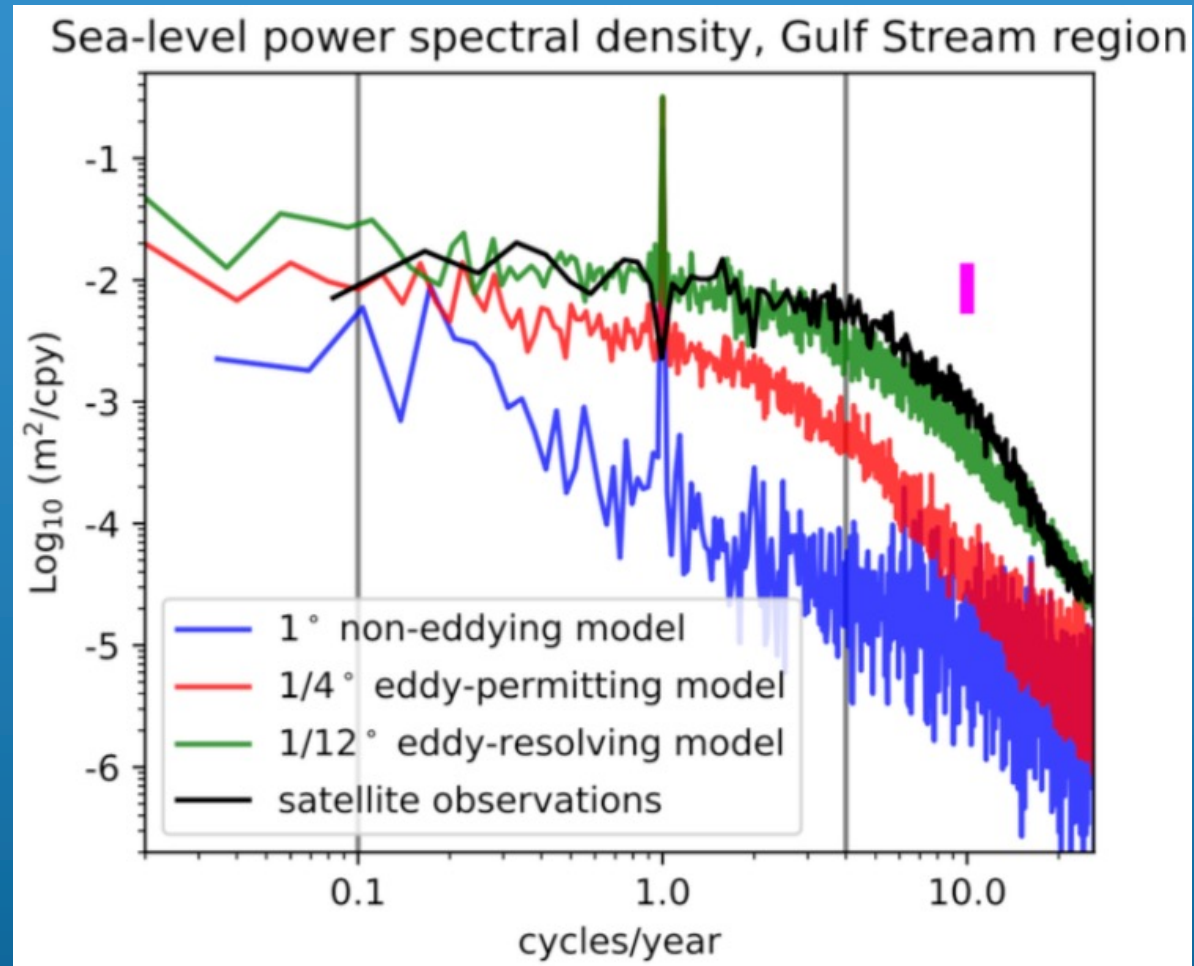


“Eddy-resolving” models compare more closely with observations, but are computationally expensive

For decadal-scale or ensemble climate modelling (CMIP7), we still must rely on mesoscale “eddy-permitting” models

