Intermittency, stochastic Universal Multifractals and the deterministic Scaling Gyroscope Cascade model

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1. Introduction

- **Intermittency**, which corresponds to the concentration of the activity of a field, e.g. flow vorticity, is one of the most fundamental features of turbulence and more generally of geophysics. It generates strongly non-Gaussian fluctuations.

- The log-normal model had been considered as a good approximation for intermittency in hydrodynamic turbulence. However, many works show that probability distribution of intermittency deviates from the log-normal distribution.

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- Based on the multiplicative cascade models and a multiplicative generalization of the central limit theorem, **Universal Multifractal** (**UM**) framework [1] was introduced to describe the **multifractality** of a field by two physically meaningful parameters:

- **The mean intermittency C_1:** measures the mean concentration of the activity**.**
- **The multifractality index α (Levy index):** how quickly the intermittency increases with the activity level .

The **UM parameters** for the atmospheric turbulence [2] yielded by various turbulence data is $\boldsymbol{\alpha \approx 1.5, C_1 \approx 0.25}$.

- UM parameters can be obtained from the **double trace moment (DTM)** analysis.

$K(q,\eta) = \eta^{\alpha} \frac{C_1}{\alpha - 1}(q^{\alpha} - q)$ where q is the moment order and η is the power.

[1] Schertzer, D., Lovejoy, S. (1987). Physical modeling and analysis of rain and clouds by anisotropic scaling multiplicative processes. *Journalof Geophysical Research: Atmospheres*, *92*(D8), 9693-9714.

[2] Schmitt, F., Schertzer, D., Lovejoy, S., & Brunet, Y. (1994). Estimation of universal multifractal indices for atmospheric turbulent velocity fields. In *Fractals in Natural Sciences* (pp. 274-281).

2. Approach

- The **Scaling Gyroscope Cascade** (**SGC**) model [3]:

$$(\frac{d}{dt} + \nu k_m^2)u_m^i = k_{m+1}\left[\left|u_{m+1}^{2i}\right|^2 - \left|u_{m+1}^{2i+1}\right|^2\right] + (-1)^{i+1}k_m u_m^i u_{m-1}^{a(i)}$$

is the evolution equation (Fig 1) of the superposition coupling of two equations of gyroscope type, which is obtained from the Bernoulli's form of the Navier-Stokes equation. **Triad interactions** preserve detailed energy conservation.

- The maximum cascade step: n (Cascade step starts from 0).
- The simulated step is m $(0 \leq m \leq n)$, i is location $(0 \leq i \leq 2^m - 1)$.
- The location of ancestor is $a(i) = int(i/2)$.

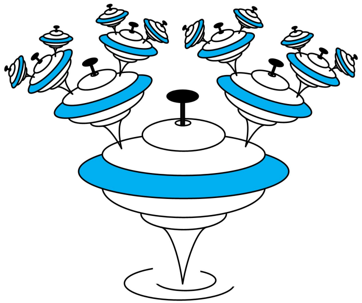


Fig 1. Figure of gyroscope cascade.

- **Forced turbulence**: It corresponds to a quasi-equilibrium between forcing and dissipation, but with high fluctuations.

- **The energy flux** at k_m: the sum of energy transfer rate from all wave number $k < k_m$ to other wave number $k \geq k_m$.

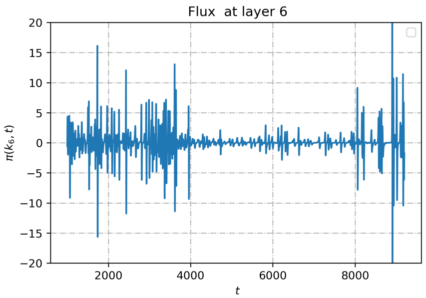
- $\Pi(k_m) = \int_{k_m}^{\infty} T(k,t) \propto \sum_{m'=m}^{n}\sum_{i=0}^{2^{m'}-1} u_{m'}^i \cdot (\frac{\partial u_{m'}^i}{\partial t} + \nu k_{m'}^2 u_{m'}^i)$

$$\Pi(k_m) \propto \sum_{m'=m}^{n}\sum_{i=0}^{2^{m'}-1} u_{m'}^i \cdot (k_{m'+1}\left[\left|u_{m'+1}^{2i}\right|^2 - \left|u_{m'+1}^{2i+1}\right|^2\right] + (-1)^{i+1}k_{m'}u_{m'}^i u_{m'-1}^{a(i)})$$

[3] Chigirinskaya, Y., Schertzer, D. (1997). Cascade of scaling gyroscopes: Lie structure, universal multifractals and self-organized criticality in turbulence. In Stochastic Models in Geosystems (pp. 57-81). Springer, New York, NY.

