Variance component estimation for co-estimated noise parameters in GRACE Follow-On gravity field recovery

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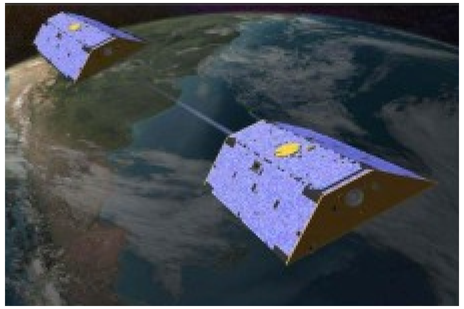
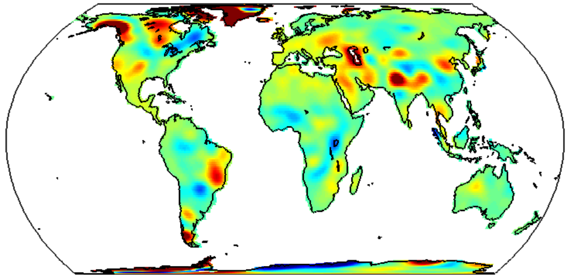
Introduction





basic parametrisation:

- initial conditions 2x(6)
- accelerometer bias 2x(3)
- accelermeter scaling 2x(3)

parameters per arc 24

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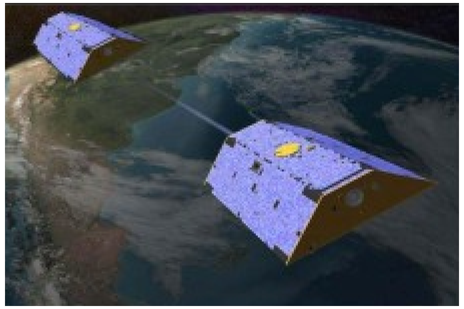

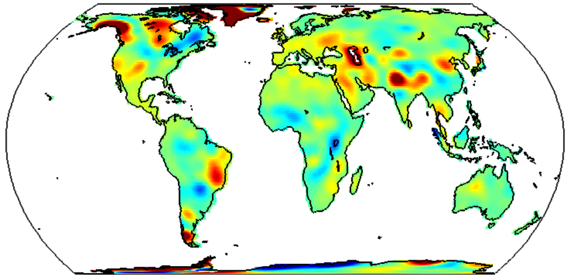

Perturbation theory [Kim, 2000]:
Errors in background models will (mostly) sum up in 1/rev

→ frequently used in the Celestial Mechanics Approach
[Beutler et al., 2010]

basic parametrisation:

• initial conditions	2x(6)
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parameters per arc	24

additional parameters:
- 15 min PCA per satellite in

→ radial	2x(96)
→ along track	2x(96)
→ cross-track	2x(96)
parameters per arc	576

in daily arcs (30 days):
- 18000 parameters,
- 17280 for the noise model
- + gravity field

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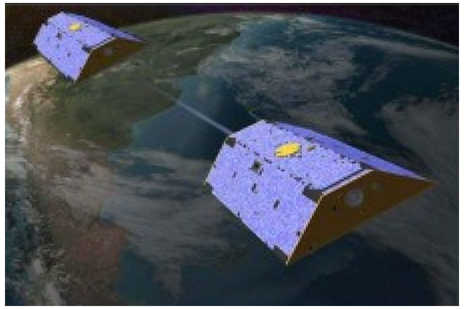

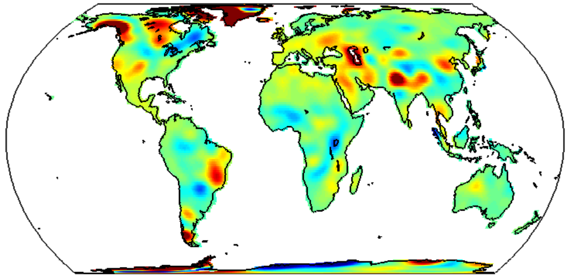

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How to constrain their impact to the correct magnitude?

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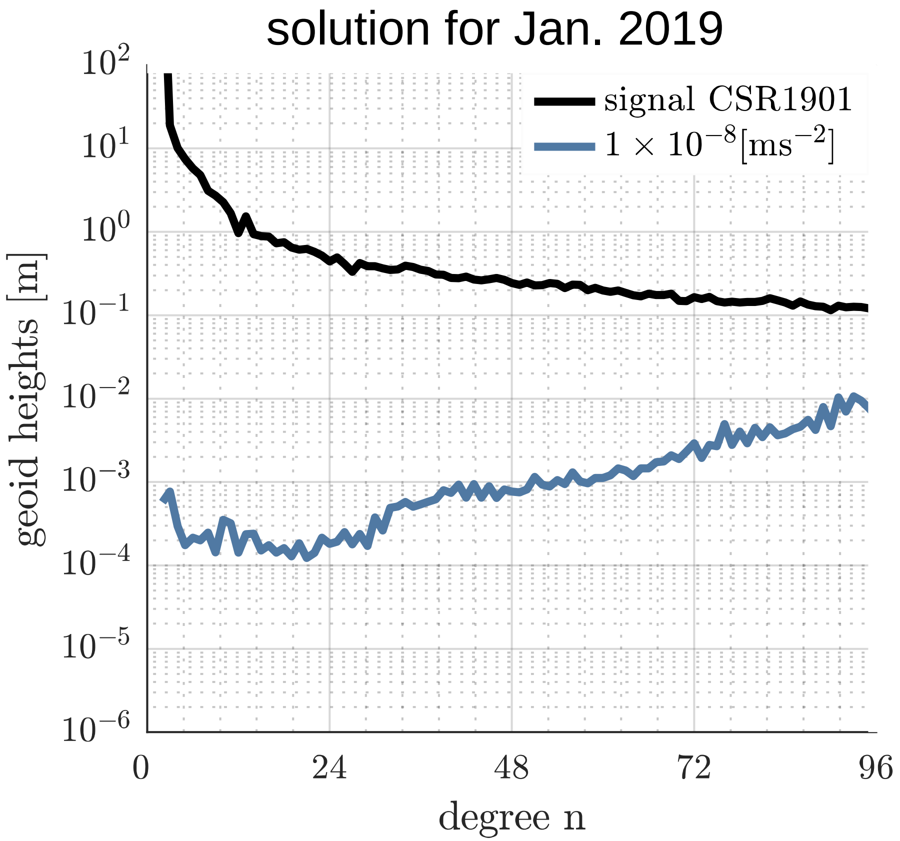
Impact of different constraints

