Nonlinear time series based forecast modelling of groundwater flooding at karst region in Ireland

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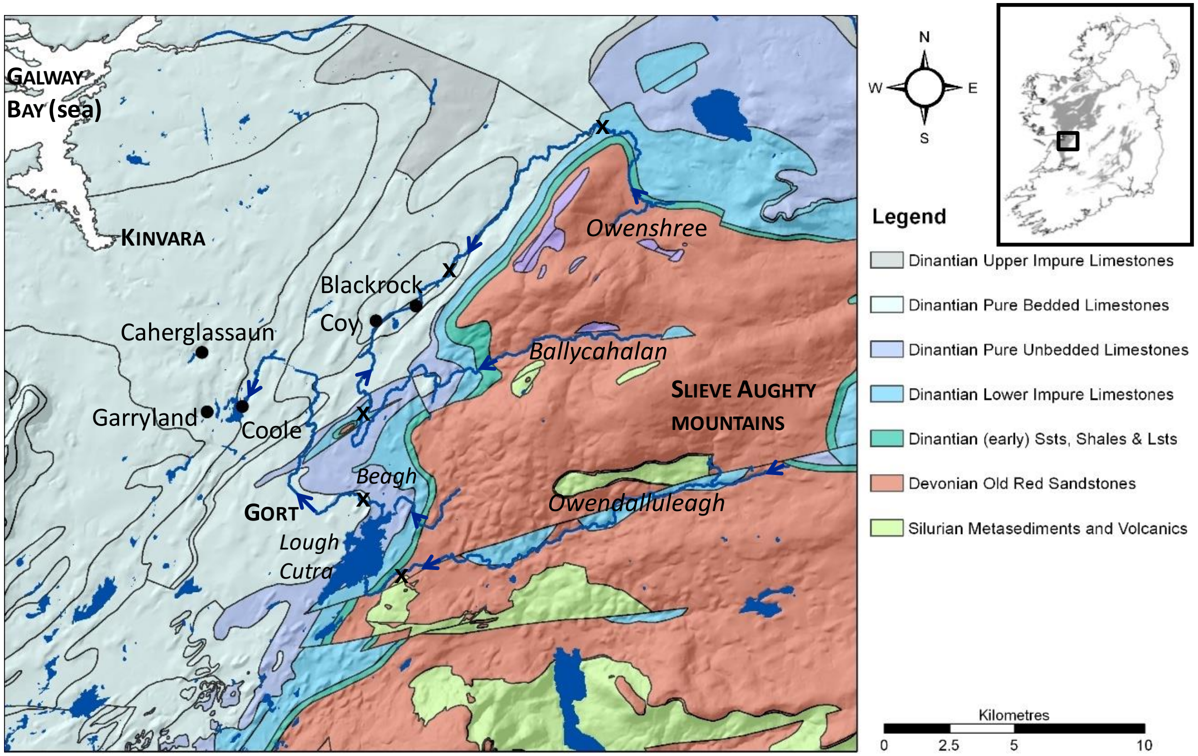
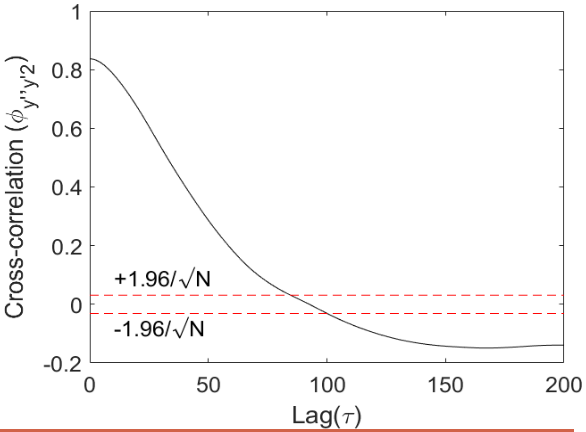
KARST