However, “eddy-permitting” models have too much viscous dissipation of KE

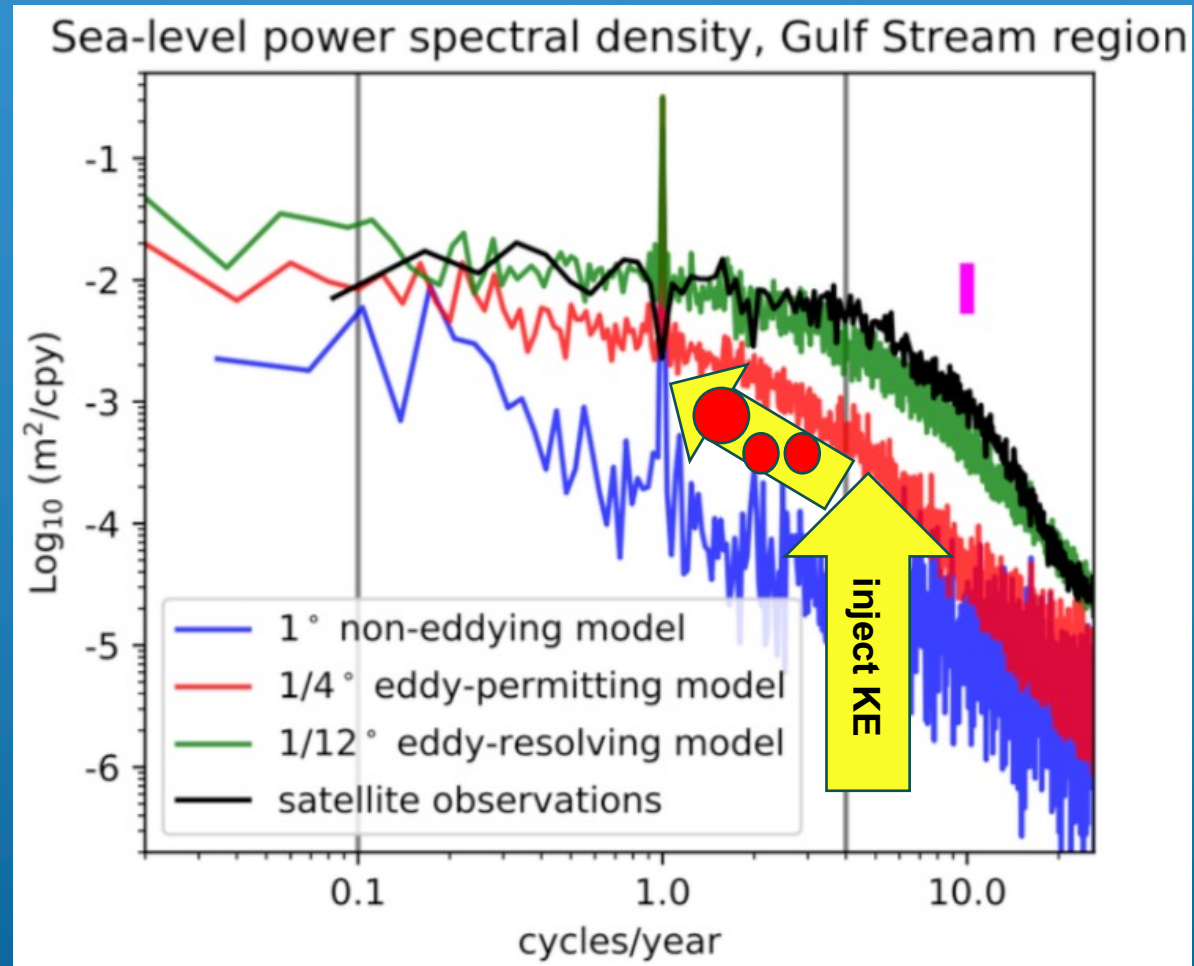


Unrealistic climate variability on seasonal and longer scales – loss of potential predictability

“Eddy-resolving” models compare more closely with observations, but are computationally expensive

For decadal-scale or ensemble climate modelling (CMIP7), we still must rely on mesoscale “eddy-permitting” models

However, “eddy-permitting” models have too much viscous dissipation of KE



Use a Kinetic Energy Backscatter parameterization to energise the partially-resolved eddies



## Control simulations

NEMO “GYRE” :

“Eddy-resolving” (top)  
1/12°  
(coarse-grained to 1/4°)

and

“eddy-permitting” 1/4°  
(bottom)

flat bottom, seasonal  
surface wind and  
buoyancy forcing

each with 30 z-levels

bi-Laplacian viscosity  
constant diffusivity

Lévy et al. (2010)

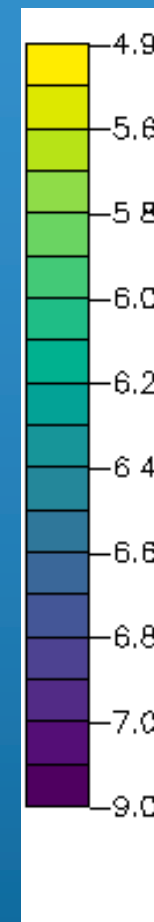
lat

lon

R12 control  
coarse-grained

R4 control

$\log_{10}$  (depth-mean KE ( $\text{m}^2 \text{s}^{-2}$ ))



**1. Why adiabatic?** Mesoscale eddies are predominantly adiabatic in the ocean interior. Adiabatic eddy forcing preserves watermass structure in multidecadal climate runs.

**2. Considerations for a KE backscatter forcing/parameterization:** Desirable to respect both the **numerics** (undoing the action of excessive viscous dissipation) and the **physics** (capture the scales, processes of the unresolved eddies).

**3. How do others approach the problem?** a) by applying a negative viscosity (usually a slightly different formulation to the actual viscous damping, e.g. Laplacian forcing for bi-Laplacian dissipation);  
b) stochastic forcing – to mimic the missing effects of nonlinear interactions due to geophysical turbulence

**4. What is our approach?** Generate stochastic forcing designed around the **numerics** and around the **physics**, based on autoregressive modelling order-n (AR(n)).

