# A consistent full waveform inversion scheme for imaging heterogeneous isotropic elastic media

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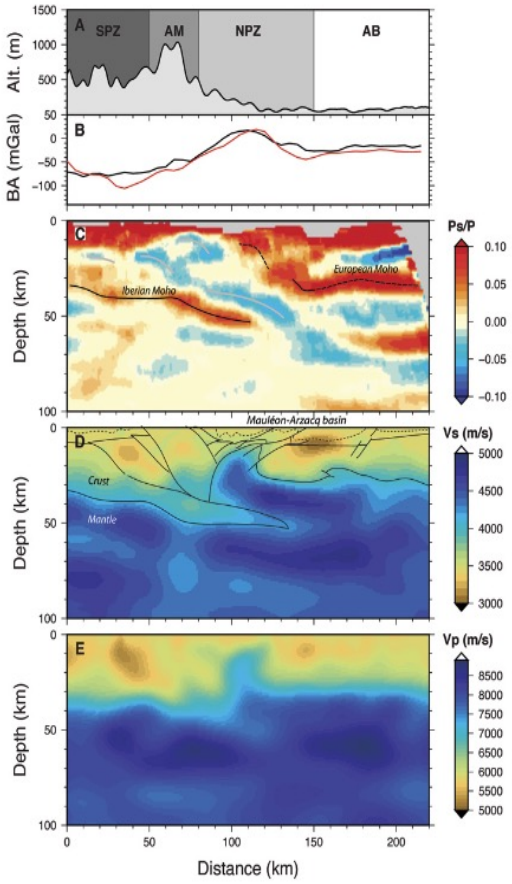
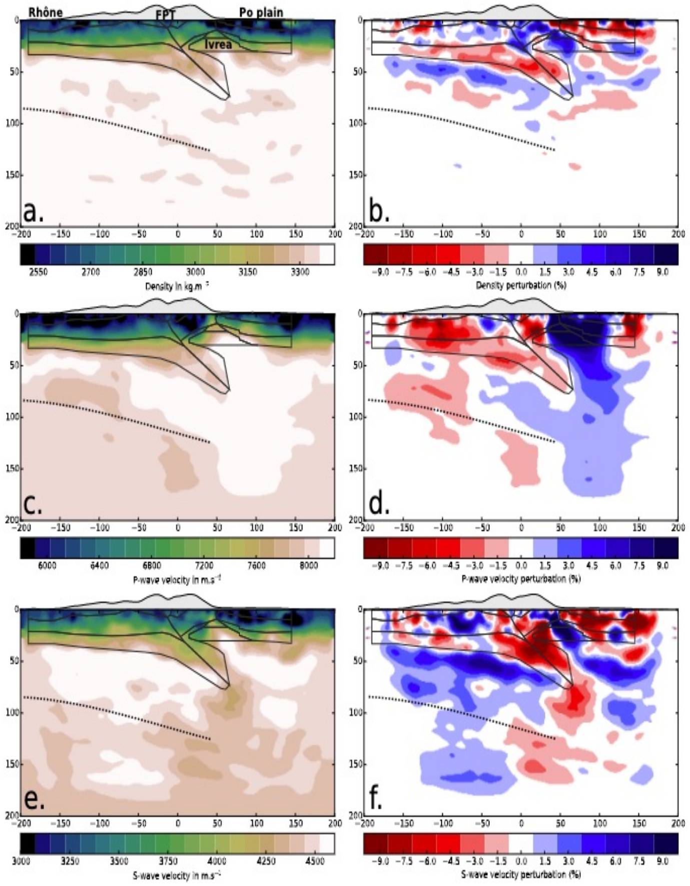
# Teleseismic full waveform inversion (FWI)

- Previous applications assume the model parameters are **independent**.

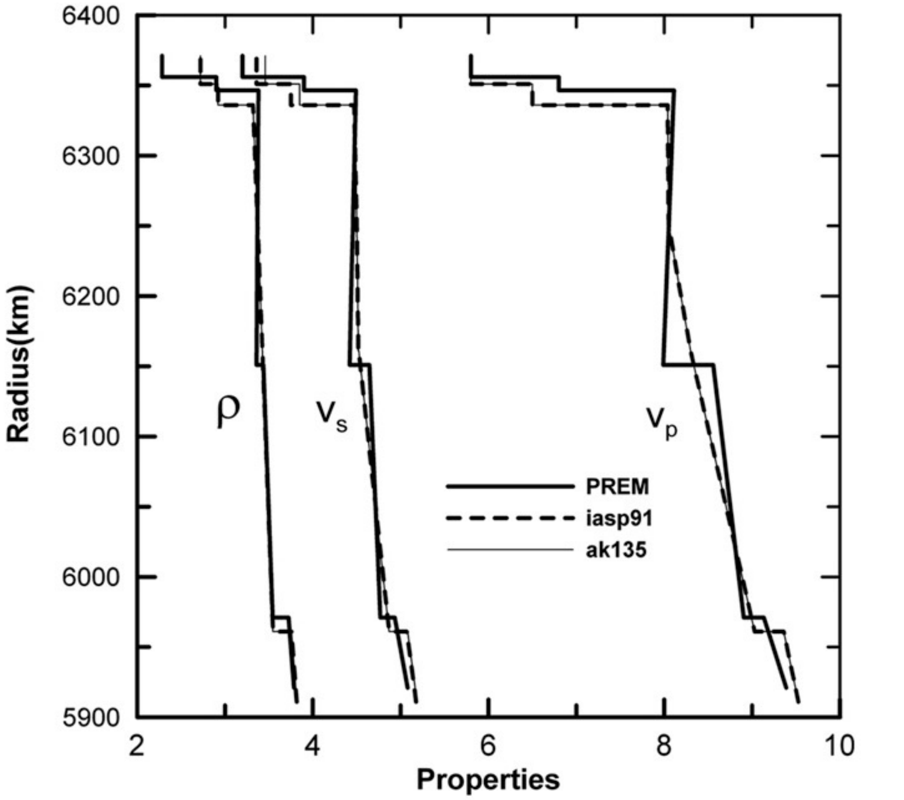



Wang et al. (2016)




Beller et al. (2017)

- In the Earth, the variation of these parameters are more or less **related**.