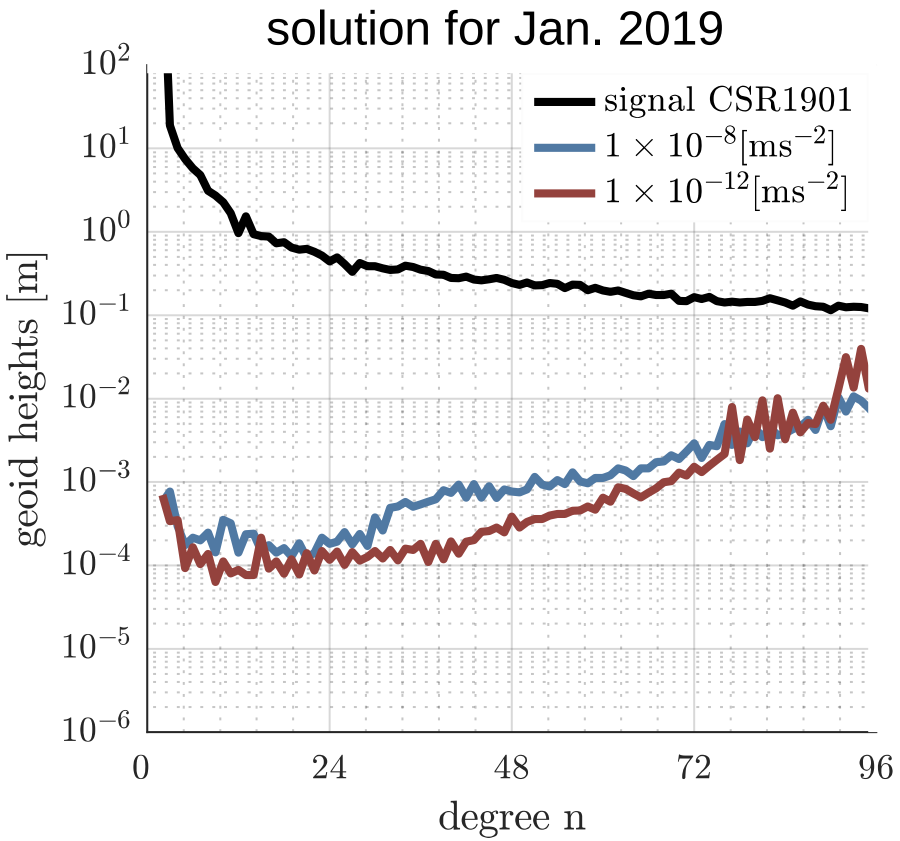


$1\times10^{-8}\ ms^{-2}$

«loose» constraint

(gravity field signal absorbed in PCAs)

