Exploiting the combined GRACE/GRACE-FO solutions to determine gravimetric excitation of polar motion

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EGU General Assembly 2022, 23–27 May 2022, Vienna

Introduction

- In this work, we use data from Gravity Recovery and Climate Experiment (GRACE) and GRACE Follow-On (GRACE-FO) to determine **gravimetric excitation of polar motion** (PM). Such series reflect mainly the impact of continental hydrosphere and cryosphere on PM excitation and can be also denoted as hydrological plus cryospheric angular momentum **(HAM/CAM)**.
- **We use three-cornered hat method to estimate the noise level of GRACE/GRACE-FO solutions and to create the combined HAM/CAM series.**
- We also consider **mean of GRACE/GRACE-FO datasets** as well as **combined solution** provided by the International Combination Service for Time-variable Gravity Field (**COST-G**).
- Our estimates of HAM/CAM based on the combined data are compared to the HAM/CAM determined from the sum of hydrological and cryospheric signal in observed (geodetic) excitation called **geodetic residuals (GAO).**
- As data form **Satellite Laser Ranging (SLR)** are also recommended to determine HAM/CAM, especially in the period of lack of GRACE or GRACE-FO measurements, we include SLR solutions to our analyses.
- We analyse the level of agreement between combined HAM/CAM series and GAO for trends, overall time series, annual oscillations and non-seasonal series (after removing trends and annual, semiannual and terannual oscillations).

Data: GRACE/GRACE-FO and SLR solutions

- **GRACE/GRACE–FO single solutions** – C_{21}, S_{21} coefficients of geopotential (GSM) obtained from the following solutions:
  - CSR RL06 – provided by Center for Space Research, Austin, USA
  - JPL RL06 – provided by Jet Propulsion Laboratory, Pasadena, USA
  - GFZ RL06 – provided by GeoForschungsZentrum, Potsdam, Germany
  - ITSG-Grace2018 and ITSG-Grace_op – provided by Institute of Geodesy at Graz University of Technology, Graz, Austria
  - CNES/GRGS RL05 – provided by the Centre National d'Etudes Spatiales (CNES), Toulouse, France

- **GRACE/GRACE-FO combined solution** – C_{21}, S_{21} coefficients of geopotential (GSM) obtained from the following solution:
  - COST-G – provided by International Combination Service for Time–variable Gravity Field. The solution is determined from the combination of data from CSR, JPL, GFZ, ITSG, CNES, AIUB and LUH

- **SLR solution** – C_{21}, S_{21} coefficients of geopotential (GSM) obtained from the following solution:
  - CSR_Monthly_5x5_Gravity_Harmonics – provided by Center for Space Research, Austin, USA

Data: observations and models for GAO computation

- **χ_1 and χ_2 components of GAM** computed from x and y coordinates of the pole obtained from time series of Earth Orientation Parameters (EOP 14 C04)
- **χ_1 and χ_2 components of AAM** (mass + motion terms) based on ECMWF (European Center for Medium–Range Weather Forecasts) model
- **χ_1 and χ_2 components of OAM** (mass + motion terms) based on MPIOM (Max Planck Institute Ocean Model) model

GAO series are computed as a difference between geodetic (GAM) and sum of atmospheric (AAM) and oceanic (OAM) excitation:

$$\mathbf{GAO} = \mathbf{GAM} - \mathbf{AAM} - \mathbf{OAM}$$

Combined HAM/CAM series from GRACE/GRACE-FO

- χ_1, χ_2 components of HAM/CAM were computed from **C_{21}, S_{21} coefficients of geopotential** delivered by GRACE and GRACE-FO (*Gross, 2015*):

$$\chi_1 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} C_{21}, \quad \chi_2 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} S_{21}$$

- A one-year gap between HAM/CAM from GRACE and HAM/CAM from GRACE-FO was filled by forecasting using the **seasonal ARIMA** (Autoregressive integrated moving average) method.