3. Simulation

Case	n	p	Force	dt	Tmax
1	12	10	0.5	0.0004	9200



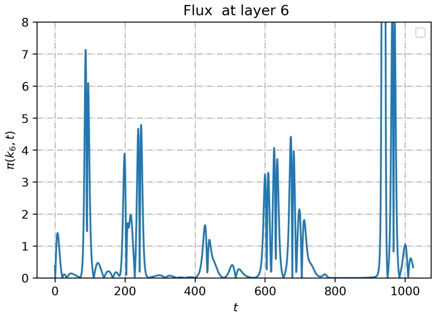
X label: The energy flux $\Pi(k_6)$
Y label: Time (1000-9200)
Get rid of initial condition

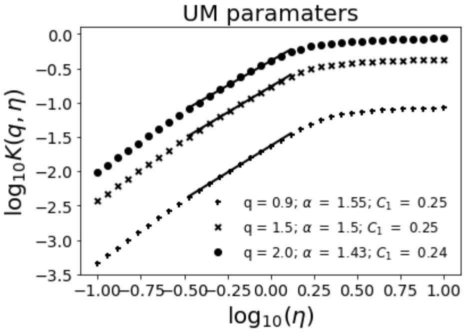
One of time series (7970 – 9170, 1200 eddy turn over time)
The energy flux $\Pi(k_6)$ is normalized by average and its absolute value is analyzed.

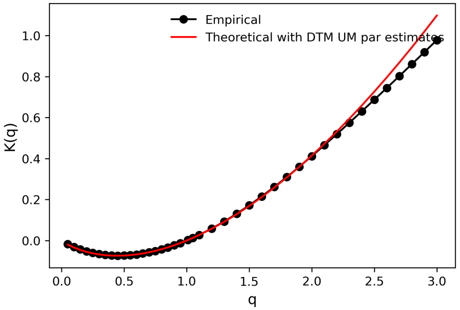
- Dissipation step: p
- Time step: dt ($dt \ll \frac{2}{\nu k_n^2}$ to ensure the stability of Euler computing method)
- Tmax: Normalized by the large eddy turn over time τ which is the time to fully develop the large eddies and enough for small eddies.

($\tau = \frac{L(0)}{\sqrt{K(0)}}$, where $K(\mathrm{t}) = \sum_{m=0}^{n} \sum_{i=0}^{2^m-1} (u_m^i)^2$ is the **system energy** and $L(t) = \frac{\int_{m=0}^{n} k_m^{-1} E(k_m,t)dk}{\int_{m=0}^{n} E(k_m,t)dk}$ is the integral scale.)

- Initial energy spectrum : $E(k,t) = C_K \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$. The Kolmogorov constant is assumed as $\boldsymbol{C_k} = 1$ and $\boldsymbol{\varepsilon = 1}$.



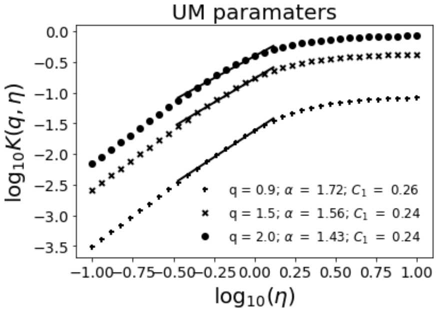


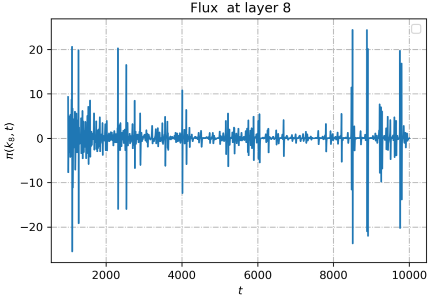


The multifractality index $\boldsymbol{\alpha \approx 1.5}$
The mean intermittency $\boldsymbol{C_1 \approx 0.25}$
The critical moments is $\boldsymbol{q_s \approx 2.51}$ after which the empirical curve becomes linear.