Impact of different constraints

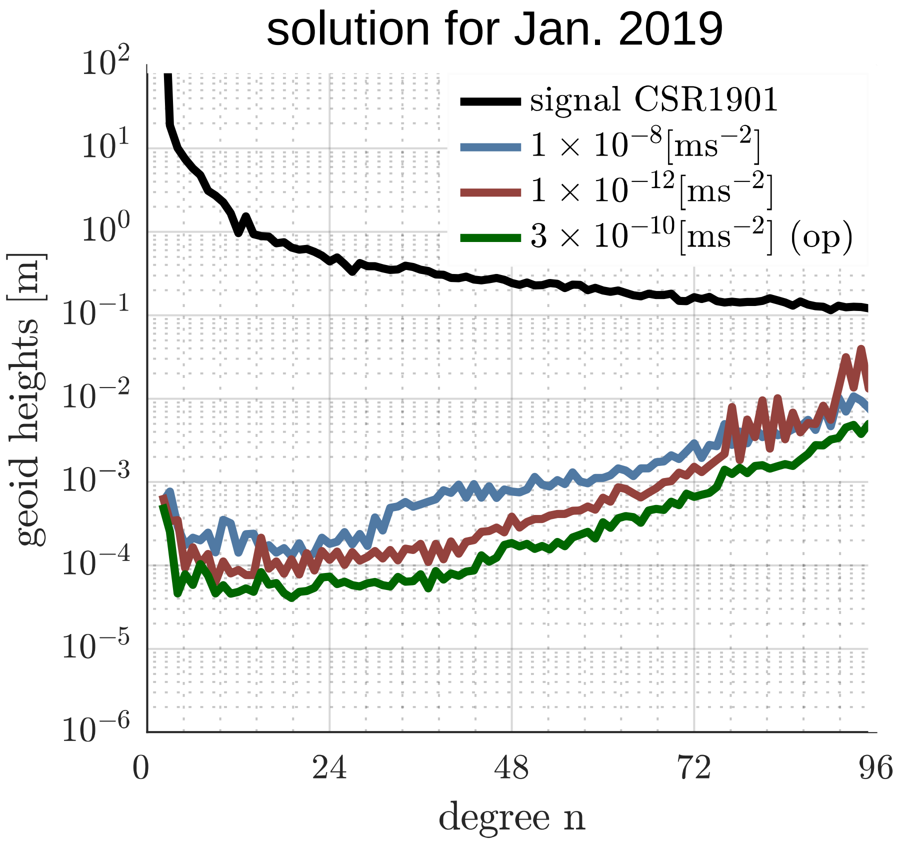


$1 \times 10^{-12}\ ms^{-2}$

«tight» constraint

(not enough to absorb mis-modellings)

Impact of different constraints



$3\times10^{-10}\,\mathrm{ms}^{-2}$

«reasonable» balance

(applied in the operational solutions)

Constraining

$$\mathbf{N}=(\mathbf{A}^T\mathbf{P}\mathbf{A})$$ and $$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l} \longrightarrow \hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

design matrix

weight matrix

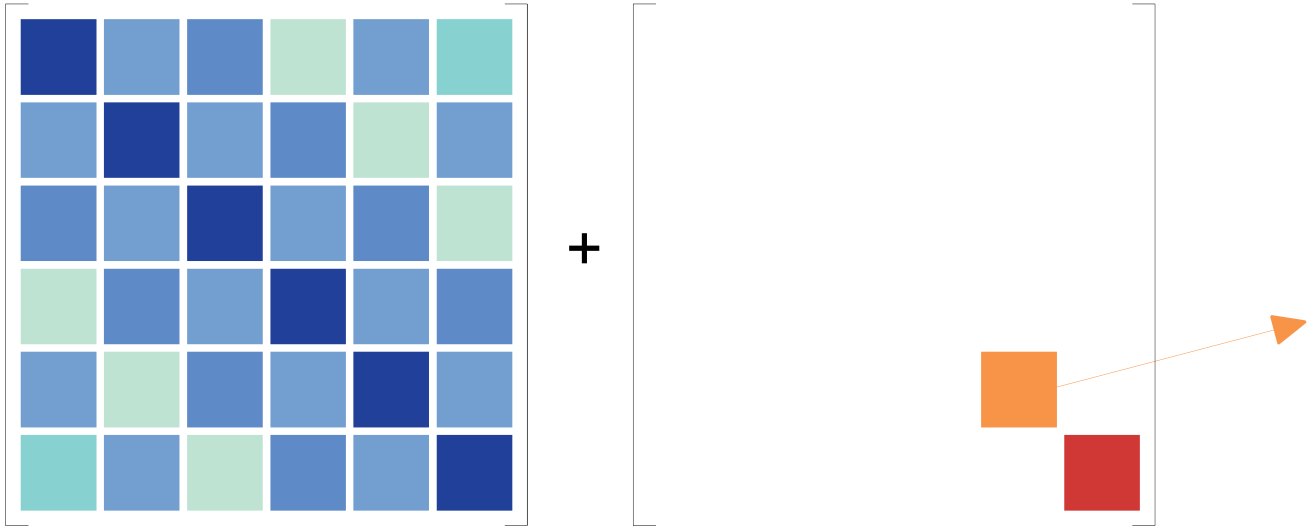
vector of observations

Constraining

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right) \quad \text{and} \quad \boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l} \longrightarrow \hat{\mathbf{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\mathbf{A}+\mathbf{W}\right)$$

$N=$



$$\frac{\sigma_0^2}{\sigma_{PCA}^2},$$

$\sigma_{PCA}^2=$ e.g., $3\times10^{-10}\ \mathrm{ms}^{-2}$

Variance Component Estimation

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right) \quad \text{and} \quad \boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l} \longrightarrow \hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

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$$\boldsymbol{l}=\begin{bmatrix} \\ \\ \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

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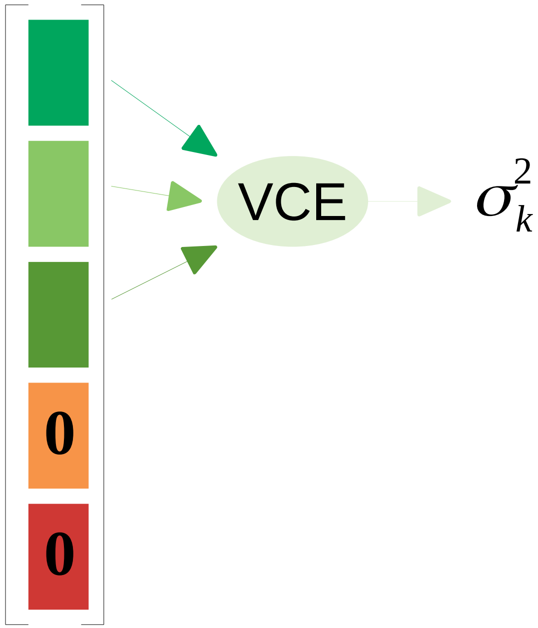
VCE: Each group of observations gets a weight based on its contribution to the final solution

