Navigation of micro swimmers in steady flow: the importance of symmetries

B. Mehlig

Department of Physics, University of Gothenburg, Sweden

University of Gothenburg, Sweden

Based on J. Qiu, N. Mousavi, K. Gustavsson, C. Xu, B. Mehlig & L. Zhao,
Journal of Fluid Mechanics **932** (2022) A10.

The research was supported by *Vetenskapsrådet.*

Question

How can an active particle learn to navigate a complex flow in an optimal way?

Zooplankton (copepods): accelerate to escape predation.

Ardeshiri, Schmitt, Souissi, Toschi & Calzavarini, J. Plankton. Res. **39** (2017) 878

Control problem: minimise time for active particle to move from A to B in complex flow. Control variable: steering angle. Buzzicotti, Biferale, Bonaccorse, Clark Di Leoni & Gustavsson (2019)

Optimal strategy not obvious. Depends on the *signals* the particle picks up as it moves, and upon its possible *actions*. Proof-of-principle demonstration: use machine-learning to find strategies minimising time it takes bottom-heavy plankton to reach water surface. Colabrese, Gustavsson, Celani & Biferale, Phys. Rev. Lett. **118** (2017) 158004

This and many follow-up studies use global signals & actions that refer to the laboratory frame (up, down, left, right). Plankton measure *local hydrodynamic signals* and take *local actions*. Kiorboe & Visser, Marine Ecol. Prog. Ser. **179** (1999) 81

Can one learn from local signals with local actions?

Hydrodynamic signals

Kiorboe & Visser, Marine Ecol. Prog. Ser. **179** (1999) 81

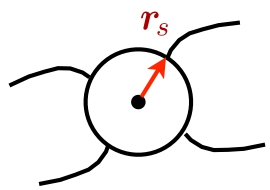
Hydrodynamical signals measured by setal bending patterns.

1. Deformation rate $\boldsymbol{\delta}_s$ of setae due to *strain* $\mathbb{S}$: $\boldsymbol{\delta}_s = \mathbb{S}\,\boldsymbol{r}_s$

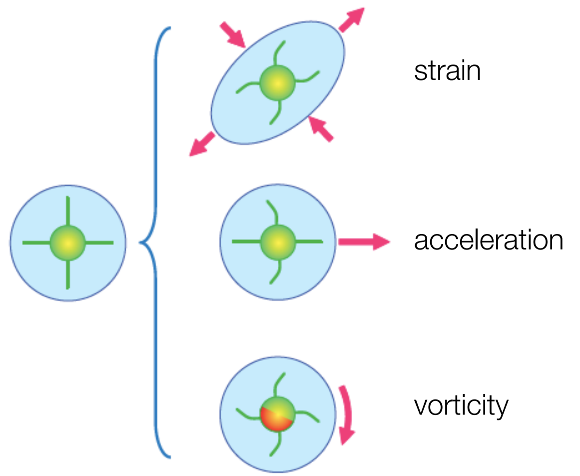
2. Deformation rate due to *fluid acceleration*.
Stokes equation $\dot{\boldsymbol{v}} = \alpha(\boldsymbol{u}-\boldsymbol{v}) + \beta\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}$ with

$$\alpha^{-1} = \frac{2a^2}{9\nu}(2\varrho_\mathrm{p}+\varrho_\mathrm{f}) \quad \text{and} \quad \beta = 3\varrho_\mathrm{p}/(2\varrho_\mathrm{p}+\varrho_\mathrm{f})$$

Steady state $\boldsymbol{\delta}_a \sim \boldsymbol{u}-\boldsymbol{v} = \frac{\beta-1}{\alpha}\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \frac{2a^2}{9\nu}\frac{\varrho_\mathrm{p}-\varrho_\mathrm{f}}{2\varrho_\mathrm{p}+\varrho_\mathrm{f}}\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}$
Must have $\varrho_\mathrm{p} \neq \varrho_\mathrm{f}$ to measure acceleration.



$|\boldsymbol{r}_s| = a$
(particle size)

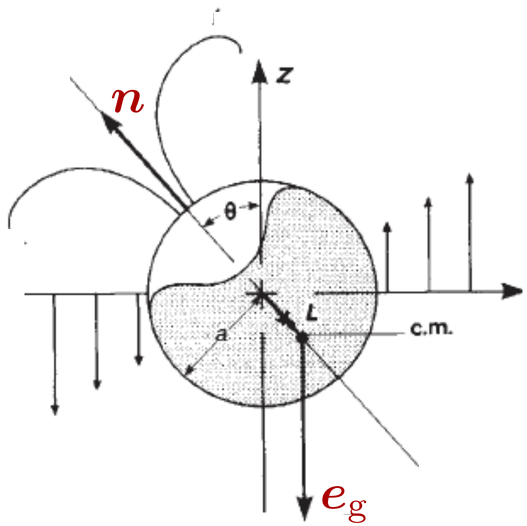


Visser (2011)