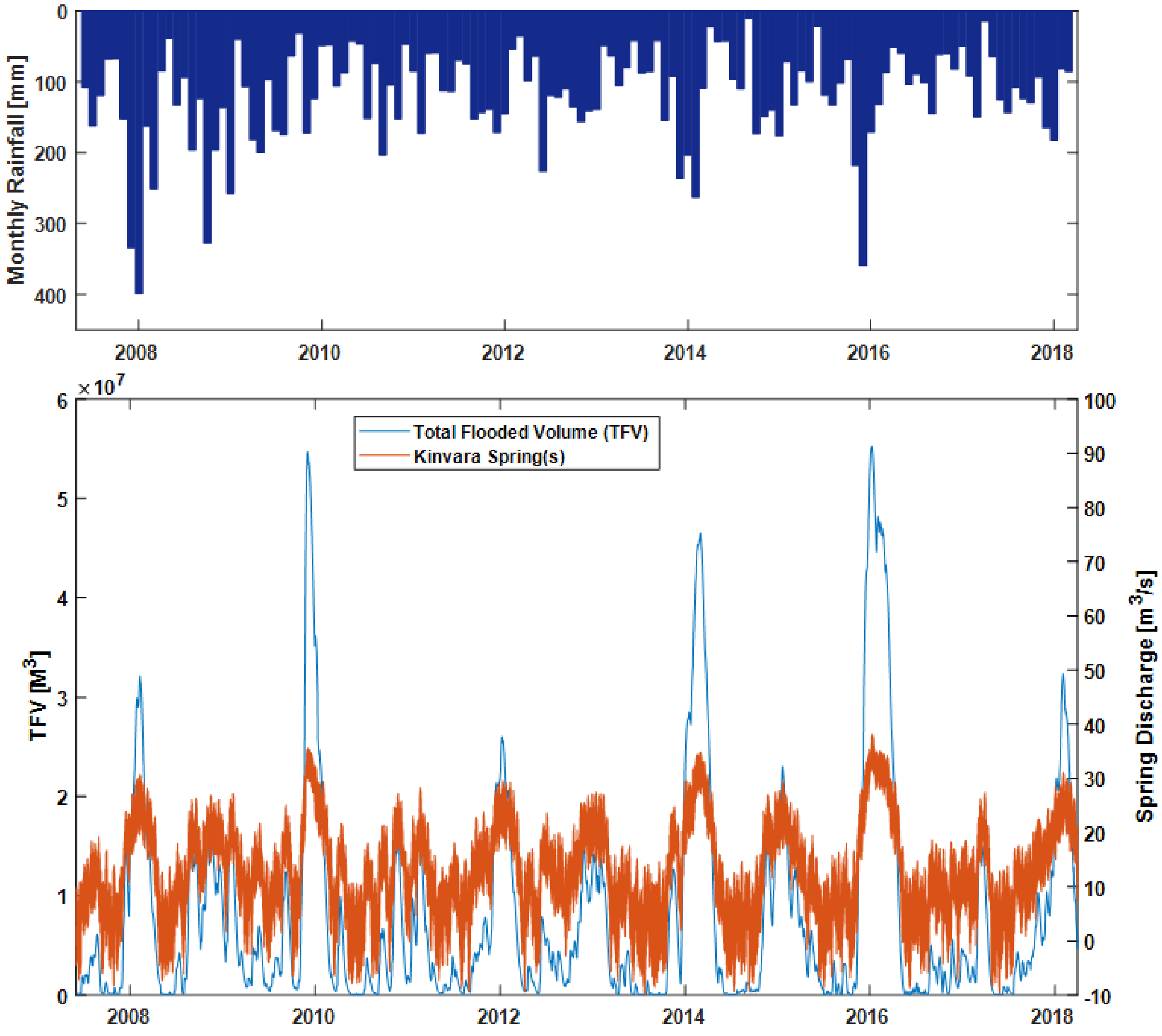
- Karst is generated by the dissolution of bedrock
- Create sinkholes, caves, springs, sinking streams
- Karst is associated with soluble rocks: limestone, marble, and gypsum
- Intense/prolonged rainfall results in recurrent groundwater flooding