$\boldsymbol{l}=$



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Variance Component Estimation

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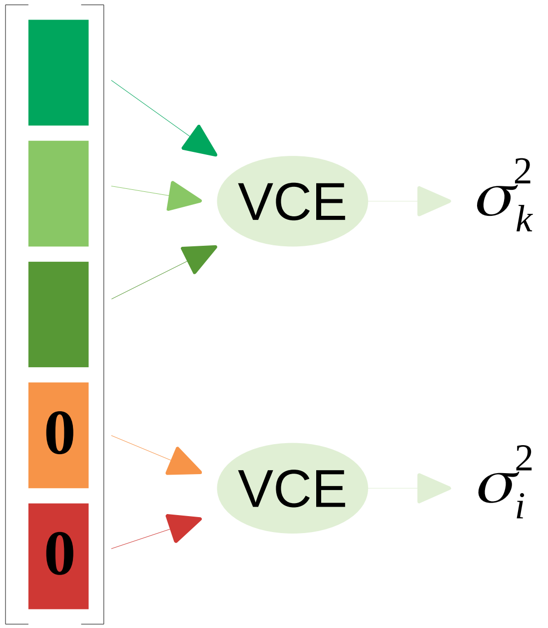

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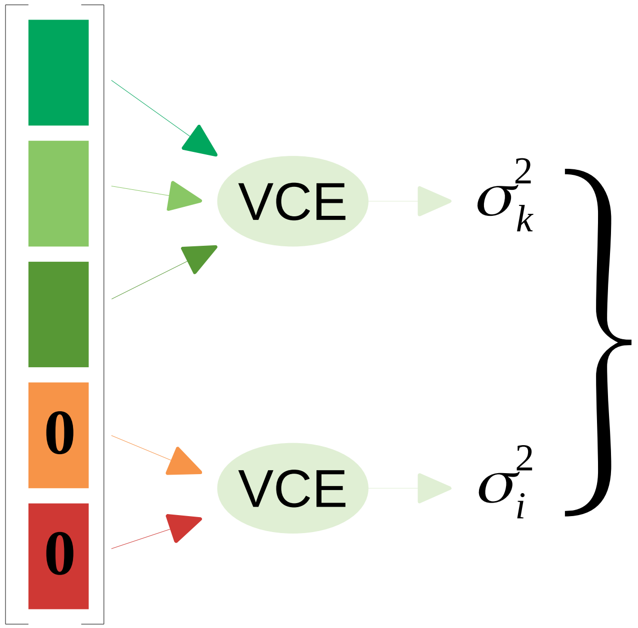

$$\hat{\boldsymbol{x}}=\left(\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\mathbf{N}_k+\sum_{i=1}^{I=2}\frac{\sigma_0^2}{\sigma_i^2}\mathbf{W}_i\right)^{-1}\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\boldsymbol{b}_k$$

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Variance Component Estimation

$\mathbf{N}=(\mathbf{A}^T\mathbf{P}\mathbf{A})$ and $\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\boldsymbol{l}$ $\longrightarrow$ $\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$

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VCE: Each group of observations gets a weight based on its contribution to the final solution

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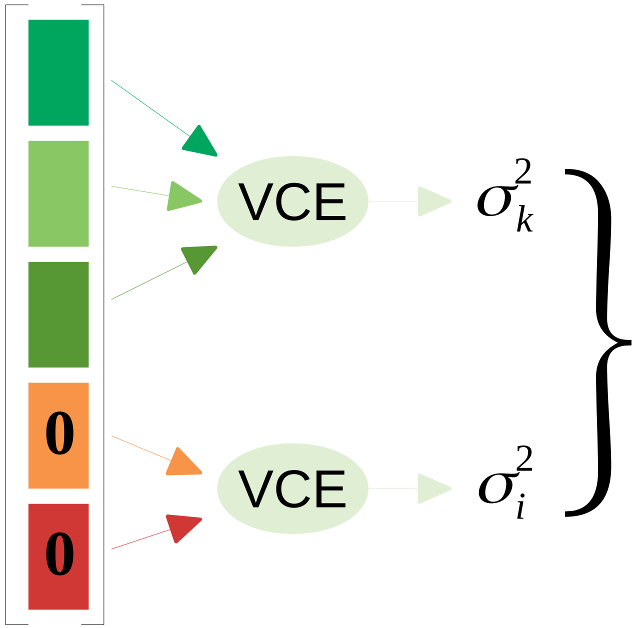

$$\hat{\boldsymbol{x}}=\left(\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\mathbf{N}+\sum_{i=1}^{I=2}\frac{\sigma_0^2}{\sigma_i^2}\mathbf{W}_i\right)^{-1}\sum_{k=1}^{K=3}\frac{\sigma_0^2}{\sigma_k^2}\boldsymbol{b}_k$$