3. Deformation rate due due to *vorticity*.
Torque balance: $8\pi a^3\varrho_\mathrm{f}\nu(\boldsymbol{\Omega}-\boldsymbol{\omega}) + m_\mathrm{p}\boldsymbol{L}\wedge\boldsymbol{g} = 0$
Jeffery torque — gyrotactic torque

$$\boldsymbol{\delta}_\omega \sim a(\boldsymbol{\Omega}-\boldsymbol{\omega}) = \frac{2\varrho_p}{9\varrho_\mathrm{f}}(a/\nu)(\boldsymbol{L}\wedge\boldsymbol{g})$$

Requires density asymmetry $\boldsymbol{L} \neq 0$ (gyrotactic torque).



Kessler, Nature **313** (1985) 218

Model

Kessler, Nature **313** (1985) 218

Durham *et al.*, Nature Communications **4** (2013) 2148

Non-spherical, gyrotactic, settling swimmer

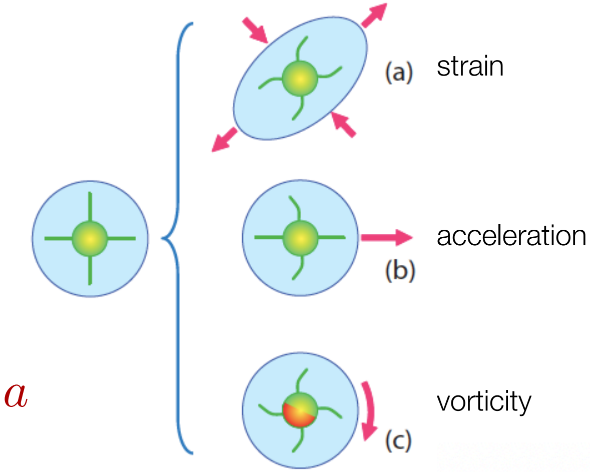
$$\dot{\boldsymbol{x}} = \boldsymbol{v}\,, \quad \boldsymbol{v} = \boldsymbol{u} + v_{\rm s}\boldsymbol{n} + \boldsymbol{v}_{\rm g}$$

swimming settling

$$\dot{\boldsymbol{n}} = \boldsymbol{\omega} \times \boldsymbol{n}\,, \quad \boldsymbol{\omega} = \boldsymbol{\Omega} + \Lambda\boldsymbol{n} \times (\mathbb{S}\boldsymbol{n}) + \boldsymbol{\omega}_{\rm s} - \frac{1}{2B}\boldsymbol{n} \times \boldsymbol{e}_{\rm g}$$

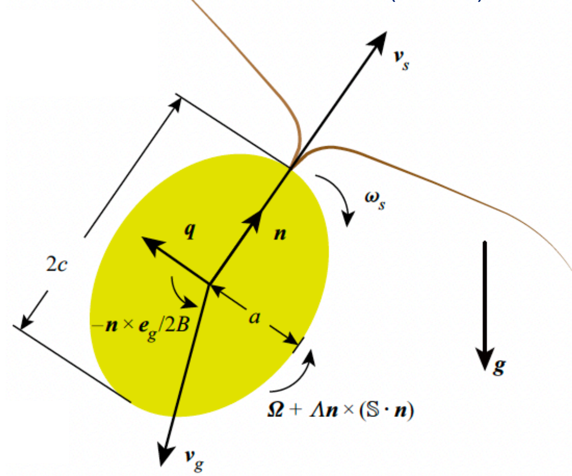
vorticity strain steering gyrotactic torque

with shape parameter $\Lambda = (\lambda^2-1)/(\lambda^2+1)$ (aspect ratio $\lambda = c/a$



Two-dimensional navigation & flow.

Visser (2011)



Signals: local strain ($S_{nn} = \boldsymbol{n}\cdot\mathbb{S}\boldsymbol{n}$, $S_{nq} = \boldsymbol{n}\cdot\mathbb{S}\boldsymbol{q}$), vorticity ($\Delta\Omega = (\boldsymbol{\Omega}-\boldsymbol{\omega})\cdot\boldsymbol{e}_z$), and velocity ($\Delta u_q = (\boldsymbol{u}-\boldsymbol{v})\cdot\boldsymbol{q}$), each discretised into three states.

Actions: steering with angular velocity $\boldsymbol{\omega}_{\rm s}$, discretised into three actions.

Q-learning to find strategy for upwards navigation.