Rainfall-Flooded Volume in KARST

- Nonlinearity
- Non-stationarity
- Stochasticity





$$y(t) = F\big[y(t-1), y(t-2)\cdots, y(t-n_y), u_1(t-d_1), u_1(t-d_1-1), \cdots, u_1(t-d_1-n_{u_1}+1), u_2(t-d_2), u_2(t-d_2-1), \cdots, u_2(t-d_2-n_{u_2}+1), \cdots, u_K(t-d_K), u_K(t-d_K-1), \cdots, u_K(t-d_K-n_{u_K}+1)\big] + e(t)$$

$$y(t) = \theta_0 + \sum_{i_1=1}^{n} \theta_{i_1} x_{i_1}(t) + \sum_{i_1=1}^{n} \sum_{i_2=i_1}^{n} \theta_{i_1 i_2} x_{i_1}(t) x_{i_2}(t) + \cdots + \sum_{i_1=1}^{n} \cdots \sum_{i_\ell=i_{\ell-1}}^{n} \theta_{i_1 i_2 \cdots i_\ell} x_{i_1}(t) x_{i_2}(t) \cdots x_{i_\ell}(t) + e(t)$$

$$x_m(t) = \begin{cases} y(t-m) & 1 \le m \le n_y \\ u_1\big(t-(m-n_y+d_1-1)\big) & n_y+1 \le m \le n_y+n_{u_1} \\ u_2\big(t-(m-n_y-n_{u_1}+d_2-1)\big) & n_y+n_{u_1}+1 \le m \le n_y+n_{u_1}+n_{u_2} \\ \vdots & \\ u_K\big(t-(m-n_y-n_{u_1}-\cdots-n_{u_{K-1}})\big) & n_y+n_{u_1}+\cdots+n_{u_{K-1}}+1 \le m \le n \end{cases}$$

NARX

- F[] as nonlinear polynomial
- Fixing Error-to-Signal ratio threshold
- Reduce the number of terms in the equation