# Complete non-diagonal model covariance matrix

- Objective function

$$\chi(\mathbf{m}) = \frac{1}{2}(\mathbf{g}(\mathbf{m})-\mathbf{u}_{obs})^t\mathbf{C}_\mathbf{D}^{-1}(\mathbf{g}(\mathbf{m})-\mathbf{u}_{obs})+\frac{\lambda}{2}(\mathbf{m}-\mathbf{m}_{prior})^t\mathbf{C}_\mathbf{M}^{-1}(\mathbf{m}-\mathbf{m}_{prior})$$

- Before: diagonal covariance matrix

$$\mathbf{C_M} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_{V_p}^2 & 0 \\ 0 & 0 & \sigma_{V_s}^2 \end{bmatrix}$$

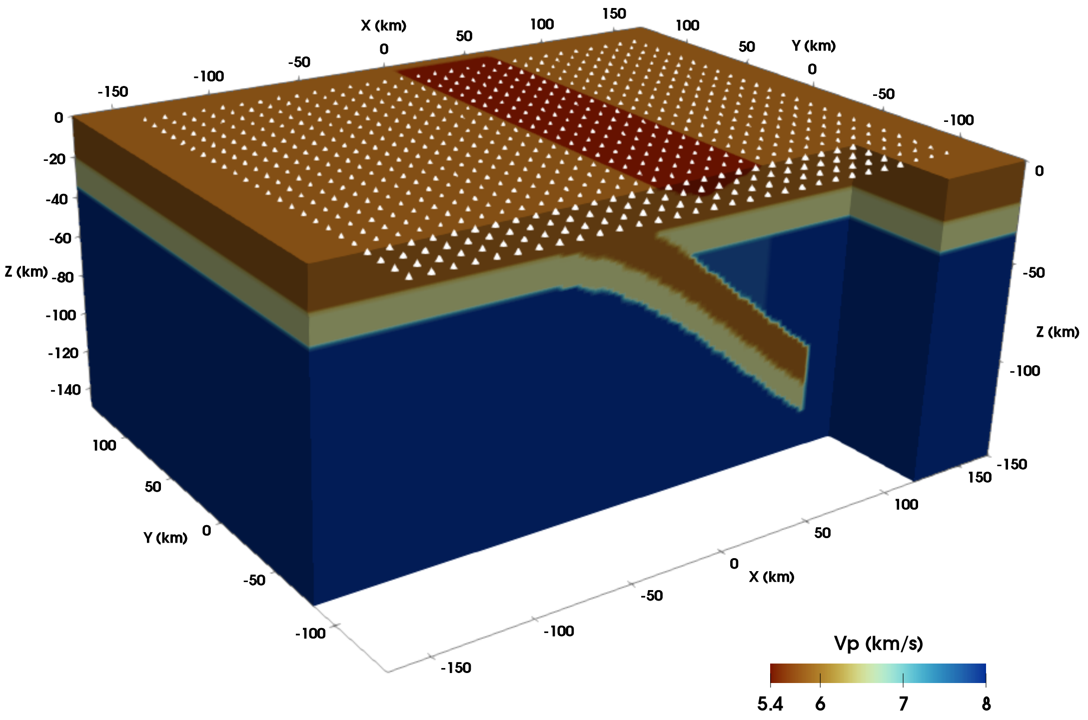
- After: non-diagonal covariance matrix (correlation included)