information about observations introduced via σ_0^2

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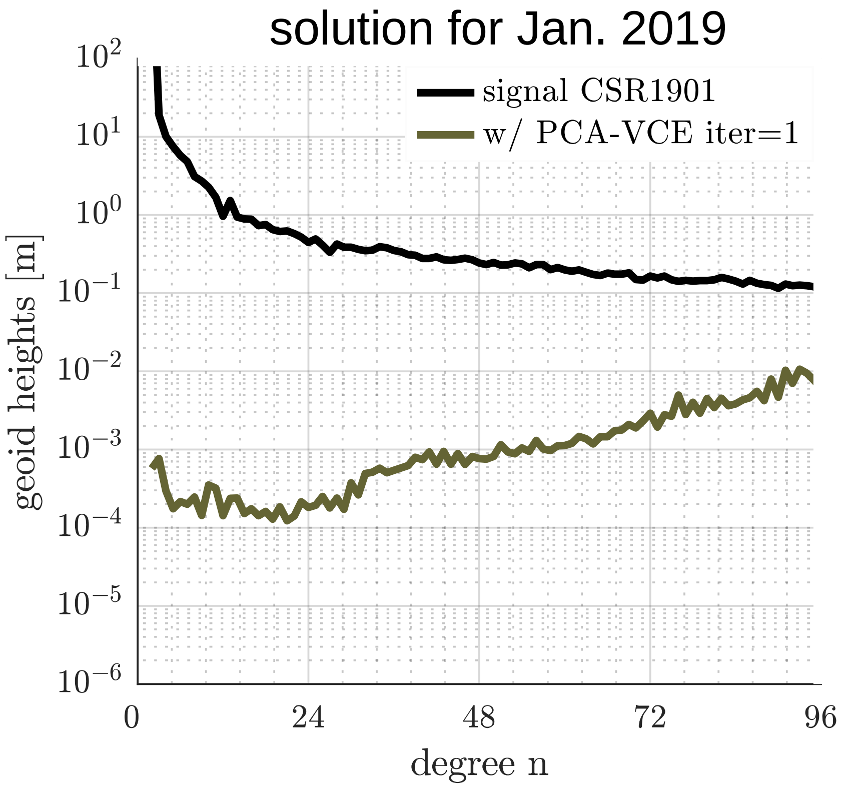
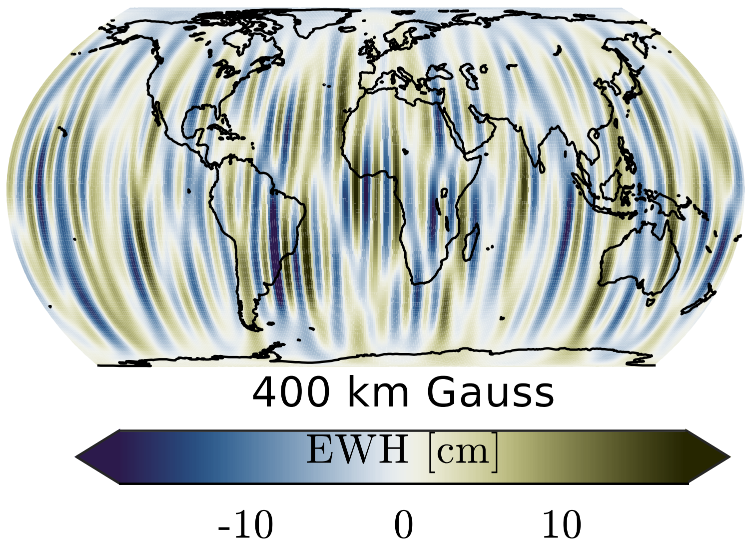
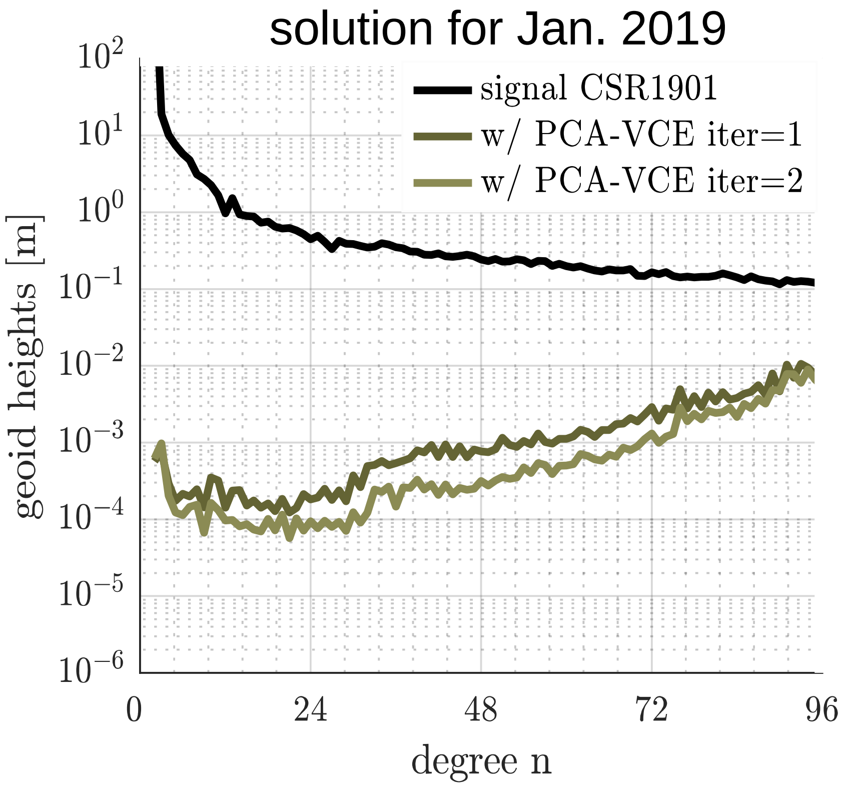
Results