Case	n	p	Force	dt	Tmax
2	14	12	0.8	0.0001	10000
3	14	12	0.7	0.0001	14000

The numerous simulations by using different forces, yield consistent UM parameters.
For example, Case 2 and Case 3 suggests that UM parameters are independent of the value of force.

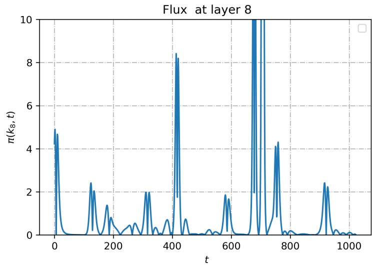


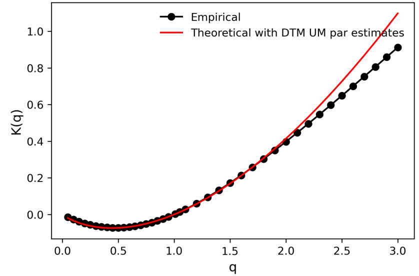


X label: The energy flux $\Pi(k_8)$
Y label: Time (1000-10000)
Get rid of initial condition

Case 2

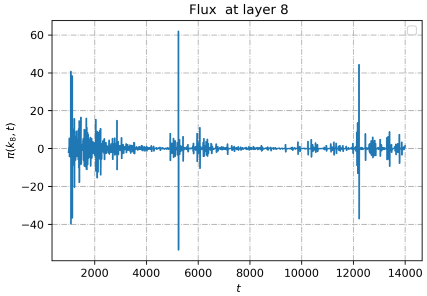
One of time series (Time: 7800, 1200 eddy turn over time)
The energy flux $\Pi(k_8)$ is normalized by average and its absolute value is analyzed.

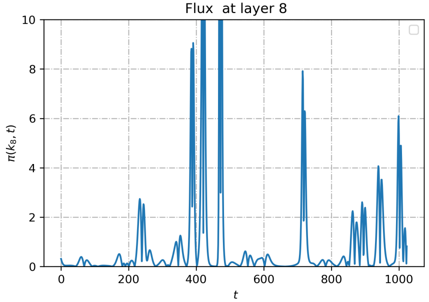


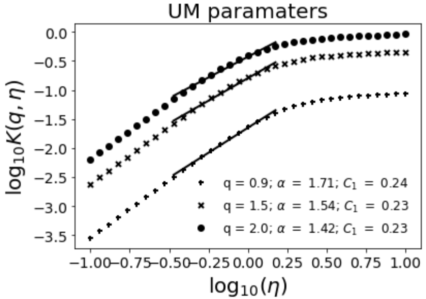


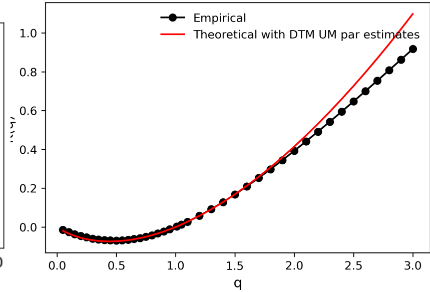
Case 3

One of time series (Time: 11750, 1200 eddy turn over time)
The energy flux $\Pi(k_8)$ is normalized by average and its absolute value is analyzed.









4. Conclusion

All simulations of SGC model display an extreme space-time intermittency. Their multifractal analysis confirms that the UM parameters are $\boldsymbol{\alpha} \approx \mathbf{1.5}$, $\boldsymbol{C_1} \approx \mathbf{0.25}$ and exhibits good approximation for the theoretical scaling moment function, which questions the log-normal approximation for the hydrodynamic turbulence.