We use the adiabatic framework of the **"semi-prognostic method"** to generate an eddy momentum forcing by changing the density "seen" by the hydrostatic equation:

Stochastic forcing, proportional to instantaneous viscous dissipation  
~ model timestep

Pressure gradient force to wiggle isopycnals up and down

$$\frac{\partial P}{\partial z} = -(\rho - \rho_c)g$$



$$\frac{\partial \mathbf{u}}{\partial t} = \dots + \frac{1}{\rho_0} \nabla P_{eddy}$$

**1. Why adiabatic?** Mesoscale eddies are predominantly adiabatic in the ocean interior. Adiabatic eddy forcing preserves watermass structure in multidecadal climate runs.

**2. Considerations for a KE backscatter forcing/parameterization:** Desirable to respect both the **numerics** (undoing the action of excessive viscous dissipation) and the **physics** (capture the scales, processes of the unresolved eddies).

**3. How do others approach the problem?** a) by applying a negative viscosity (usually a slightly different formulation to the actual viscous damping, e.g. Laplacian forcing for bi-Laplacian dissipation);  
b) stochastic forcing – to mimic the missing effects of nonlinear interactions due to geophysical turbulence

**4. What is our approach?** Generate stochastic forcing designed around the **numerics** and around the **physics**, based on autoregressive modelling order-n (AR(n)).

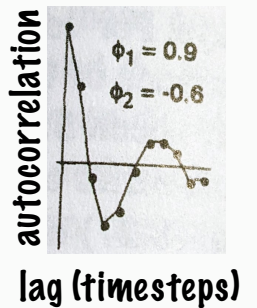
We use the adiabatic framework of the **"semi-prognostic method"** to generate an eddy momentum forcing by changing the density "seen" by the hydrostatic equation:

**Stochastic forcing, proportional to instantaneous viscous dissipation  
~ model timestep**

**Pressure gradient force to wiggle isopycnals up and down**

$$\frac{\partial P}{\partial z} = -(\rho - \rho_c)g$$

$$(\rho_c)_t = b \sum_{i=1}^n a_i \rho_{t-i\Delta t} + \epsilon_{t-i\Delta t}$$





National  
Oceanography  
Centre

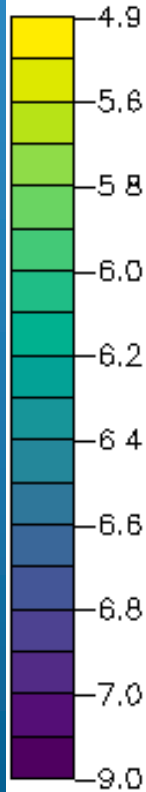
Left panels: control

Right panels: with  
stochastic, adiabatic  
forcing

lat

lon

log10 (depth-mean KE ( $\text{m}^2 \text{s}^{-2}$ ))



R12 control  
coarse-grained

R4 stochastic 0.2

R4 control

R4 stochastic 0.1

Contact:

[cwi@noc.ac.uk](mailto:cwi@noc.ac.uk)





National  
Oceanography  
Centre

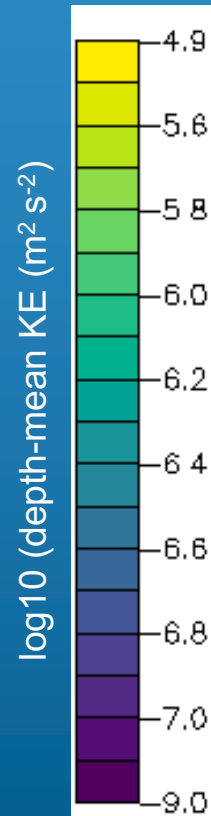
### Summary:

In this first phase, we explored the near-timestep, near-gridscale “undoing of numerical overmixing of KE” – an ADIABATIC KE backscatter forcing

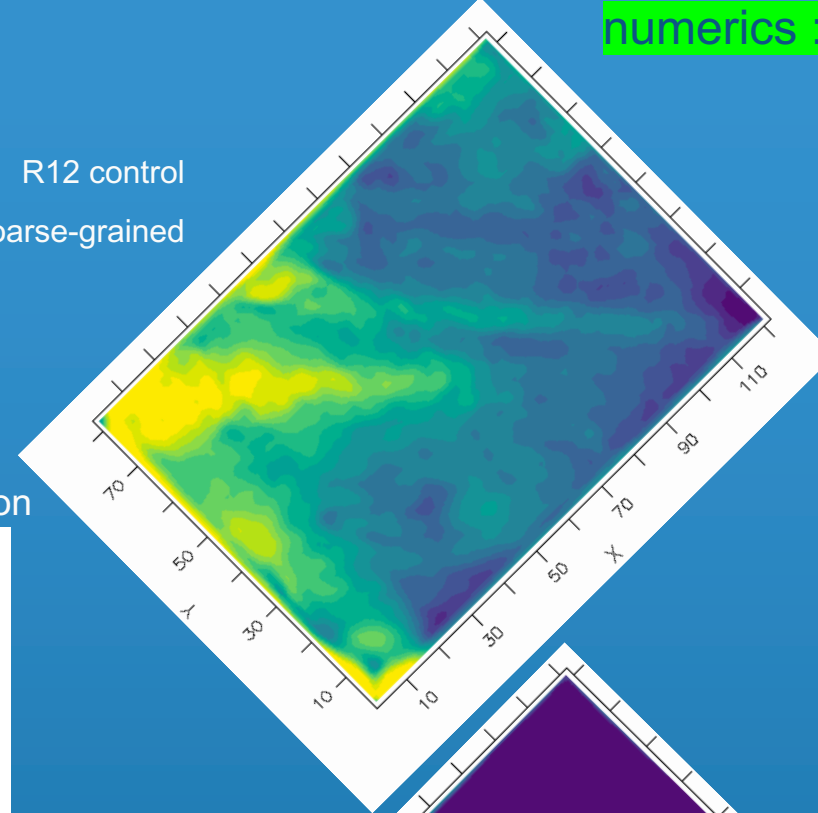
The AR(2) stochastic model can energise the flow and cause feedbacks which extend the jet