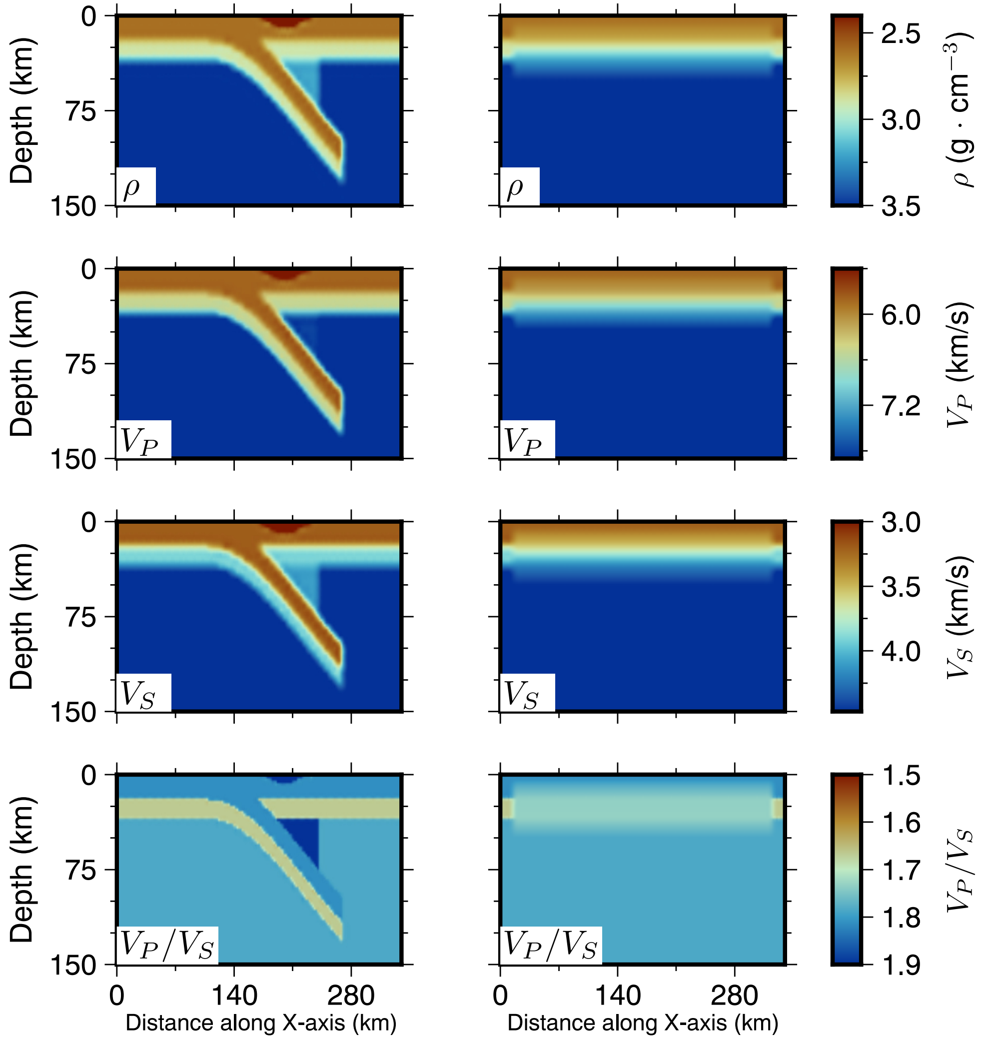
$$\mathbf{C_M} = \begin{bmatrix} \sigma_\rho^2 & r_{\rho,V_p}\cdot\sigma_\rho\sigma_{V_p} & r_{\rho,V_s}\cdot\sigma_\rho\sigma_{V_s} \\ r_{\rho,V_p}\cdot\sigma_\rho\sigma_{V_p} & \sigma_{V_p}^2 & r_{V_p,V_s}\cdot\sigma_{V_p}\sigma_{V_s} \\ r_{\rho,V_s}\cdot\sigma_\rho\sigma_{V_s} & r_{V_p,V_s}\cdot\sigma_{V_p}\sigma_{V_s} & \sigma_{V_s}^2 \end{bmatrix}$$

Two benefits

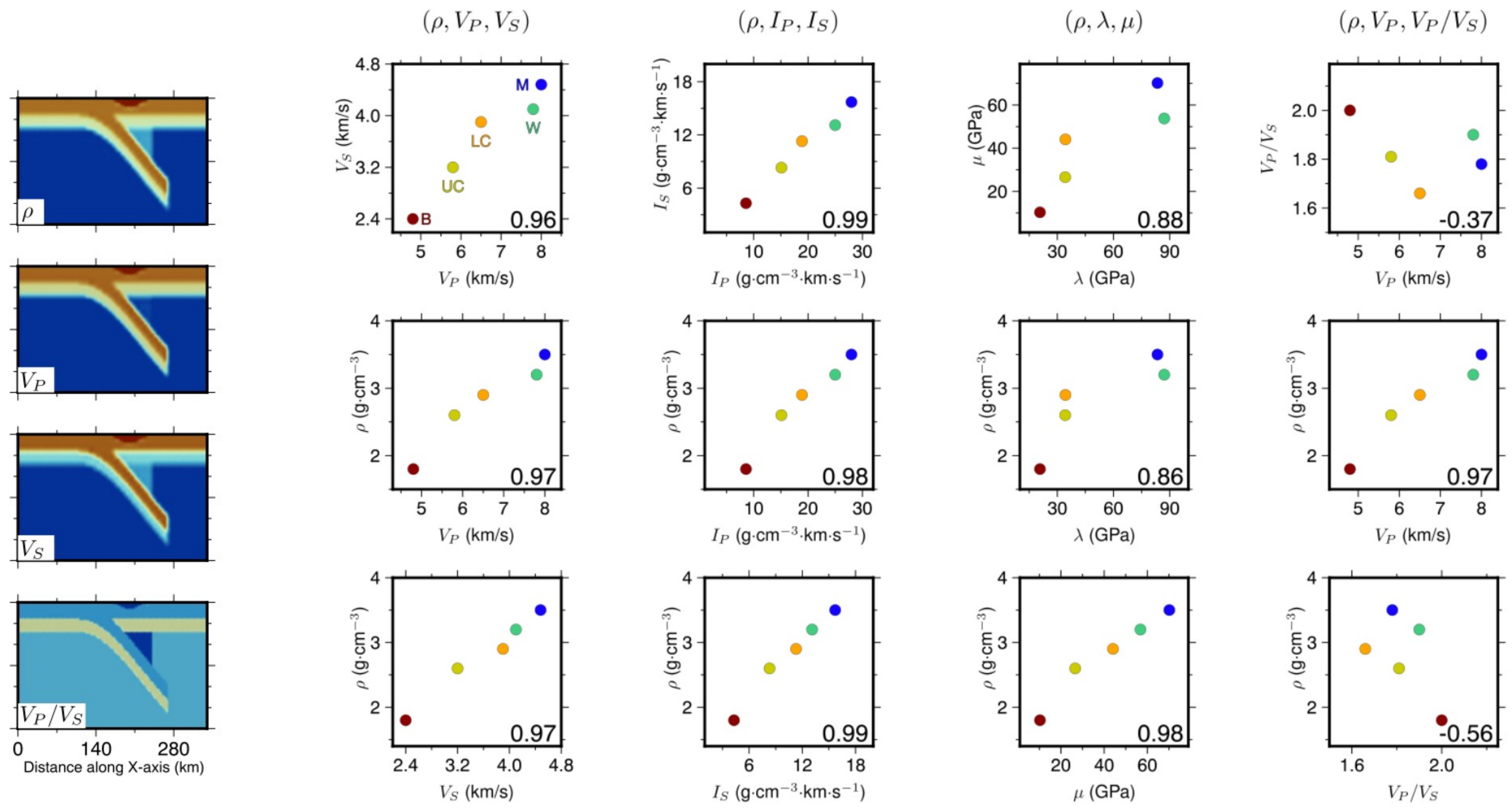
- Retrieve consistent results with different model parameterizations.
- Improve the similarity of resulting models between parameters.