- A **three cornered hat (TCH) method** (*Tavella and Premoli 1994*) was used to estimate noise level of HAM/CAM series by making their comparison with each other.
- The three-cornered hat method allows an estimation of the variance of the individual noise of each series, under some assumptions on the correlations between those noises. Different formulations of the TCH method are used, depending on the assumptions made on the correlations between the noises. In this presentation, we use a **generalized TCH method** which does not make the assumption of zero correlation between the series tested (*Koot et al. 2006, Quinn et al. 2019).*
- Here, the computation of the noise variance in HAM/CAM with the use of TCH method was based on the differences between individual series and the selected series treated as a reference (here CSR RL06), and then minimizing the global correlation among the noises of the individual time series.
- After finding the noise level of each HAM/CAM series, we computed the **combined HAM/CAM series** as a weighted mean where the weights calculated for each series were inversely proportional to the noise variance of the series.

Combined HAM/CAM series

- **COMB1** – combination of HAM/CAM from CSR, JPL, ITSG, CNES, COST-G (without GFZ) based on weights determined with TCH method:

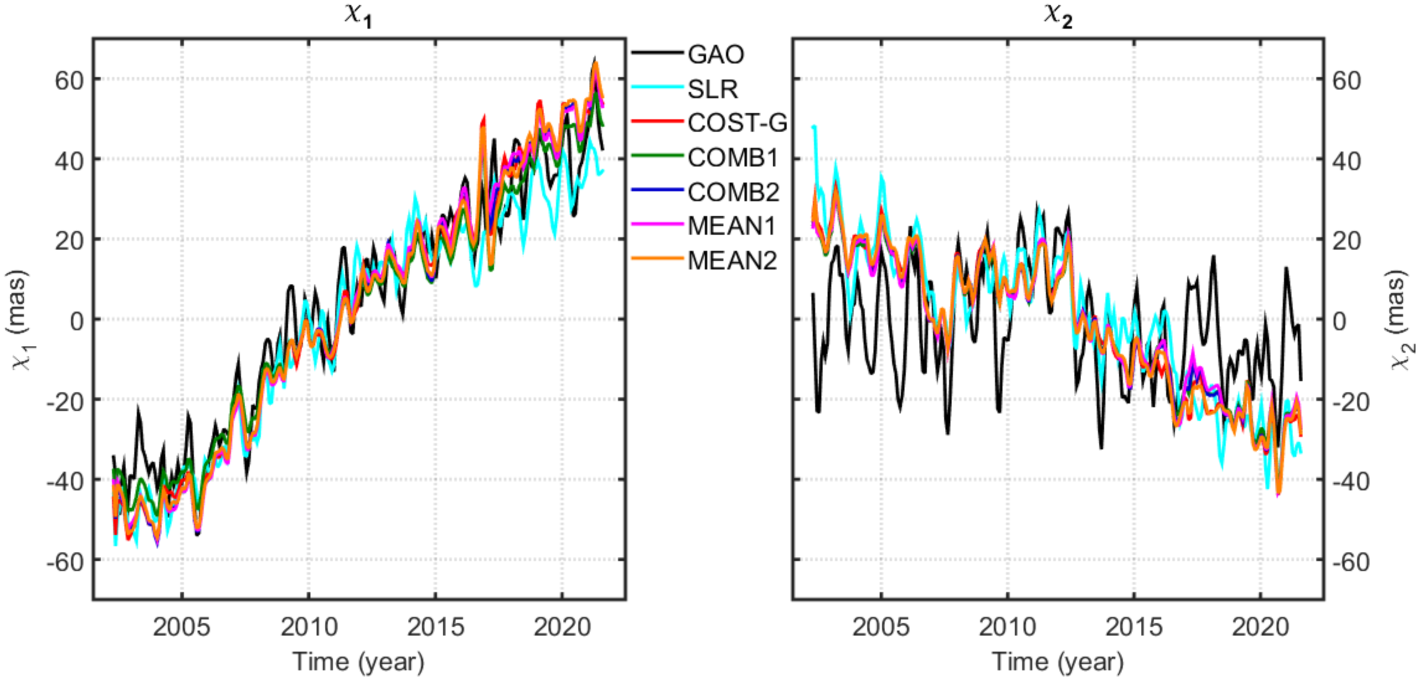
	CSR	JPL	ITSG	CNES	COST-G
Weight for χ_1	0.0064	0.2837	0.2211	0.1312	0.3577
Weight for χ_2	0.0175	0.1754	0.2760	0.1269	0.4042