Further development is required including exploring parameterization of eddy physical scales

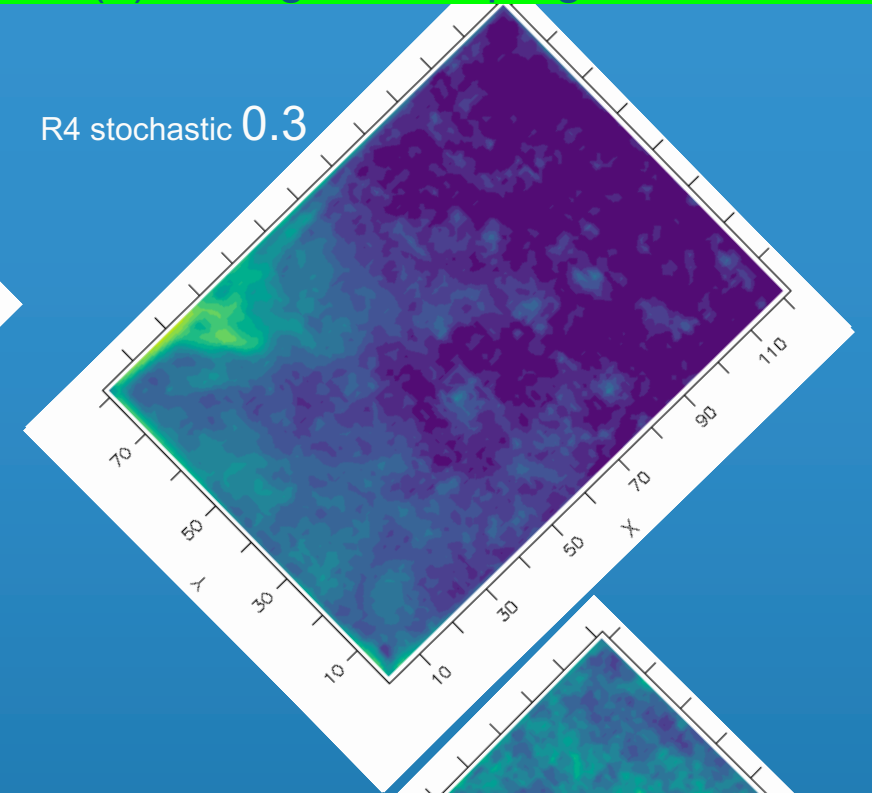
lat  
lon



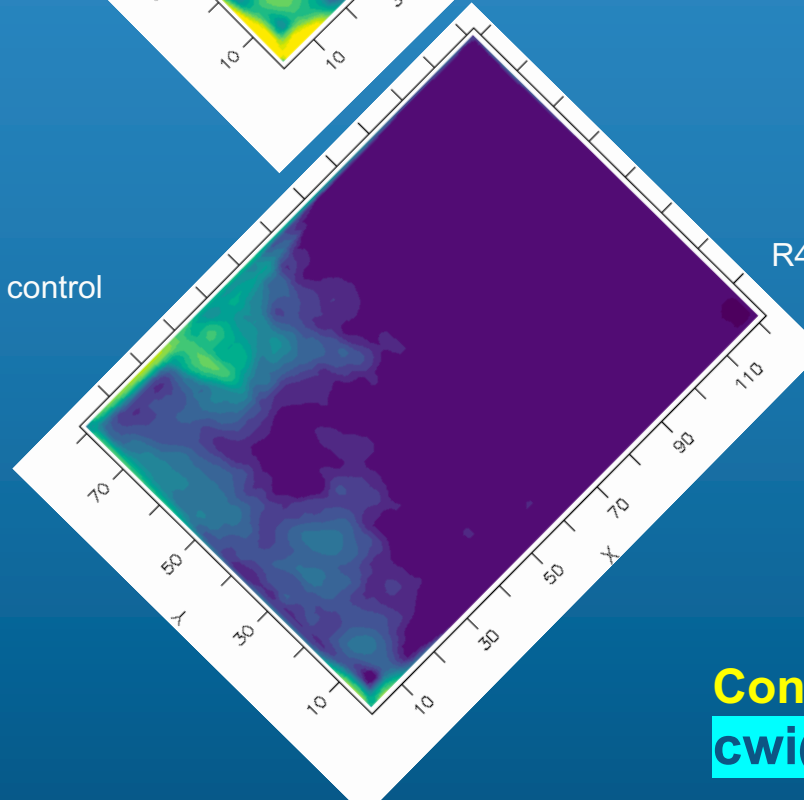
R12 control  
coarse-grained



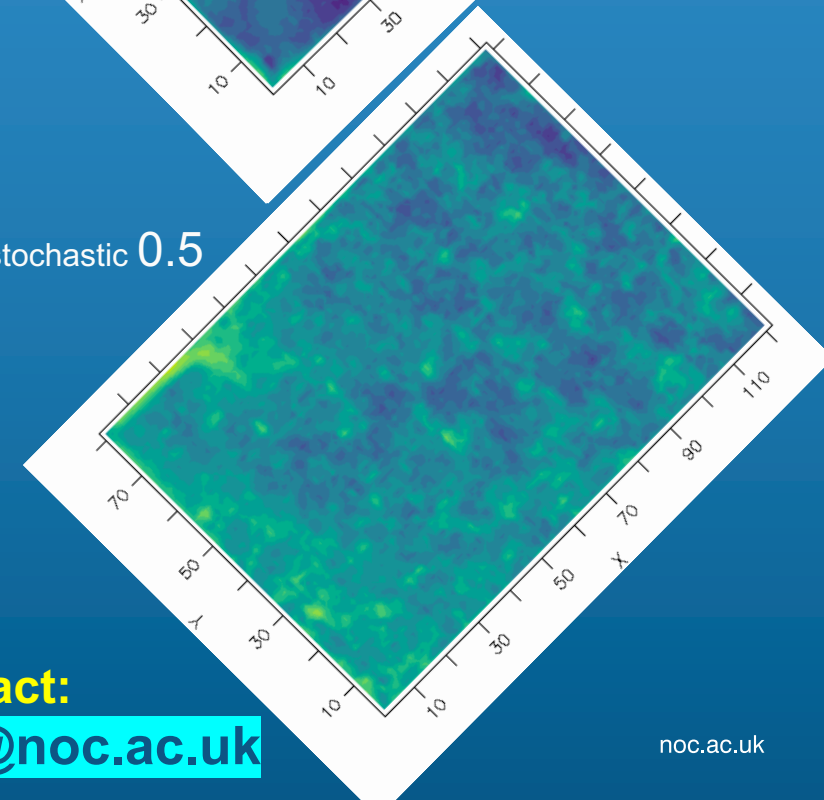
R4 stochastic 0.3



R4 control



R4 stochastic 0.5



Contact:

[cwi@noc.ac.uk](mailto:cwi@noc.ac.uk)

**How is our method different from the stochastic equation of state in NEMO?**

**(Brankart et al., 2013; 2015)**

- **We do not directly alter the density or tracer field**
- **We generate a perturbation to the  $dP/dz$  term in the hydrostatic equation, which is then integrated in the vertical and differentiated in the horizontal to generate a momentum forcing, which is like the grad P term from the standard hydrostatic equation.**
- **Within the loop that calculates  $dP/dz$  in the hydrostatic equation, a dummy density term is used on the r.h.s. but then replaced after the loop.**