# Synthetic experiment: a simple subduction model



# Synthetic experiment: different model parameterizations



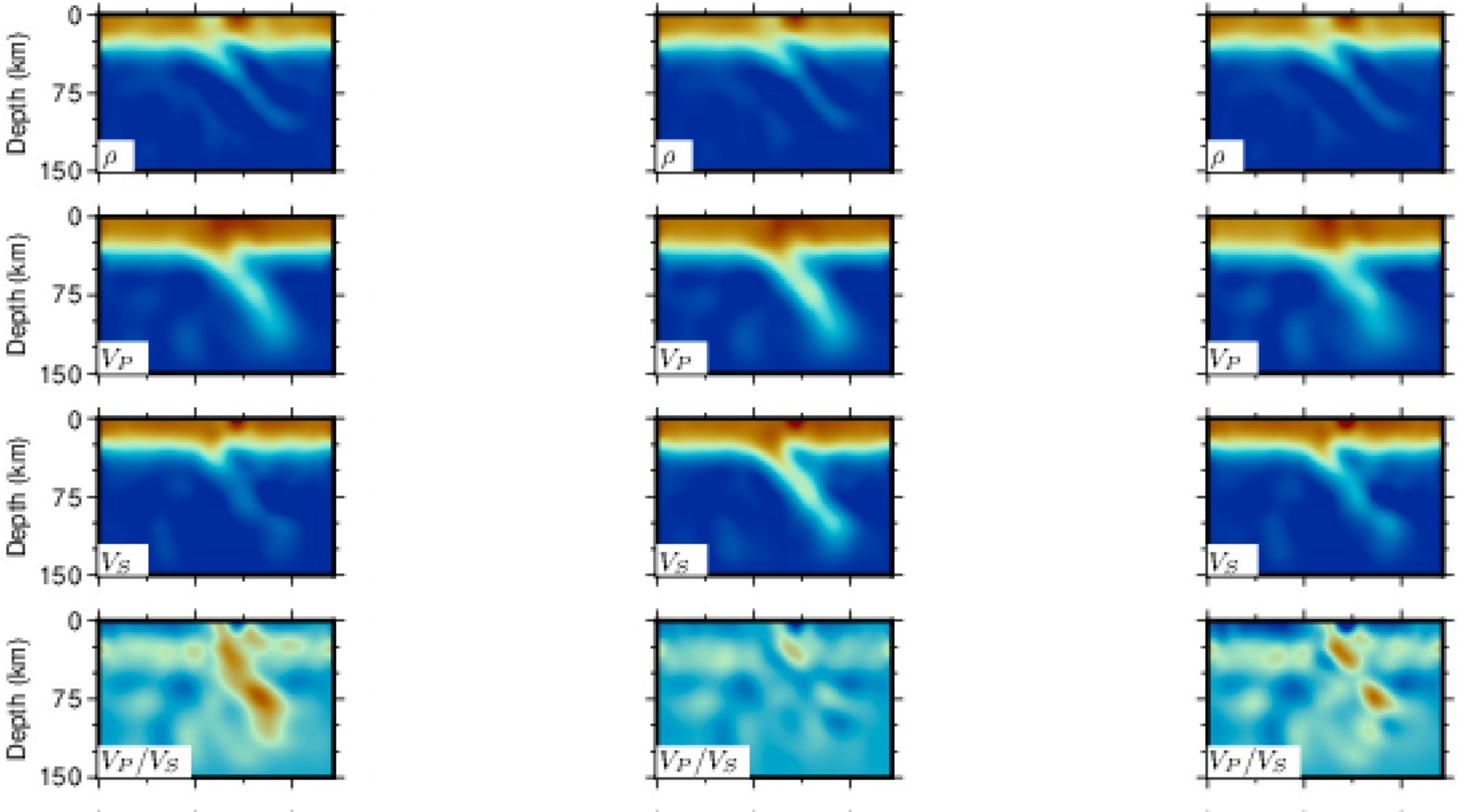
# Result: consistent result with different parameterization

- Retrieve consistent results with different model parameterizations.

$$\mathbf{C}_{(\rho,V_p,V_s)} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_{V_p}^2 & 0 \\ 0 & 0 & \sigma_{V_s}^2 \end{bmatrix}$$