- **COMB2** – combination of HAM/CAM from CSR, JPL, GFZ, ITSG, CNES, COST-G based on weights determined with TCH method:

	CSR	JPL	GFZ	ITSG	CNES	COST-G
Weight for χ_1	0.0062	0.2761	0.0269	0.2151	0.1277	0.3481
Weight for χ_2	0.0170	0.1701	0.0301	0.2677	0.1231	0.3920

- **MEAN1** – mean of HAM/CAM from CSR, JPL, ITSG, CNES, COST-G (without GFZ)
- **MEAN2** – mean of HAM/CAM from CSR, JPL, GFZ, ITSG, CNES, COST-G

Results: overall time series and trends



Trends (mas/year)

	χ_1	χ_2
GAO:	5.05±0.42	–0.65±0.68
COST-G:	5.98±0.28	–2.93±0.37
COMB1:	5.32±0.27	–2.70±0.35
COMB2:	6.01±0.29	–2.82±0.37
MEAN1:	5.98±0.29	–2.74±0.38
MEAN2:	5.94±0.30	–2.93±0.37
SLR:	5.05±0.41	–3.01±0.47

Correlation with GAO

	χ_1	χ_2
COST-G:	0.61	0.64
COMB1:	0.55	0.68
COMB2:	0.64	0.69
MEAN1:	0.69	0.72
MEAN2:	0.56	0.66
SLR:	0.59	0.50

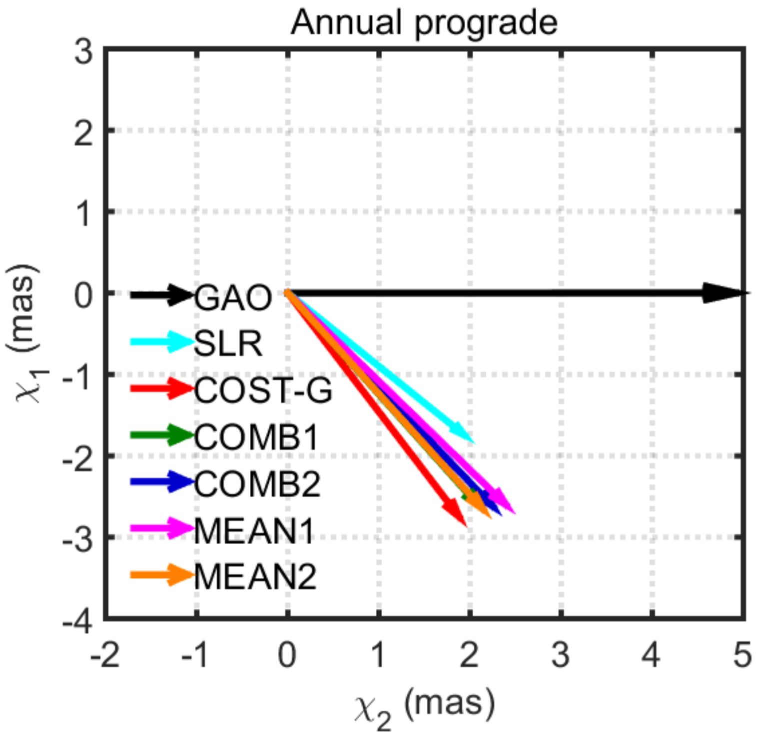
RMSE [mas]