**NEMO****SST** 15N : 50N

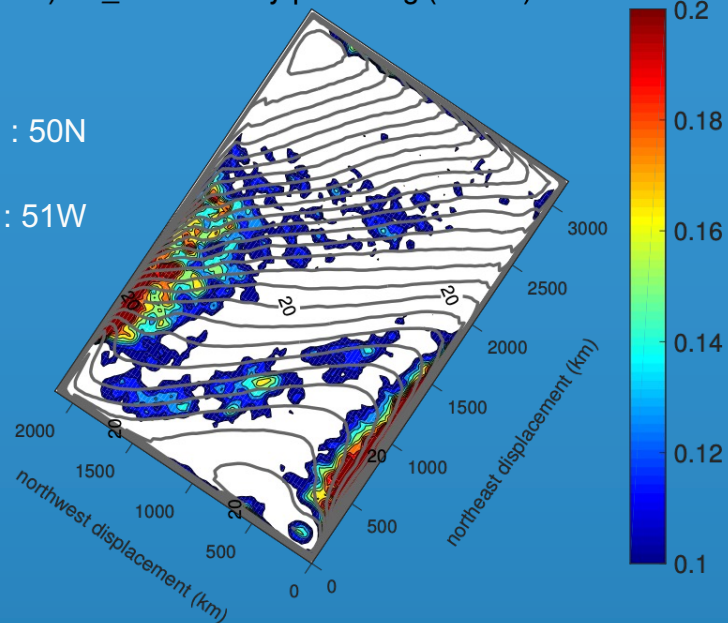
86W : 51W

**1/4 deg.  
& 1/9 deg.**

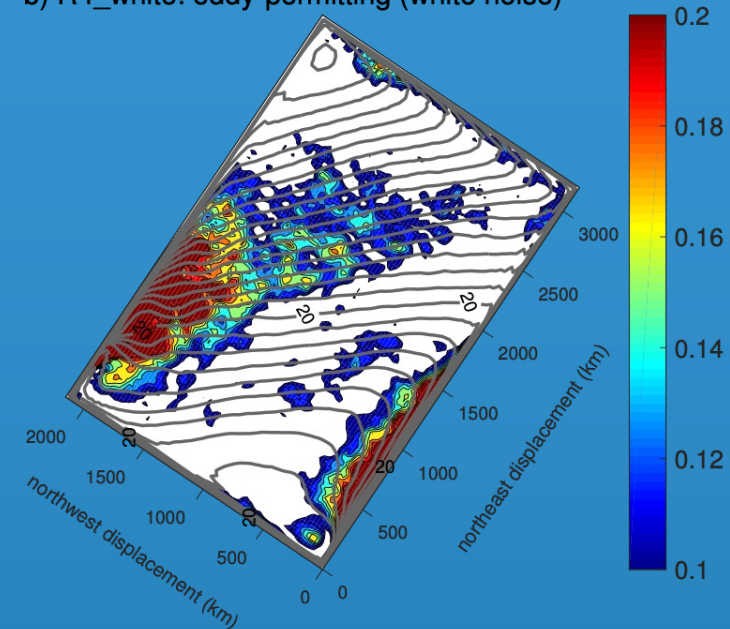
lat

lon

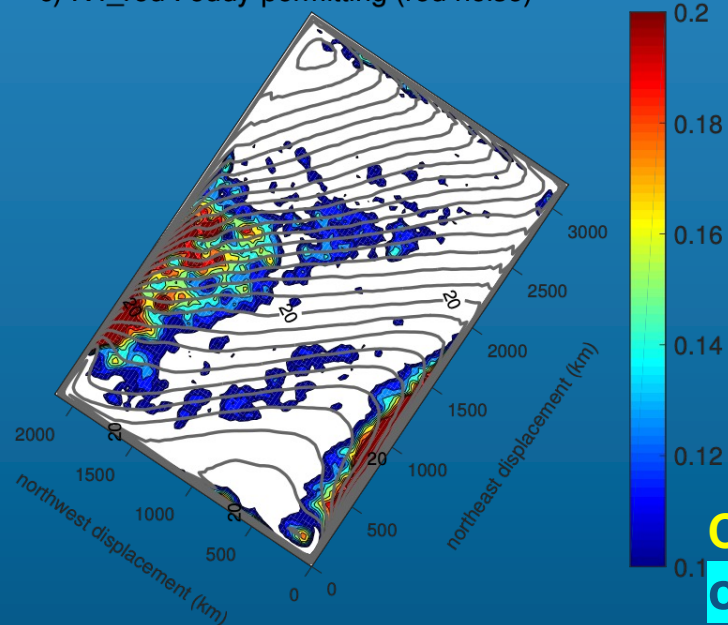
a) R4\_control: eddy-permitting (control)



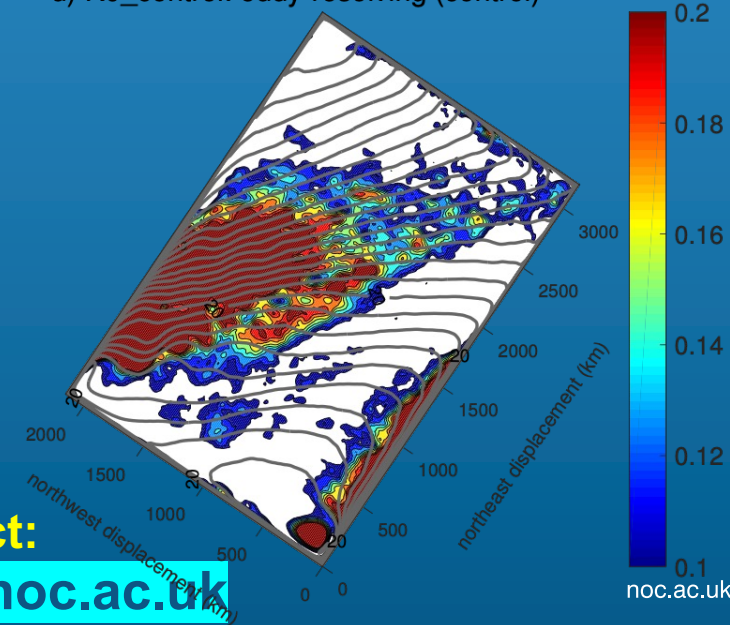
b) R4\_white: eddy-permitting (white noise)



c) R4\_red : eddy-permitting (red noise)



d) R9\_control: eddy-resolving (control)