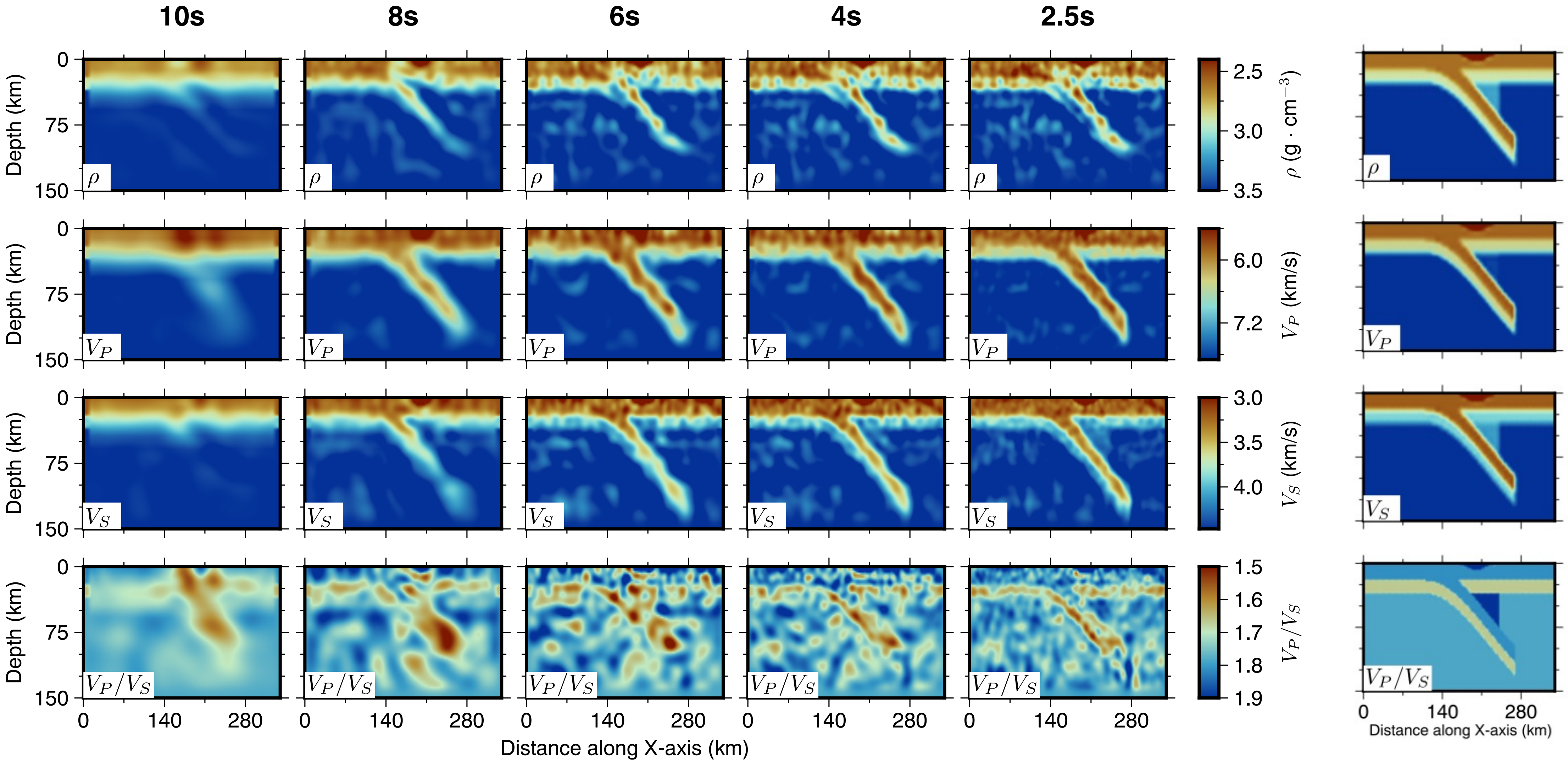
$$\mathbf{C}_{(\rho,V_p,\frac{V_p}{V_s})} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_{V_p}^2 & 0 \\ 0 & 0 & \sigma_{\frac{V_p}{V_s}}^2 \end{bmatrix}$$

$$\mathbf{C'}_{(\rho,V_p,\frac{V_p}{V_s})} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_{V_p}^2 & \frac{1}{V_s}\sigma_{V_p}^2 \\ 0 & \frac{1}{V_s}\sigma_{V_p}^2 & \frac{1}{V_s^2}\sigma_{V_p}^2 + \frac{V_p^2}{V_s^4}\sigma_{V_s}^2 \end{bmatrix}$$



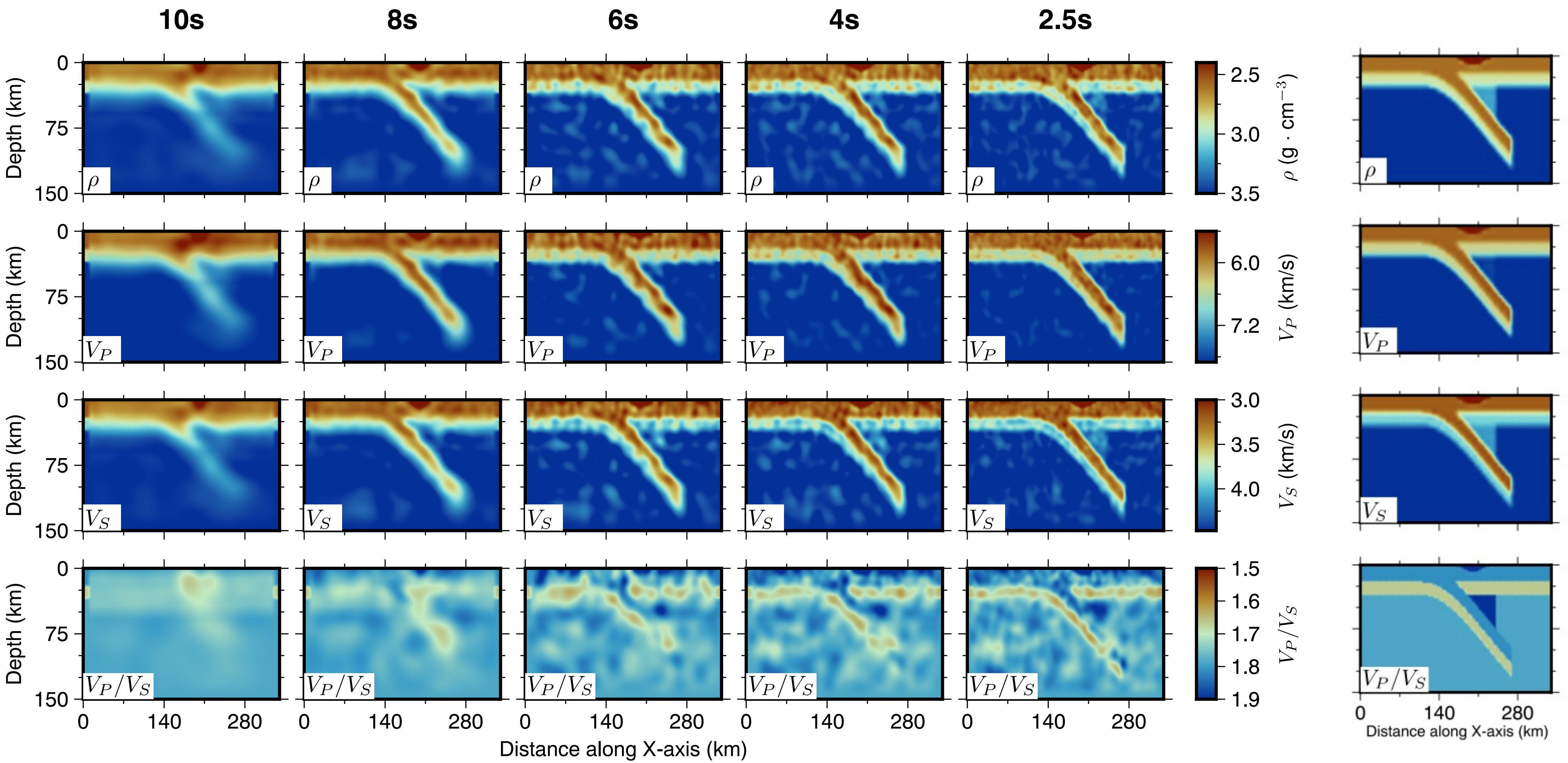
# Result: hierarchical inversion with diagonal **CM** (no correlation)