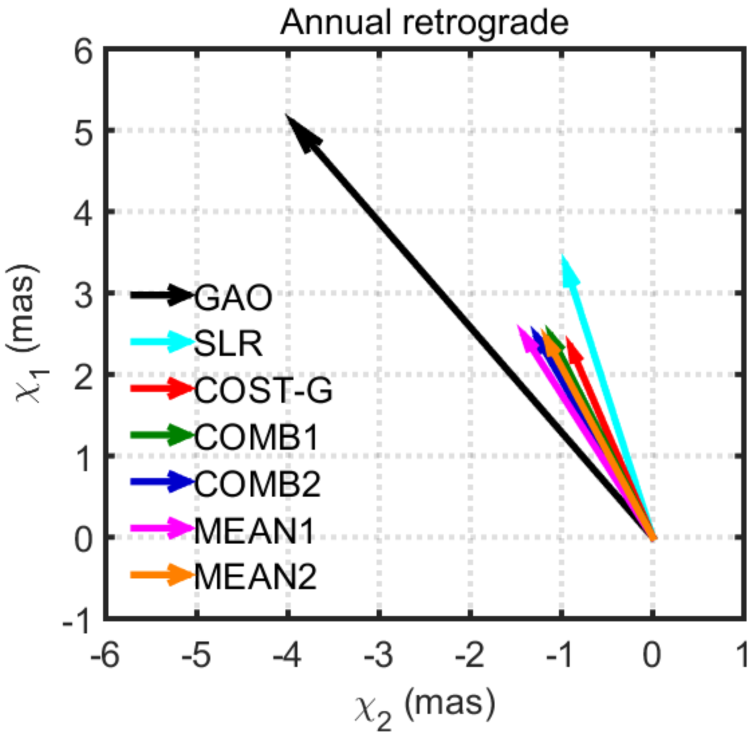
	χ_1	χ_2
COST-G:	6.42	10.22
COMB1:	6.79	9.88
COMB2:	6.20	9.72
MEAN1:	5.89	9.44
MEAN2:	6.85	9.99
SLR:	7.25	11.66

The critical value of the correlation coefficient for 80 independent points and a 95% confidence level is 0.17 and the standard error of the difference between the two correlation coefficients is 0.16

- In terms of overall series, MEAN1 provides the highest consistency with GAO
- In terms of trends, the use of COMB1 enables to obtain highest consistency with GAO
- COMB2 provides quite higher agreement with GAO than COST-G

Results: annual oscillations





- MEAN1 provides the highest consistency between HAM/CAM and GAO for amplitudes and phases of annual oscillations
- Each of COMB1, COMB2, MEAN1 and MEAN2 enables higher agreement with GAO than COST-G