Results

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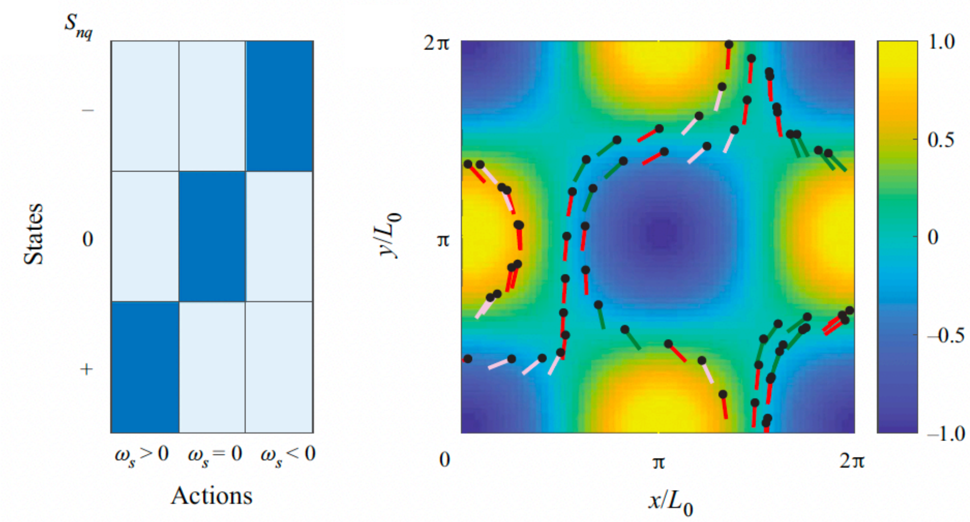
Case	State	Action	Settling	Gyrotaxis	Vertical reflection symmetry	Training
S1	S_{nq}, S_{nn}	$\omega_s = [-1, 0, 1]$	No	No	Unbroken	Failure
S4	S_{nq}, S_{nn}	$\omega_s = [-1, 0, 1]$	No	Yes	Broken (gyrotaxis)	Success
S5	S_{nq}, S_{nn}	$\omega_s = [-1, 0, 1]$	Yes	No	Broken (settling)	Success
S6	S_{nq}, S_{nn}	$\omega_s = [-1, 0, 1]$	Yes	Yes	Broken (gyrotaxis and settling)	Success

Dynamics must break symmetry (gyrotaxis, settling) for the active particle to learn to navigate upwards.



What does the swimmer learn?
In two dimensions (x - y -plane), the swimmer learns

$$\omega_s > 0 \quad \text{if} \quad S_{nq} > 0$$
$$\omega_s = 0 \quad \text{if} \quad S_{nq} = 0$$
$$\omega_s < 0 \quad \text{if} \quad S_{nq} < 0$$

Why is this strategy successful?

Interpretation of optimal strategy

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What does the swimmer learn? In two dimensions (x- y-plane)

$$S_{nq} = [\boldsymbol{n} \times (S\boldsymbol{n})] \cdot \boldsymbol{e}_z$$

Compare angular equation of motion

$$\dot{\boldsymbol{n}} = \boldsymbol{\omega} \times \boldsymbol{n}\,, \quad \boldsymbol{\omega} = \boldsymbol{\Omega} + \Lambda \boldsymbol{n} \times (\mathbb{S}\boldsymbol{n}) + \boldsymbol{\omega}_{\rm s} - \frac{1}{2B}\boldsymbol{n} \times \boldsymbol{e}_{\rm g}$$

Conclusion: the swimmer adopts a strategy that emulates a slender particle ($\Lambda = 1$).

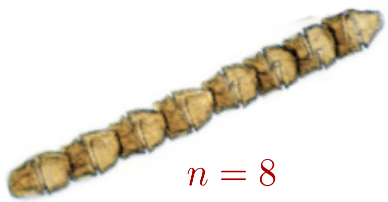
Why? Because slender swimmers tend to sample upwelling regions of the flow.

Gustavsson, Berglund, Jonsson & Mehlig, Phys. Rev. Lett. **116** (2016) 108104

Borgnino, Boffetta, De Lillo & Cencini, J. Fluid. Mech. **856** (2018) R1

Chain formation allows cells to move upwards more efficiently.

Lovecchio, Climent, Stocker & Durham, Science Advances **5** (2019)



$n = 8$

Open questions

Here two-dimensional steady flow. Role of symmetry in three spatial dimensions? Unsteady flow?

We discretised signals and actions using simplified estimates for threshold values. Expect: different thresholds may change strategy.

How can the organisms distinguish undisturbed flow-signal from the disturbance they themselves create?

Upwards navigation is a fairly simple task, here used to illustrate the role of symmetry. In reality avoidance of predators more important, or a combination of different targets to optimise.