$$\mathbf{C}_{(\rho,V_p,V_s)} = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_{V_p}^2 & 0 \\ 0 & 0 & \sigma_{V_s}^2 \end{bmatrix}$$



# Result: hierarchical inversion with non-diagonal **C<sub></sub>M** (strong correlation)

$$\mathbf{C_M} = \begin{bmatrix} \sigma_\rho^2 & r_{\rho,V_p}\cdot\sigma_\rho\sigma_{V_p} & r_{\rho,V_s}\cdot\sigma_\rho\sigma_{V_s} \\ r_{\rho,V_p}\cdot\sigma_\rho\sigma_{V_p} & \sigma_{V_p}^2 & r_{V_p,V_s}\cdot\sigma_{V_p}\sigma_{V_s} \\ r_{\rho,V_s}\cdot\sigma_\rho\sigma_{V_s} & r_{V_p,V_s}\cdot\sigma_{V_p}\sigma_{V_s} & \sigma_{V_s}^2 \end{bmatrix}$$



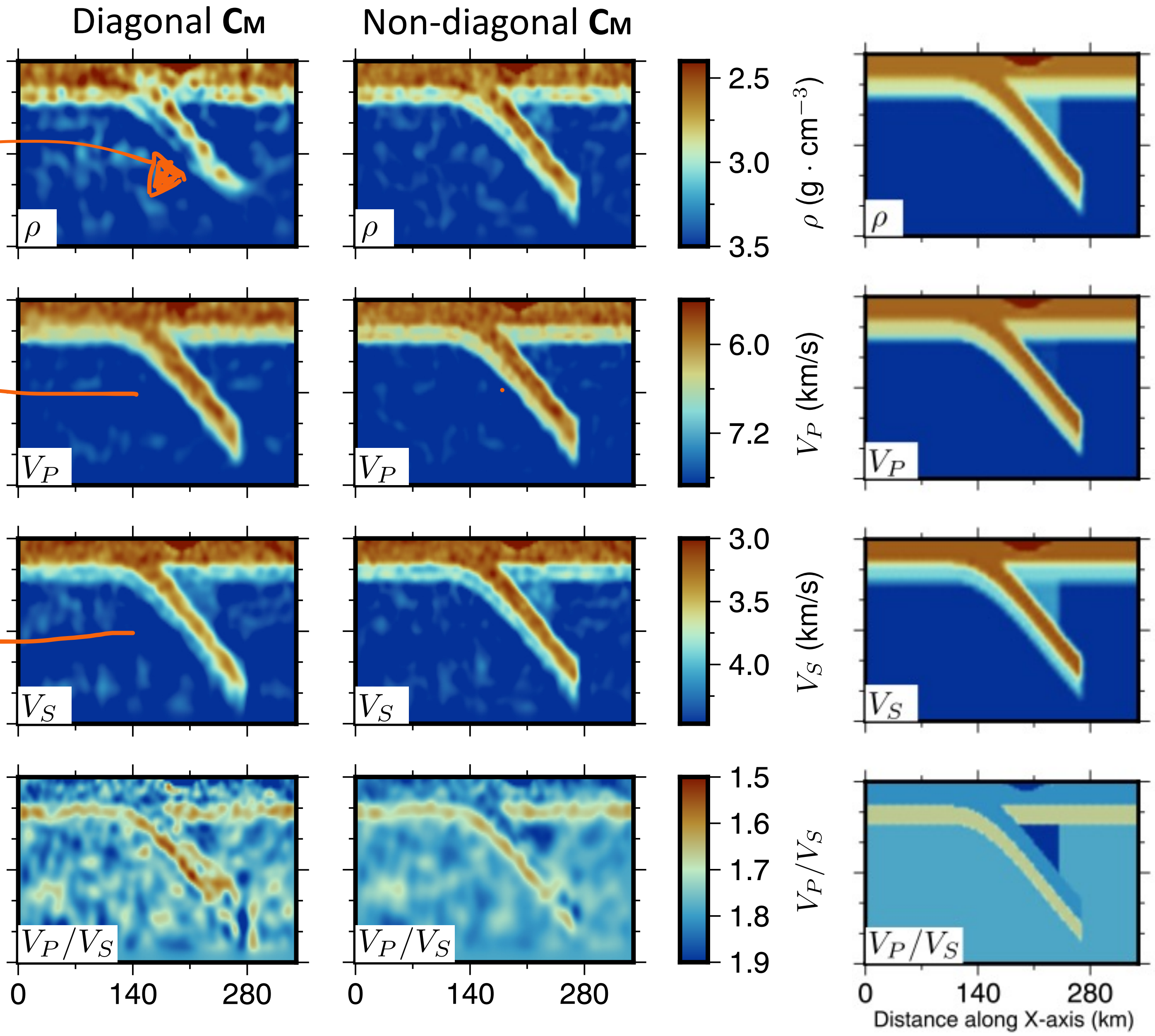
# Result: improvements

- Improve the similarity of resulting models between parameters.