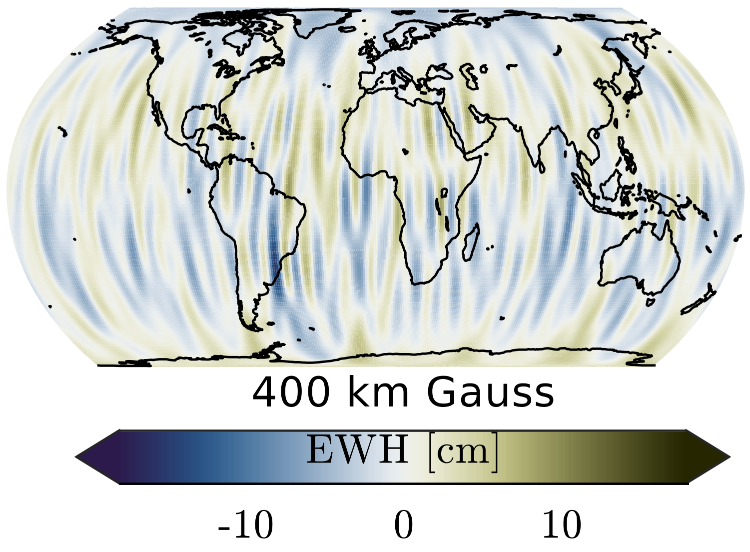
	ms^{-2}
radial	1×10^{-8}
along-track	1×10^{-8}
cross-track	1×10^{-8}



Results



	ms^{-2}
radial	1.9×10^{-8}
along-track	9.8×10^{-9}
cross-track	8.6×10^{-9}



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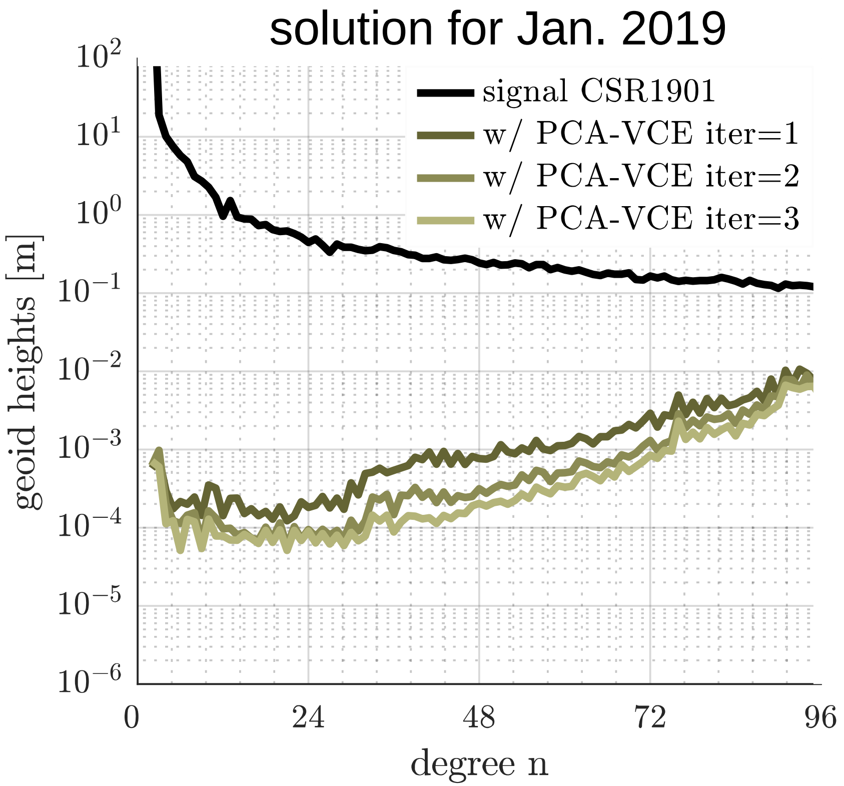
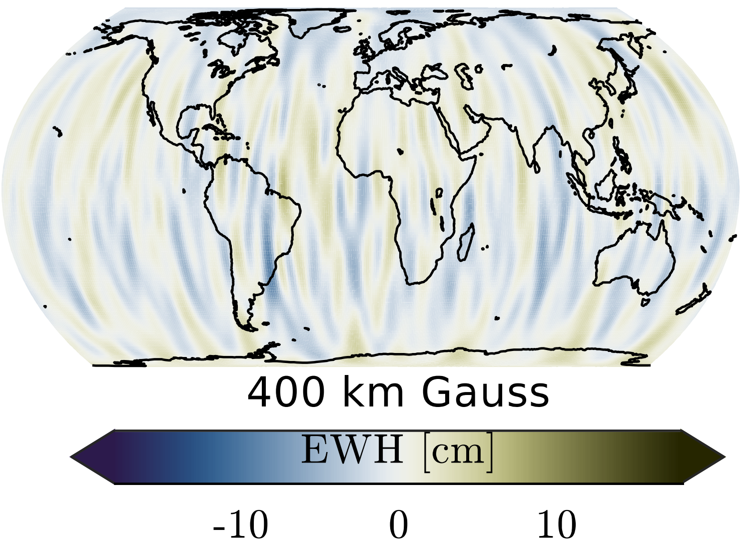