$$\boxed{y(t) = \sum_{i=1}^{M} \theta_i p_i(t) + e(t); \quad t = 1, \cdots, N}$$

SIMULATION

- Rainfall-Runoff Model

$$Q = \frac{(P-0.2S)^2}{P+0.8S}$$

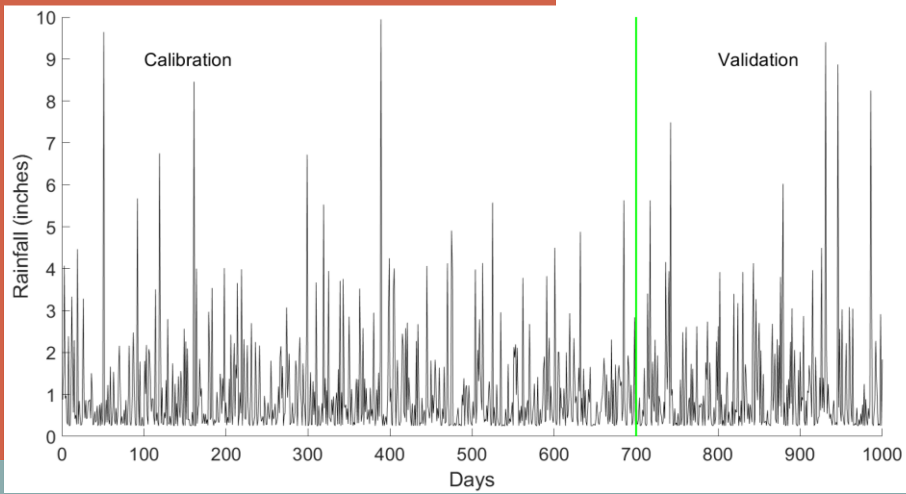
Potential Maximum Retention

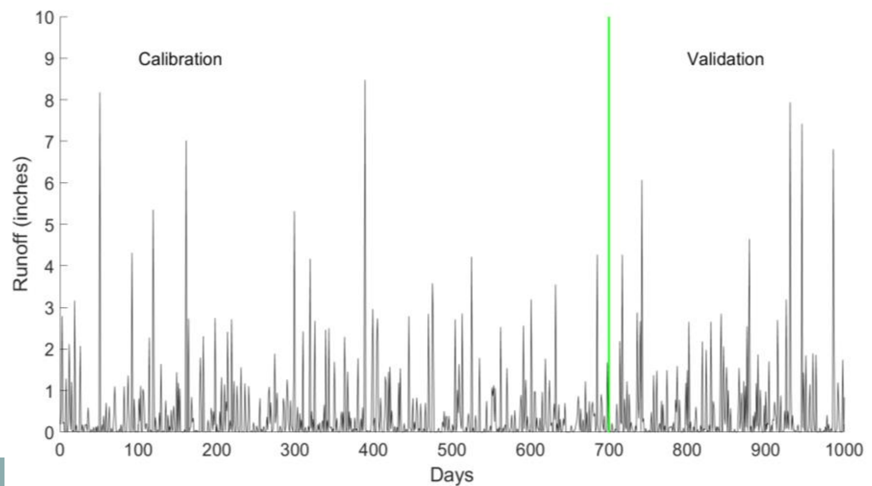
$$S = \frac{1000}{CN} - 10$$

- Rainfall simulation

$$f(x) = \frac{(x-\xi)^{\alpha-1} e^{-(x-\xi)/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

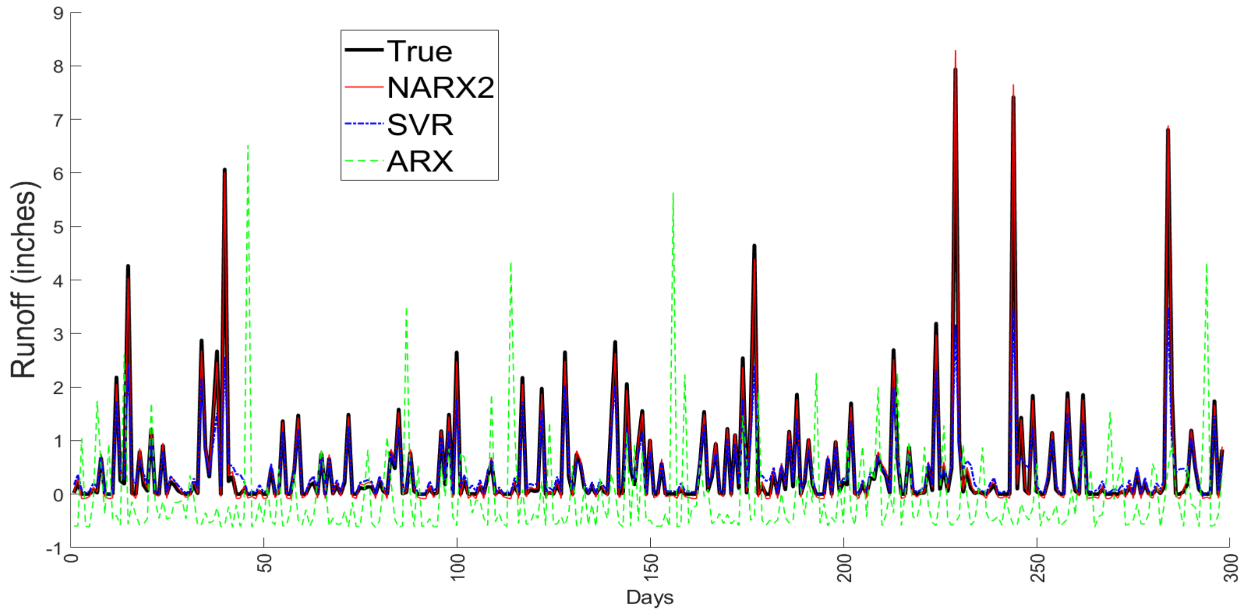
where $\alpha = 1/\gamma^2$, $\beta = \frac{1}{2}\sigma|\gamma|$ and $\xi = \mu - 2\sigma/\gamma$