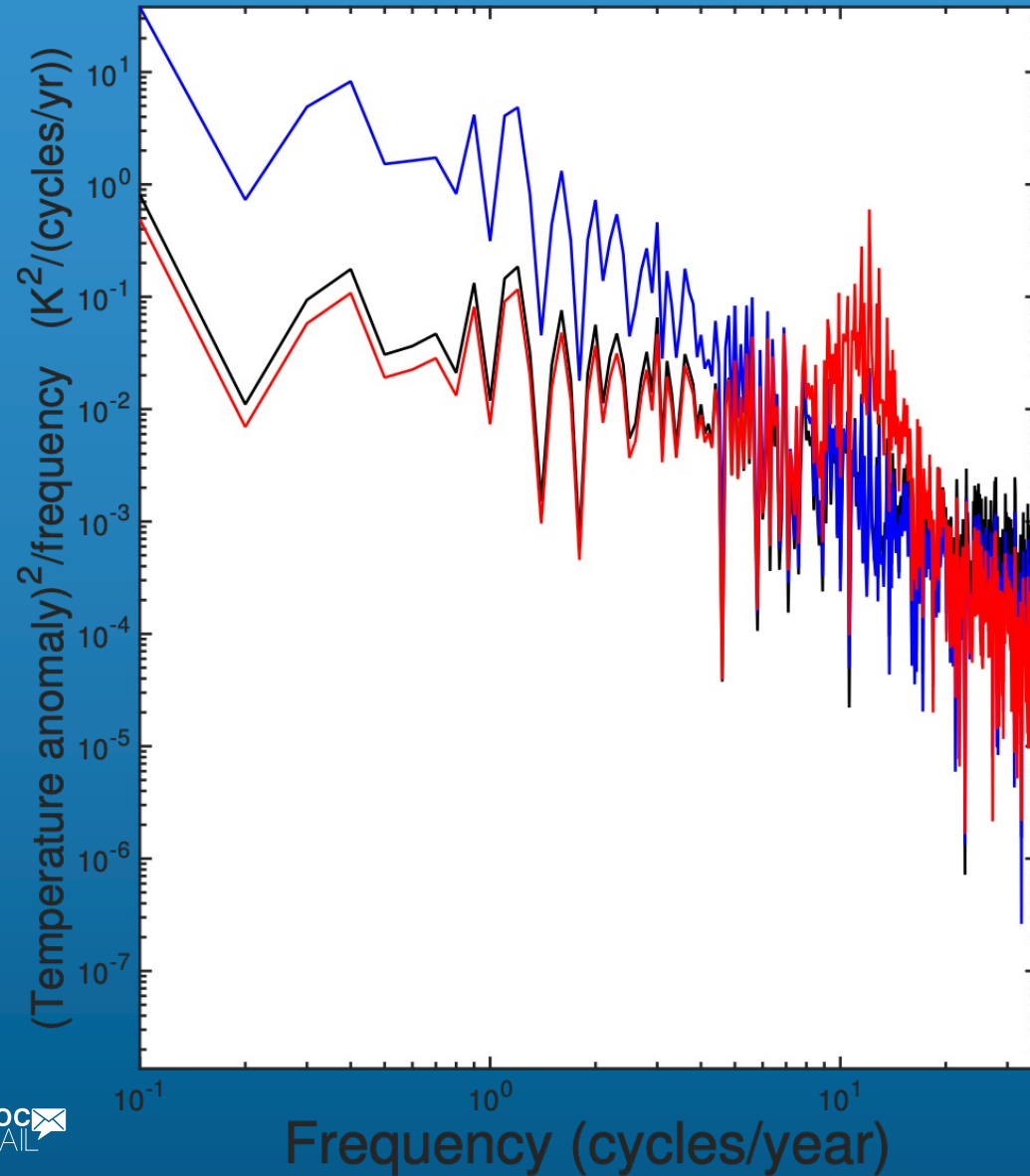
**Contact:**[cwi@noc.ac.uk](mailto:cwi@noc.ac.uk)