Density: depth resolution

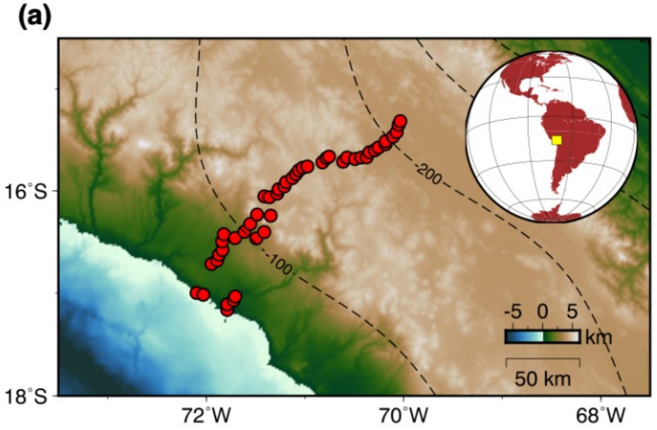
Vp, Vs: sharpness of layers

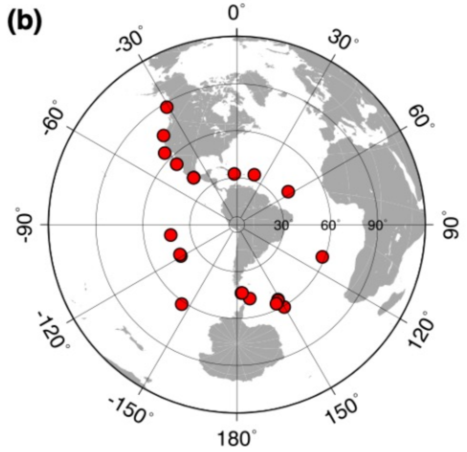
Vp/Vs: artefacts



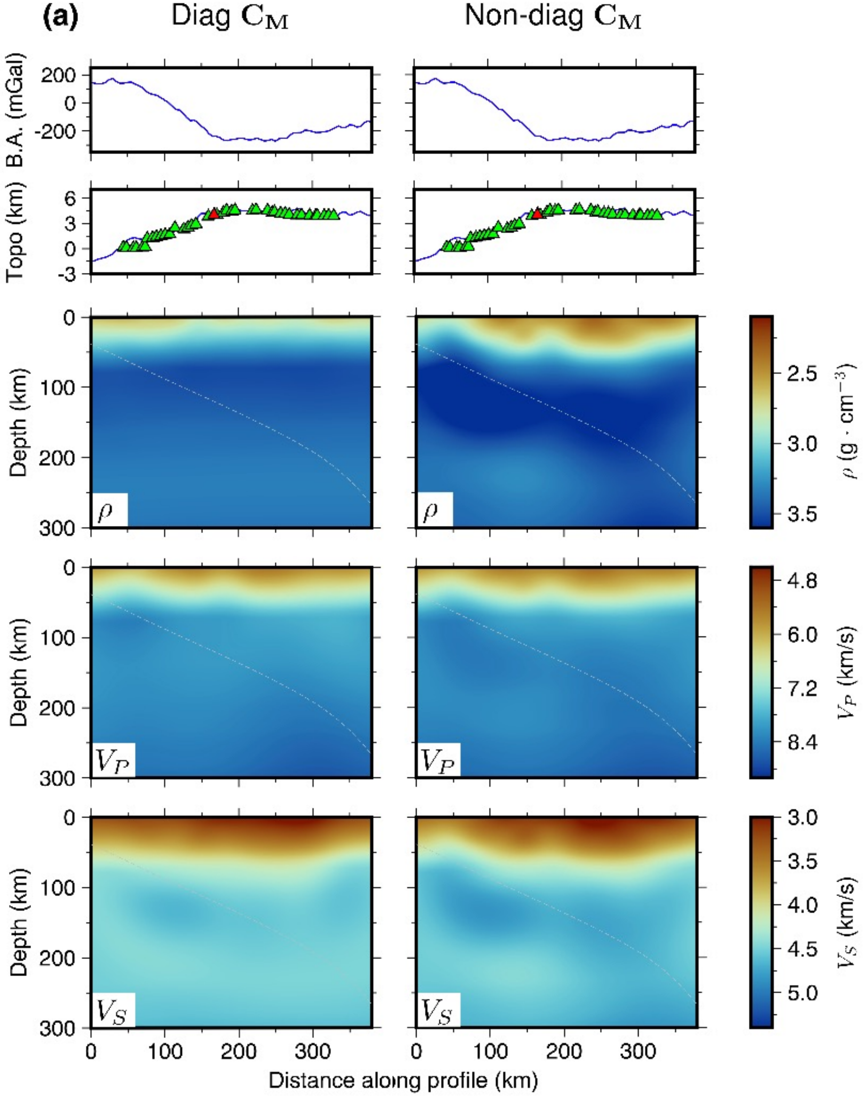
# Real data application

## A profile at southern Peru

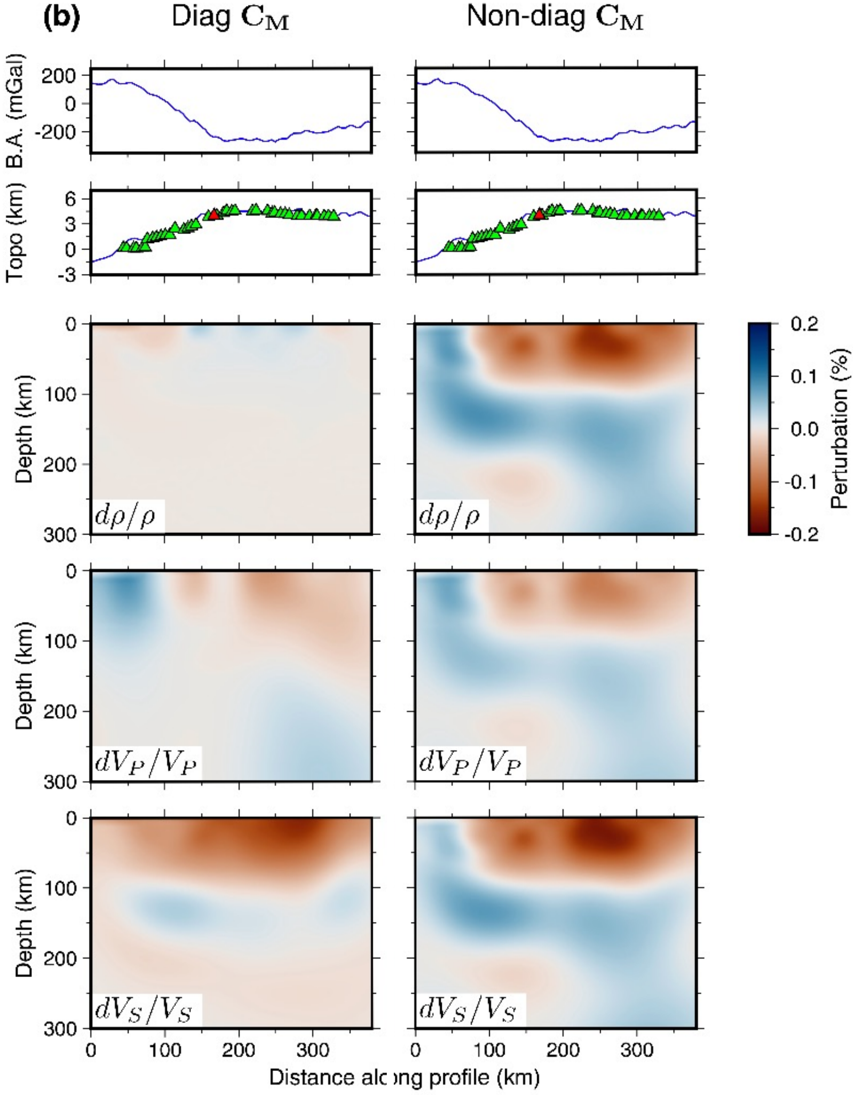



### Absolute value

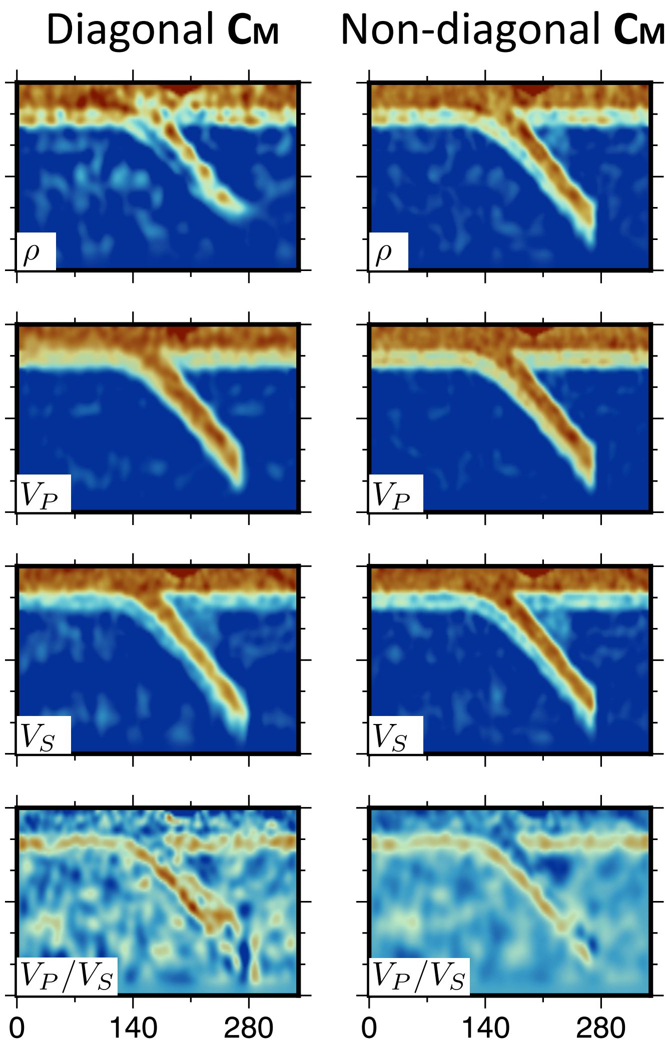



### Model perturbation



# Summary

- We introduce a priori information to FWI algorithm through full 3-D model covariance matrix.
- By assuming correlations between model parameters during inversion,
  - The reconstruction of the models are improved, in particular the density and Vp/Vs ratio.
  - The artifacts are suppressed.
- The choice of parameterization is not critical when model covariance matrix **CM** is accordingly transformed.




Thank you for your attention!