PERFORMANCE

- Comparison of NARX, SVR and ARX models



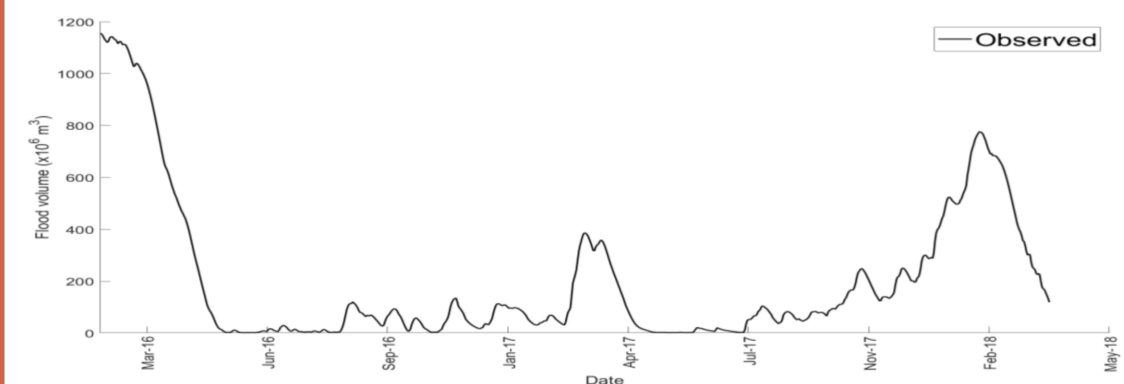
$$Q = \frac{(P-0.2S)^2}{P+0.8S}$$

$$y(t) = 0.6149 \times u_1(t) + 0.3648 + 0.0400 \times u_1(t)^2 - 0.0054 \times y(t-1) \times u_1(t) - 0.0004 \times y(t-1) + e(t)$$

Performance measure	NARX	SVR	ARX
NSE	0.994	0.742	-1.050
r	0.997	0.952	-0.038
KGE	0.992	0.532	-0.509
Bias	-0.38	-5.79	-52.11
RMSE	8.05	53.43	150.57

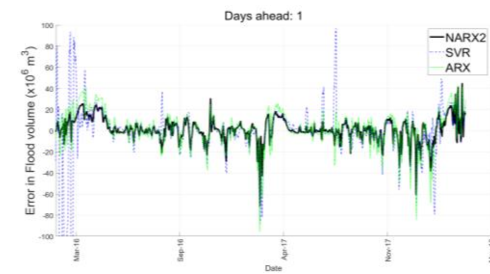
Real-World Study

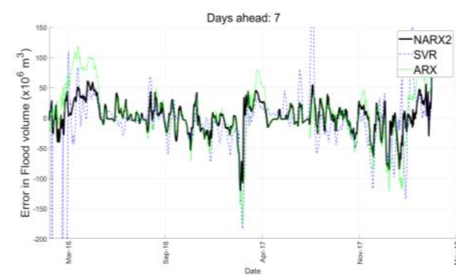
$$
\begin{aligned}
y(t) = {} & 1.7127 \times y(t-1) - 0.8462 \times y(t-2) + 0.3328 \times u_1(t-1) \\
& + 0.0063 \times u_1(t-2)^2 + 0.3135 \times u_1(t-2) \\
& + 0.0024 \times y(t-1) \times u_1(t-1) - 0.0021 \times y(t-4) \times u_1(t-1) \\
& + 0.0243 \times y(t-5) + 0.2841 \times y(t-3) - 0.1800 \times y(t-4) \\
& + 0.0081 \times u_1(t-1) \times u_1(t-2) + 0.4860 + e(t)
\end{aligned}
$$

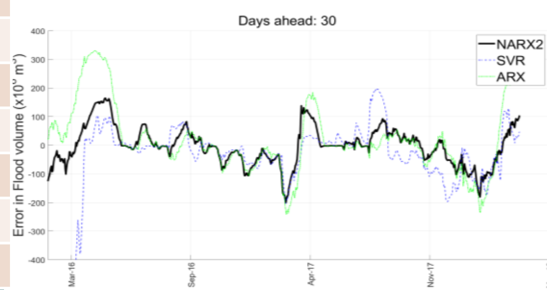


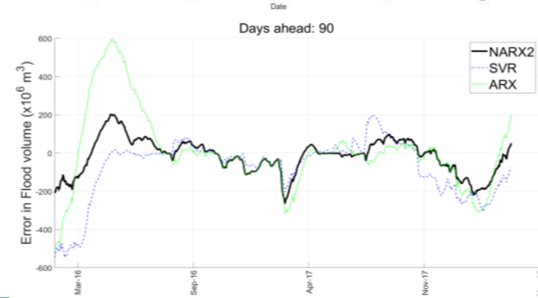
Error	Days	1	7	30	90
NSE	NARX2	1.00	0.99	0.95	0.90
	SVR	0.98	0.90	0.75	0.65
	ARX	1.00	0.98	0.83	0.50
r	NARX2	1.00	1.00	0.97	0.96
	SVR	0.99	0.96	0.91	0.93
	ARX	1.00	0.99	0.93	0.73
KGE	NARX2	0.99	0.99	0.93	0.81
	SVR	0.96	0.84	0.65	0.50
	ARX	0.99	0.97	0.88	0.72