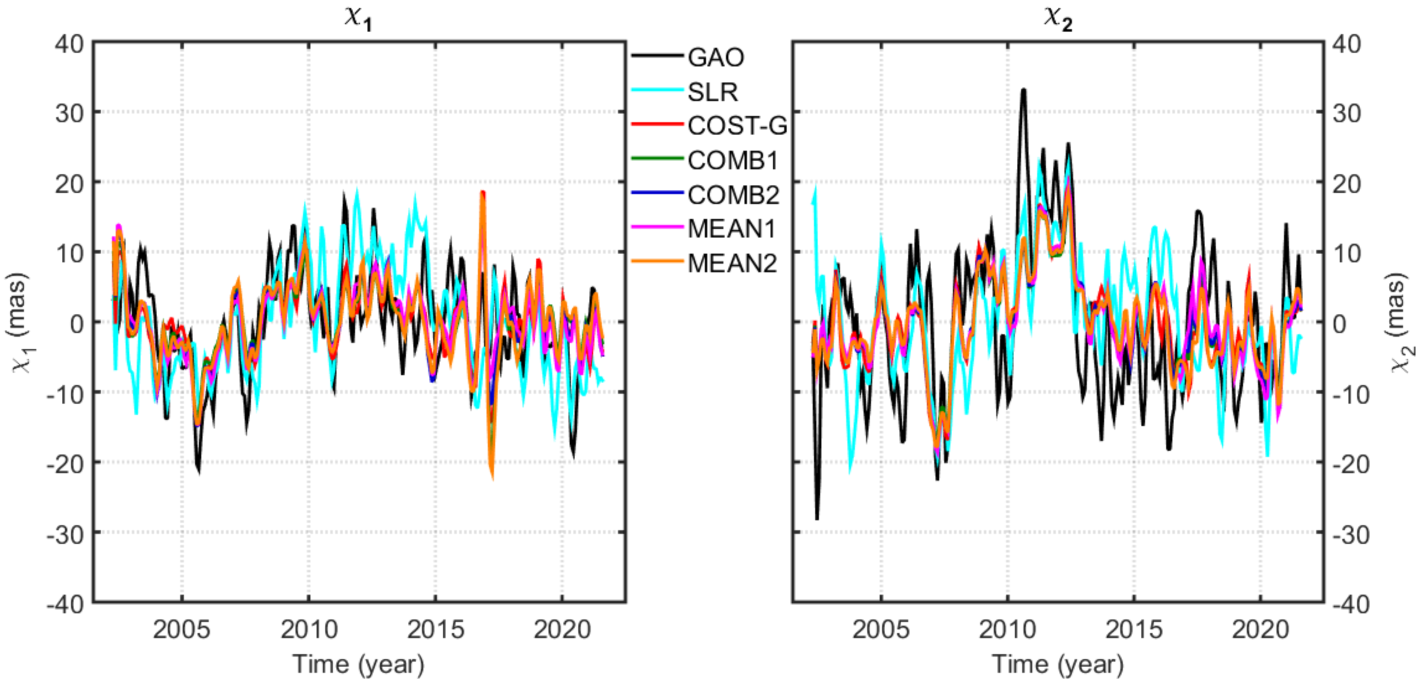
Difference vs GAO – prograde

	Amplitude (mas)	Phase (°)
COST-G:	1.59	56
COMB1:	1.57	51
COMB2:	1.46	49
MEAN1:	1.39	47
MEAN2:	1.51	51
SLR:	2.31	42

Difference vs GAO – retrograde

	Amplitude (mas)	Phase (°)
COST-G:	3.93	17
COMB1:	3.72	13
COMB2:	3.66	10
MEAN1:	3.57	8
MEAN2:	3.74	12
SLR:	2.96	22

Results: non-seasonal series



- MEAN1 provides the highest consistency between HAM/CAM and GAO for non-seasonal series
- MEAN1, COMB1 (only for χ_1) and COMB2 (for both χ_1 and χ_2) provides higher level of consistency with GAO than COST-G

Correlation with GAO

	χ_1	χ_2
COST-G:	0.58	0.64
COMB1:	0.53	0.66
COMB2:	0.61	0.67
MEAN1:	0.65	0.69
MEAN2:	0.54	0.64
SLR:	0.59	0.44

RMSE [mas]

	χ_1	χ_2
COST-G:	5.70	8.39
COMB1:	5.95	8.21
COMB2:	5.52	8.13
MEAN1:	5.24	7.89
MEAN2:	6.08	8.36
SLR:	6.60	10.52

The critical value of the correlation coefficient for 80 independent points and a 95% confidence level is 0.17 and the standard error of the difference between the two correlation coefficients is 0.16

Conclusions

- The use of TCH method enables to determine the errors of HAM/CAM series computed from GRACE/GRACE-FO based on the noise level of individual solutions.
- Based on these errors, it is possible to determine the combined HAM/CAM solution that could be characterized by minimal noise level.
- We noticed that the choice of weights calculated for each HAM/CAM series with the use of TCH method depend on the chosen reference series. The lowest weight is always for reference HAM/CAM (calculated from CSR RL06 in our case), while the highest weight is computed for the series characterized by the highest consistency with reference HAM/CAM.
- The presented results showed that such a combination, which is computed on the level of HAM/CAM series, provides quite higher consistency with GAO than COST-G solution, which is a combination performed on the solution level (combination of coefficients).
- Nevertheless, in most cases, HAM/CAM computed from the mean of solutions from CSR, JPL, ITSG and CNES is characterized by highest consistency with GAO.

