Stochastic Modelling and Location Uncertainty (LU) formalism: hydrostatic Boussinesq equations and their implementation in NEMO

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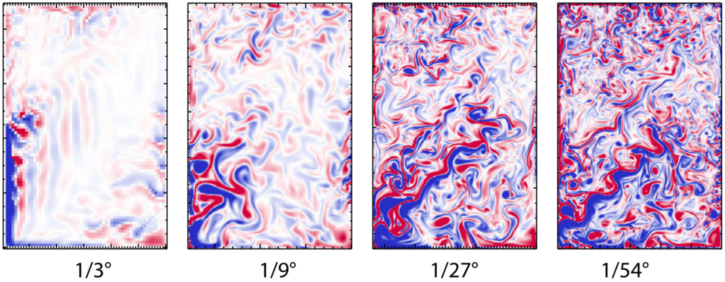
Motivation

The accuracy of a simulation is intrinsically dependent on the chosen resolution[1].



Fixed δ representing the resolution, there will always be a distance between the real solution $\boldsymbol{u}$ and its approximation $\boldsymbol{u}_\delta$.
This distance exists also in the physics that is representable by the simulation, due to non-linear interactions between scales that propagate information to scales smaller than the computed resolution.

Can we improve the solution at a coarse resolution $\boldsymbol{u}_\triangle$ by modelling what is left behind from a finer information $\boldsymbol{u}_\delta$?

[1] Image taken from Levy 2012, representing vorticity for an idealised jet stream configuration.

Boussinesq equations under Location Uncertainty

Location Uncertainty [1] decomposition principle:

$$\mathrm{d}X_t^i(\boldsymbol{x}_0) = \underbrace{u_i(\mathbf{X}_t, t)\,\mathrm{d}t}_{\text{Resolved}} + \underbrace{\eta_t^i(\mathbf{X}_t)}_{\text{Unresolved}},$$

where the resolved component is dependent on the resolution chosen, the unresolved component accounts for turbulent effects, truncation and approximation effects, uncertainty in the forcing and initial conditions;

This introduces the stochastic material derivative:

$$\mathbb{D}_t\theta = \mathrm{d}_t\theta + [\boldsymbol{u}^\star\,\mathrm{d}t + \underbrace{\boldsymbol{\eta}]\cdot\nabla\theta}_{\text{Noise advection}} - \underbrace{\frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla\theta)\,\mathrm{d}t}_{\text{Diffusion by Unresolved scales}}.$$

where

$$\boldsymbol{u}^\star = \boldsymbol{u} - \boldsymbol{u}_s,$$

with $\boldsymbol{u}_s$ accounting for heterogeneities.

LU Boussinesq

Horizontal momentum:

$$\mathbb{D}_t\boldsymbol{v} + f\mathbf{e}_3 \times \left(\boldsymbol{v}\,\mathrm{d}t + \frac{1}{2}\boldsymbol{\eta}_t^{\mathrm{H}}\right) = -\nabla_{\mathrm{H}}\left(\pi\,\mathrm{d}t + \mathrm{d}p_t^\sigma\right)$$

Vertical momentum:

$$\mathbb{D}_t w = -\partial_z\left(\pi\,\mathrm{d}t + \mathrm{d}p_t^\sigma\right) + b\,\mathrm{d}t$$

Tracer (Temperature and salinity):

$$\mathbb{D}_t\tau = \kappa_\tau\Delta\tau\,\mathrm{d}t,$$

Incompressibility:

$$\nabla\cdot[\boldsymbol{u} - \boldsymbol{u}_s] = 0, \qquad \nabla\cdot\boldsymbol{\eta} = 0,$$

Equation of state:

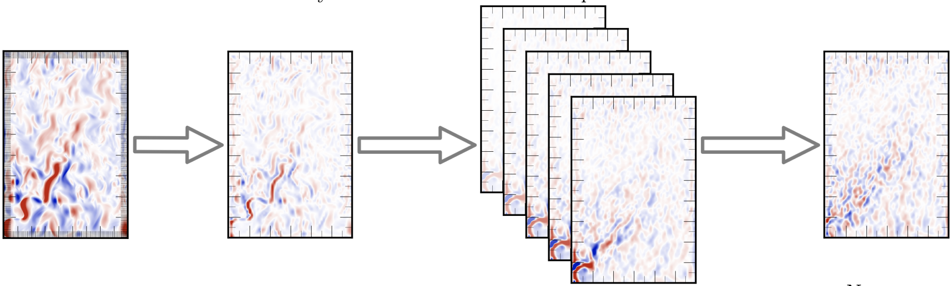
$$b = b(T, S, z).$$

with

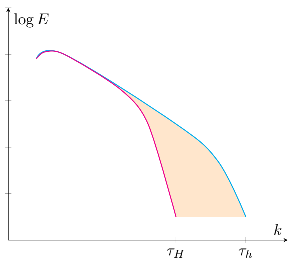
$$\pi = p' - \frac{\nu}{3}\nabla\cdot\mathbf{u}_s$$

Fluctuation model | **Noise construction**

High Resolution | High-pass filtered velocity | Proper Orthogonal Decomposition | Noise Forcing