12 terms from possible 136 terms



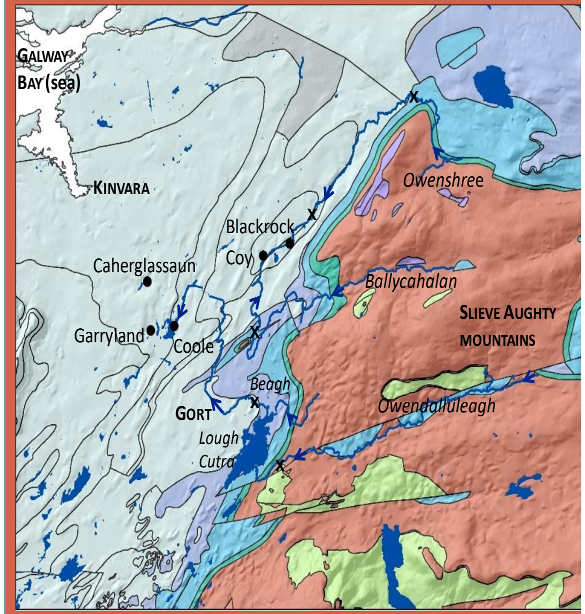


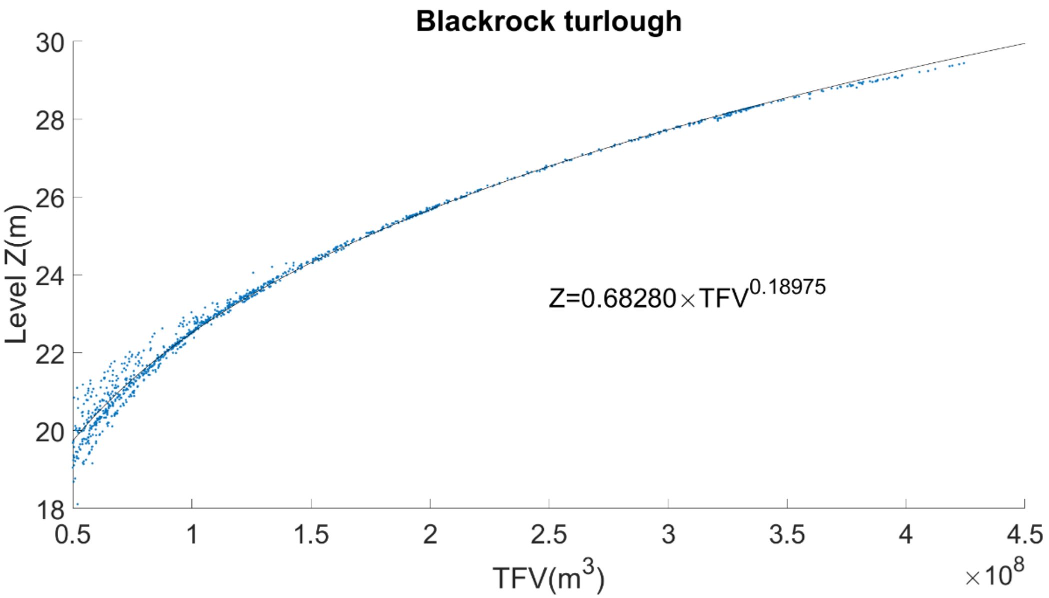


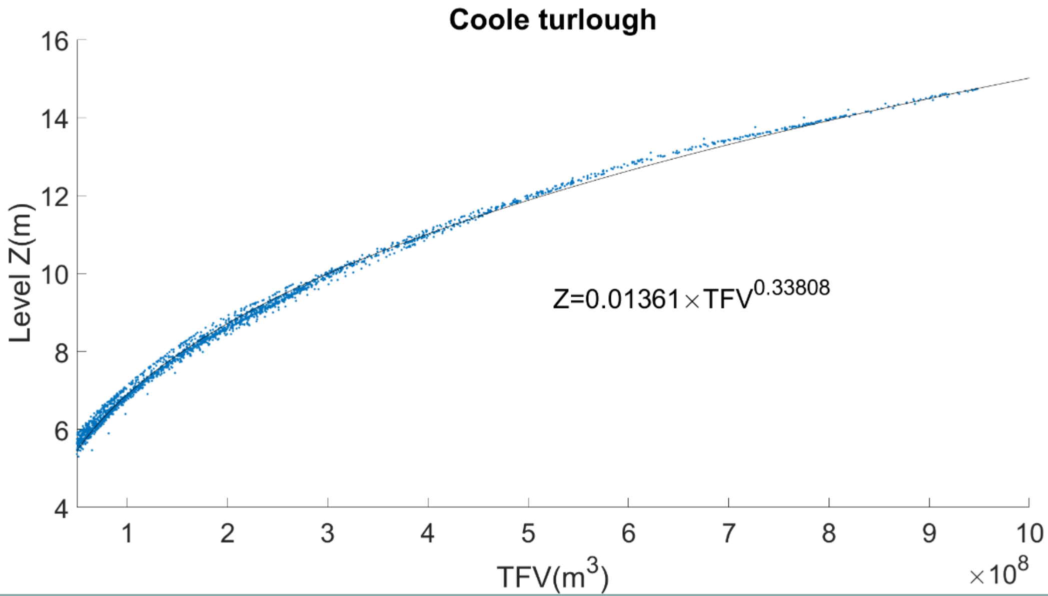


Applications

- Waterlevel estimation

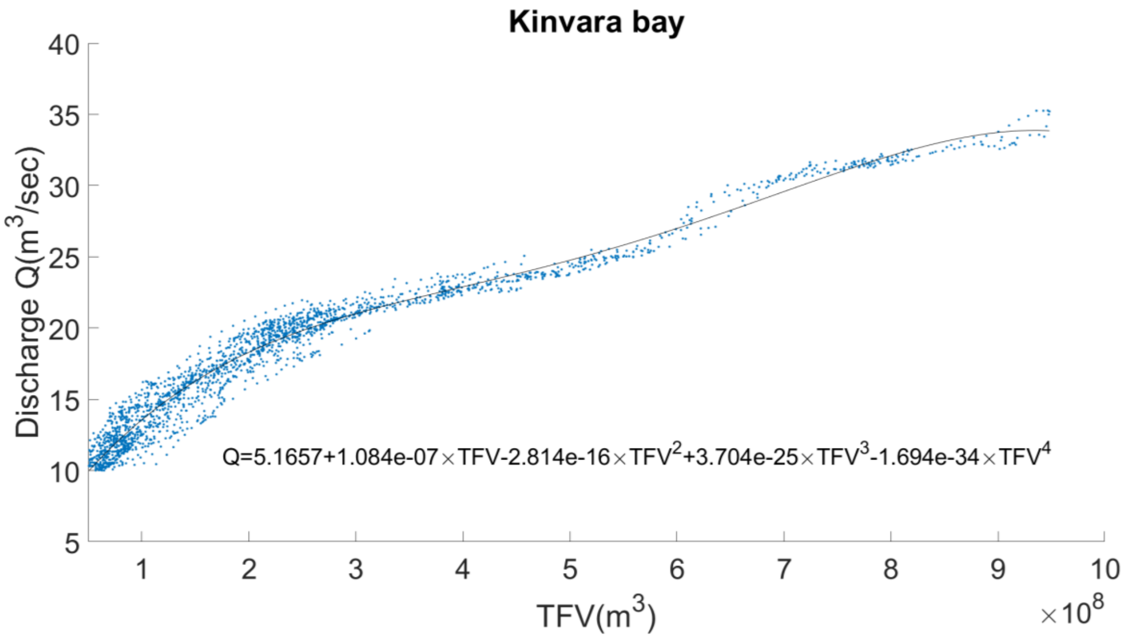






More Applications

- ❑ Discharge estimation



KEY POINTS

- ❑ Early Warning System
- ❑ Nonlinearity quantification
- ❑ Complexity in system dynamics

THANK YOU

Ref:

Bidroha Basu, Patrick Morrissey and Gill, L. W. (2022). Application of nonlinear time series and machine learning algorithms for forecasting groundwater flooding in a lowland karst area. Water Resources Research, 58(2), e2021WR029576.