Thank you

Acknowledgement. *The work of J. Śliwińska was supported by the National Science Center, Poland (NCN), grant number 2018/31/N/ST10/00209.*

Backup slides

Computation of HAM/CAM from C_{21}, S_{21} coefficients

χ_1, χ_2 components of HAM/CAM are proportional to the changes of C_{21}, S_{21} coefficients of geopotential (*Gross, 2015):*

$$\chi_1 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} C_{21}$$

$$\chi_2 = -\sqrt{\frac{5}{3}} \cdot \frac{1.608 \cdot R_e^2 \cdot M}{C - A'} S_{21}$$

where R_e and M are the Earth's mean Earth's radius and mass, respectively; A, B, and C are the principal moments of inertia for Earth (A = 8.0101×10^{37} kg·m^2, B = 8.0103×10^{37} kg·m^2, C = 8.0365×10^{37} kg·m^2); A' = (A + B)/2 is an average of the equatorial principal moments of inertia; and C_{21} and S_{21} are the normalized spherical harmonics coefficients of the gravity field (Table 1 in *Gross, 2015*). In the above equation, the load Love number and the effects of mantle anelasticity on the load Love number is taken into account.

Gross, R. (2015). Theory of earth rotation variations. In VIII Hotine–Marussi Symposium on Mathematical Geodesy, Sneeuw, N., Novák, P., Crespi, M., Sansò, F., Eds., Springer: Cham, Switzerland, 2015, p. 142

Filling a gap between GRACE and GRACE-FO series

- All geodetic and geophysical time series were interpolated with a monthly time span in order to maintain an equal time span for GRACE and GRACE Follow-On gravimetric time series.
- In addition, a one-year gap (6/2017–6/2018) between GRACE and GRACE Follow-on gravimetric excitation functions was filled by performance forecasting using the seasonal ARIMA method with the following assumptions:
  - ARIMA (3,0,2) Model Seasonally Integrated with Seasonal AR (48) and MA(12) (Gaussian Distribution),
  - seasonality of the model: 12 months.

Three cornered hat method

- To estimate noise level of time series using TCH method (*Tavella and Premoli 1994, Koot et al. 2006, Quinn et al. 2019*), we first calculate the differences between each series and the chosen one treated as a reference (X^N):

$$Y^{iN} = X^i - X^N = \varepsilon^i - \varepsilon^N, \quad i = 1, 2, \dots, N-1.$$

- The samples of the $N-1$ solution centres differences are concatenated in a $M \times (N-1)$ matrix as:

$$Y = [Y_{1N} \quad Y_{2N} \cdots \quad Y_{(N-1)N}].$$

- The covariance matrix S of the series of the differences is computed as: $S = cov(Y)$.
- Then we introduce the $N \times N$ Allan covariance matrix of the individual noises R, whose elements are the unknowns, and can be determined from:

$$S = [I - u]\begin{bmatrix} \hat{R} & r \\ r^T & r_{NN} \end{bmatrix}\begin{bmatrix} I \\ -u^T \end{bmatrix},$$ where I is the identity matrix and u is the $[1\ 1\ \cdots 1]^T$ vector.

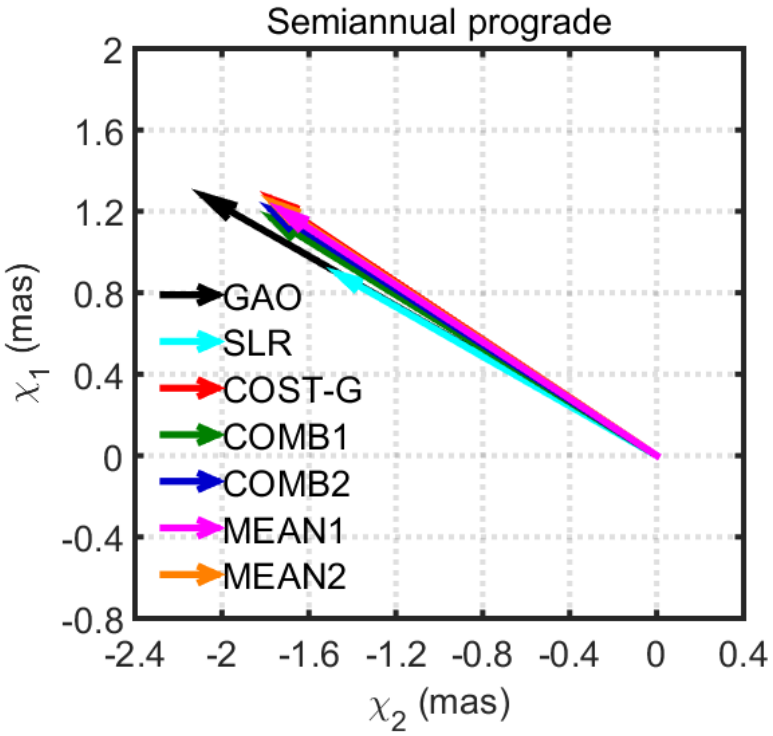
- Next we isolate the N free parameters of above equation by the minimization of the global correlation among the noises of the individual time series according to the Kuhn–Tucker theorem:

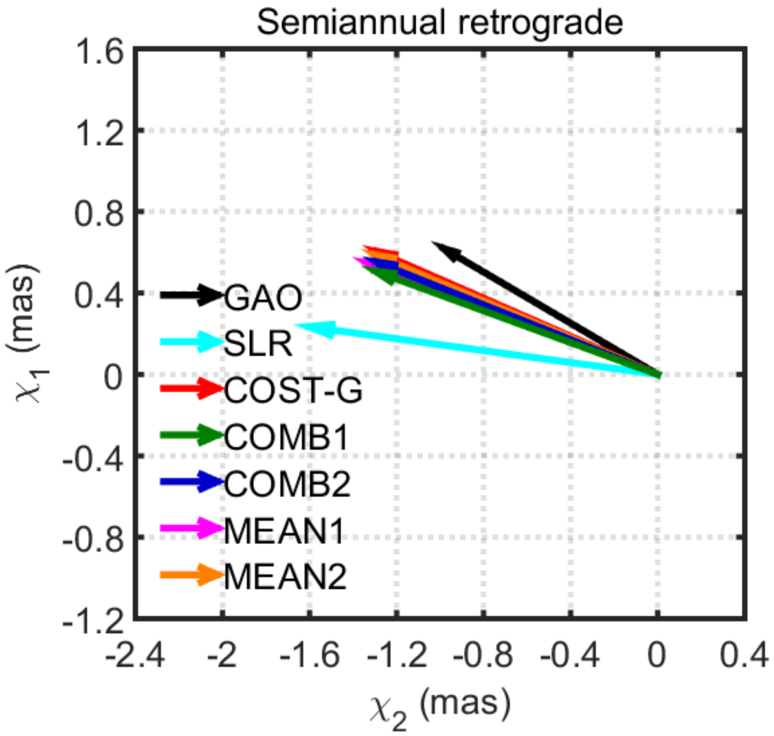
$$F(r, r_{NN}) = \sum \frac{r_{ij}^2}{(\det(S))^{\frac{2}{N-1}}}$$ with a constraint function: $$G(r, r_{NN}) = -\frac{r_{NN} - [r - r_{NN}u]^T \cdot S^{-1} \cdot [r - r_{NN}u]}{(\det(S))^{\frac{1}{N-1}}} < 0.$$

- After the noise level of each HAM/CAM time series were found, the combined HAM/CAM series were computed as:

$$\begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \sum_{i=1}^{n} w_i(t) \begin{bmatrix} \chi_1^i \\ \chi_2^i \end{bmatrix},$$ where w is weight which is inversely proportional to its noise variance: $$w_i = \frac{\frac{1}{var(\varepsilon_i)}}{\sum_{j=1}^{n} \frac{1}{var(\varepsilon_j)}}$$

Results: semiannual oscillations





Difference vs GAO – prograde

	Amplitude (mas)	Phase (°)
COST-G:	0.27	4
COMB1:	0.32	2
COMB2:	0.30	3
MEAN1:	0.34	3
MEAN2:	0.28	4
SLR:	0.73	0

Difference vs GAO – retrograde

	Amplitude (mas)	Phase (°)
COST-G:	0.27	7
COMB1:	0.23	11
COMB2:	0.24	9
MEAN1:	0.29	10
MEAN2:	0.26	8
SLR:	0.46	24

References

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