noc.ac.uk

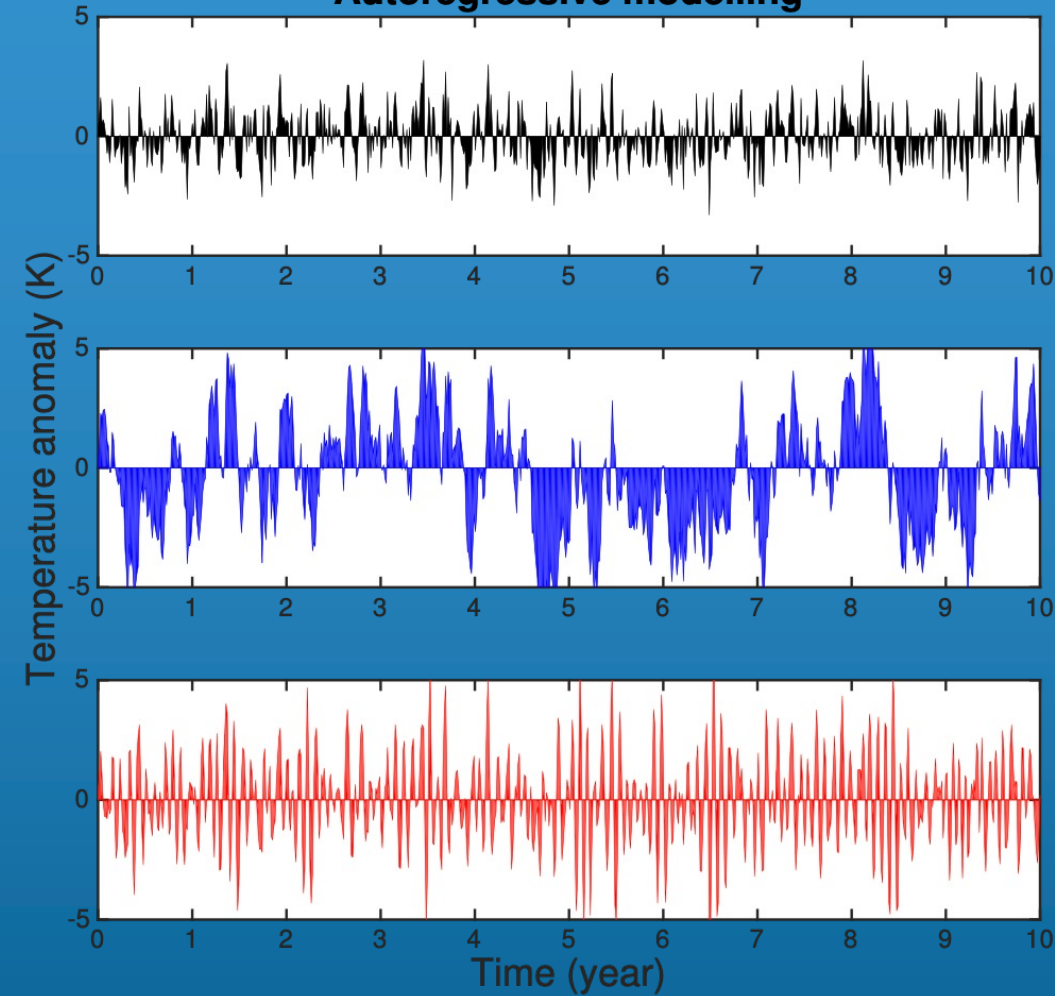


National  
Oceanography  
Centre

## Characteristic frequency spectra



## Autoregressive modelling



Contact:

[cwi@noc.ac.uk](mailto:cwi@noc.ac.uk)



A1) derive statistical models of the stochastic variability of density from the "truth" and use this to directly energise the missing mesoscale energy sources of APE and, via adjustment, KE.

**Most basic: approach of Zang and Wunsch (2001)**

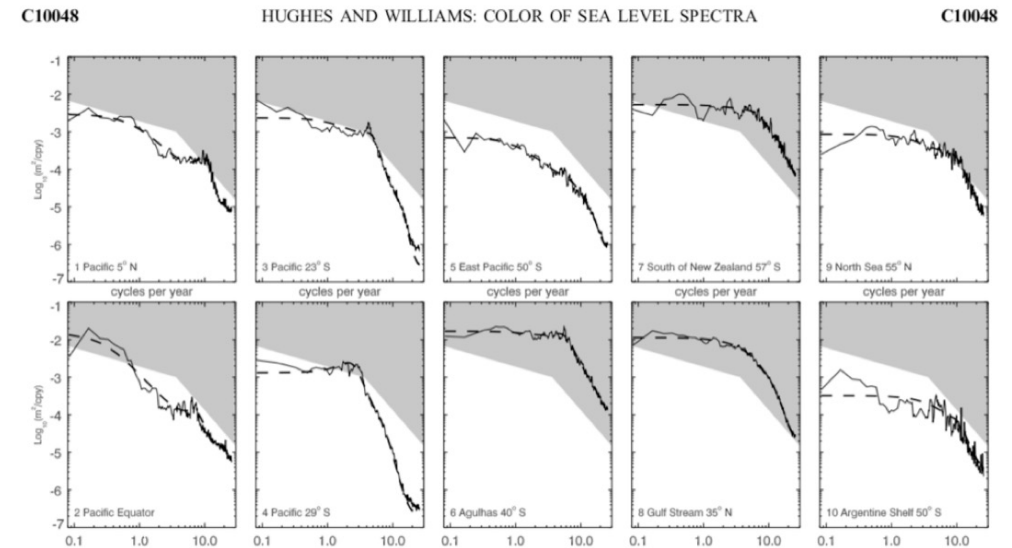
For each mode,

for any  
variable,

$$\Phi(k, l, \omega, n, \phi, \lambda) = B_n(k)C_n(l)D_n(\omega)E_0(n)I(\phi, \lambda). \quad (29)$$

**Hughes and Williams (2010)**

**Frequency domain: can improve ZW10 with AR(n) model**



**Figure 6.** Power spectral densities as a function of frequency for spectra averaged over the 10 boxes shown in Figure 5b. Dotted lines show the corresponding AR(5) fit to the data. The shading covers the region above the line represented by the Zang and Wunsch [2001] spectrum at an arbitrary chosen amplitude.