	ms^{-2}
radial	2.9×10^{-9}
along-track	1.5×10^{-9}
cross-track	1.3×10^{-9}



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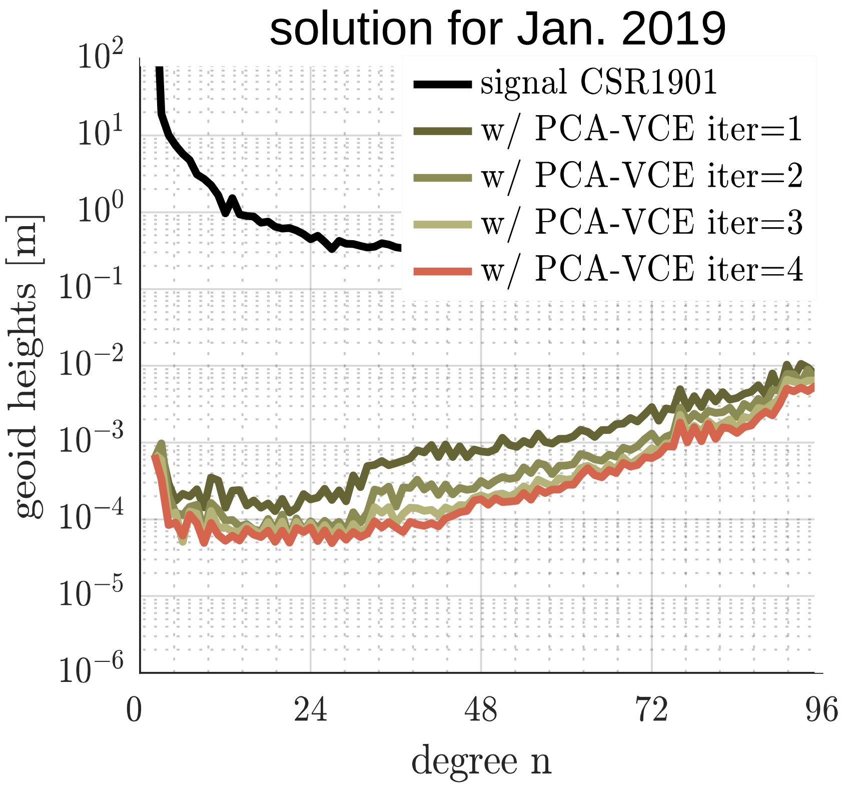
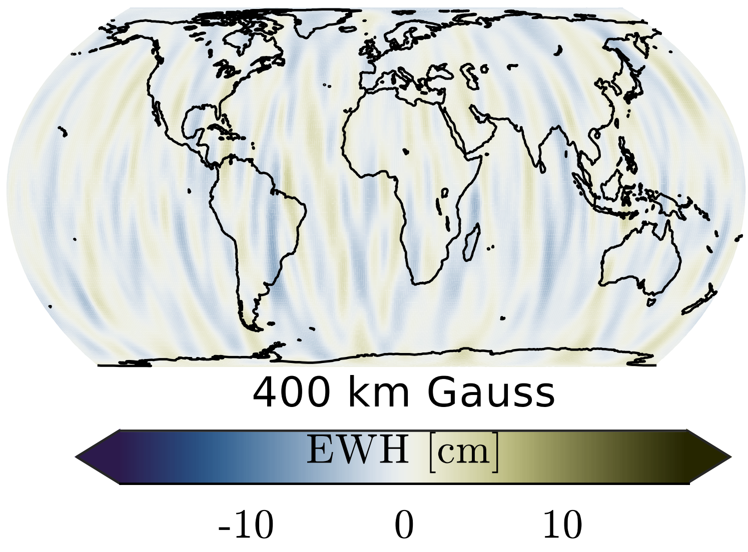


	ms^{-2}
radial	1.2×10^{-9}
along-track	6.2×10^{-10}
cross-track	6.9×10^{-10}



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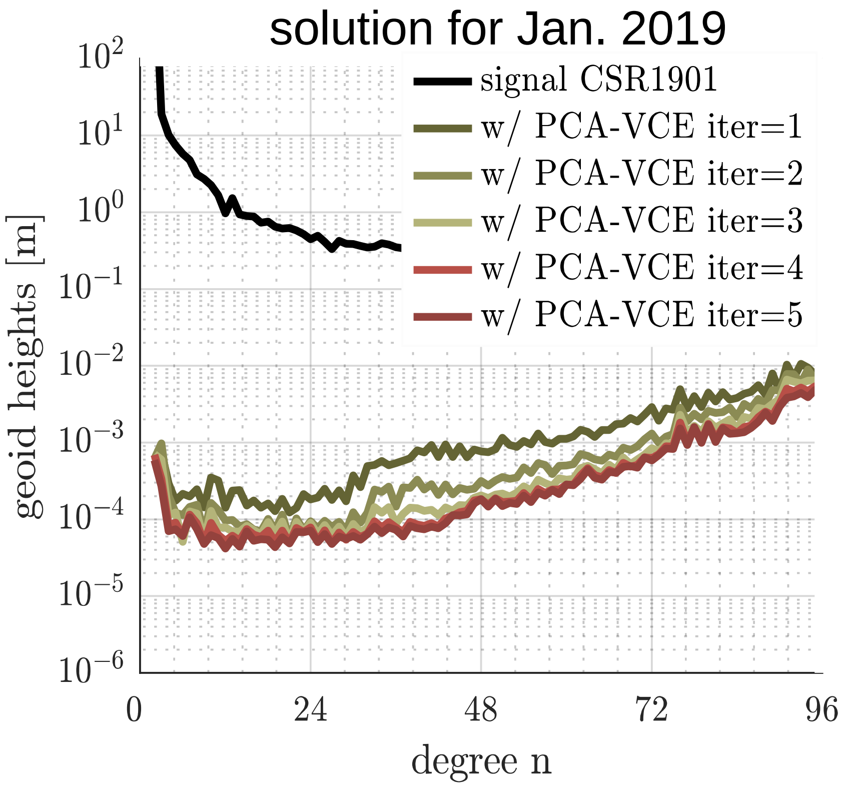
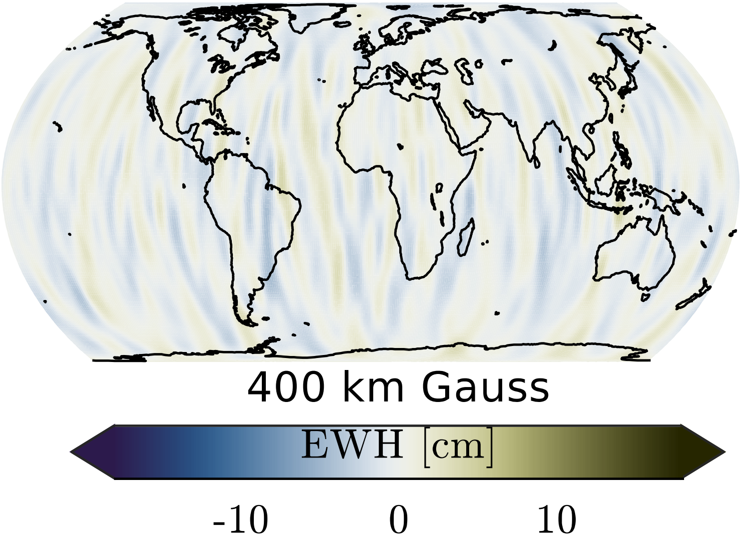