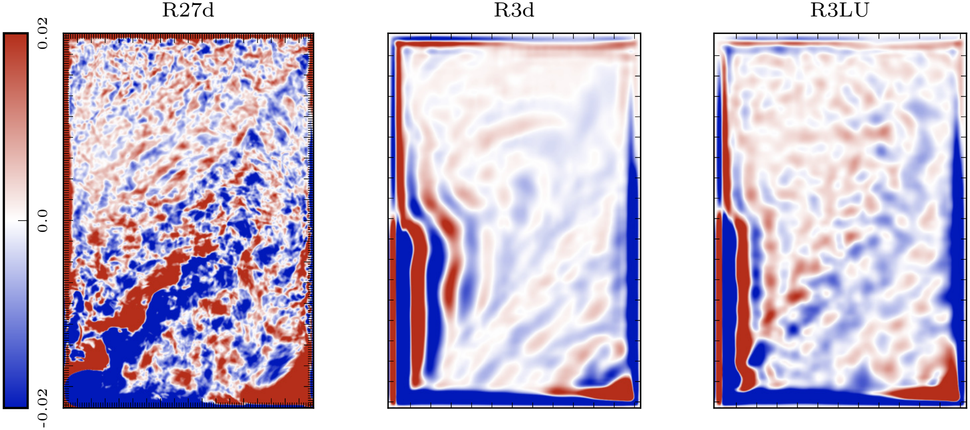


$\mathbf{u}_{\mathrm{HR}}$ $\quad$ $\mathbf{u}_{\mathrm{LR}} = \left[(1-\mathcal{G})\,\mathbf{u}_{\mathrm{HR}}\right]^{\downarrow}_{\mathrm{LR}}$ $\quad$ $\{\boldsymbol{\Phi}_k(\boldsymbol{x}),\ k \in [1, N]\}$ $\quad$ $\dot{\boldsymbol{\eta}}(\boldsymbol{x}) = \sum_{k=1}^{N} \sqrt{\lambda_k}\,\boldsymbol{\Phi}_k(\boldsymbol{x})\xi_k$



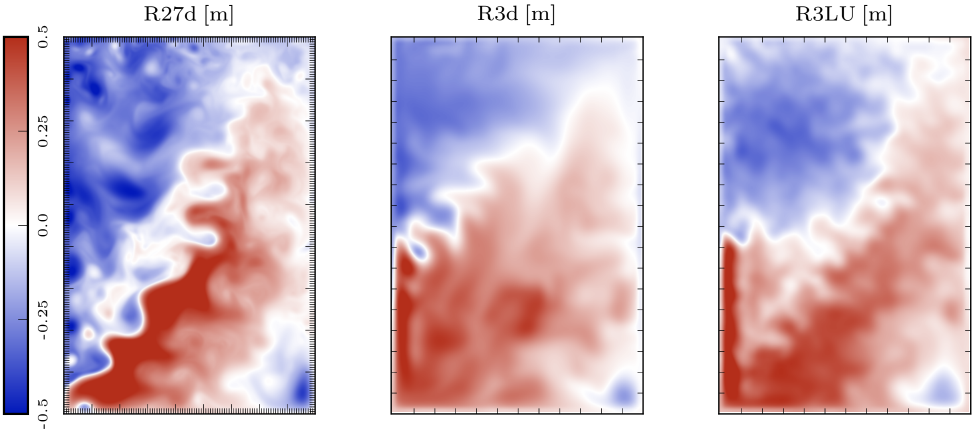
The fluctuation model should carry the information that cannot be represented with a coarse simulation, i.e. the fraction of energy represented with the shaded area. The noise should be constructed to carry this partial information rather that the whole velocity field information

10-years averaged relative vorticity $\zeta = (\partial_x v - \partial_y u) / f$ at the surface layer of the model.



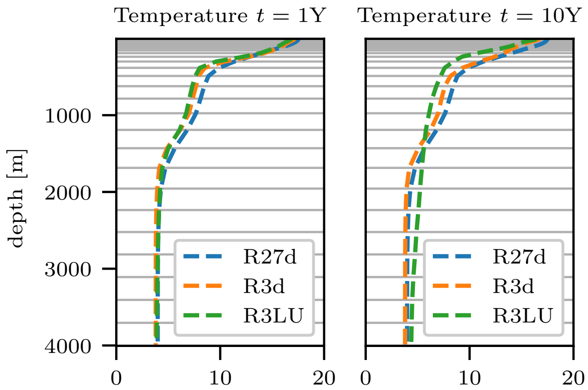
Improvements: Better resolution of Vorticity.

5-days averaged sea surface height.



Improvements: Better resolution of Sea Surface Height.

Drawbacks: Increase in vertical diffusion and transport in tracers, leading to fields far from the reference $\mathrm{R27}$ simulation.



Possible motivations to investigate:

1. No feedback from u^t velocity;
2. Two-dimensional noise not adequate;

Future works:

1. different types of noise (3D POD, DMD, Local decompositions...);
2. hydrostatic stochastic pressure model;
3. barotropic-baroclinic splitting for the noise;
4. realistic configurations.

Thank you for your attention.

References

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