	ms^{-2}
radial	5.6×10^{-10}
along-track	2.6×10^{-10}
cross-track	6.1×10^{-10}



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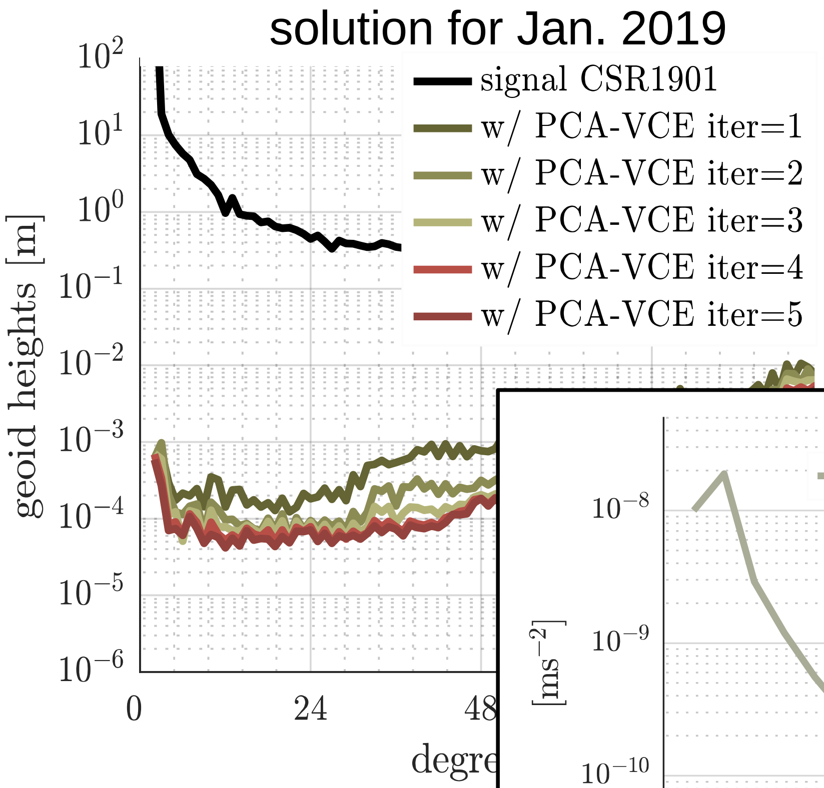
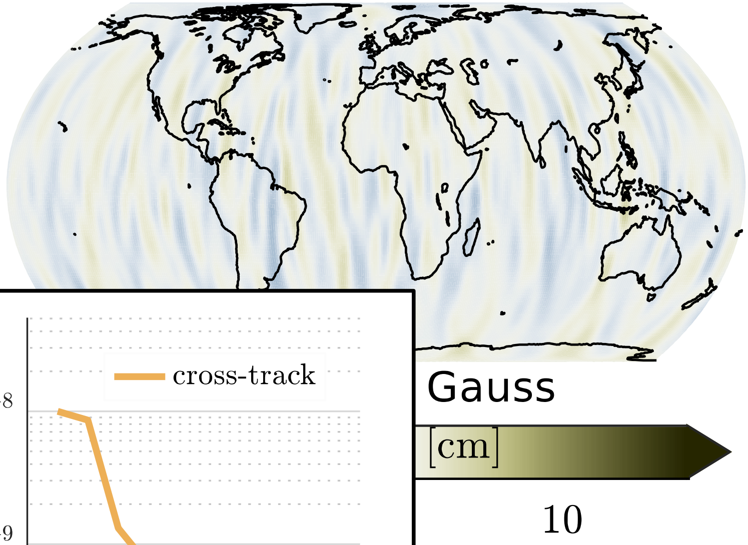
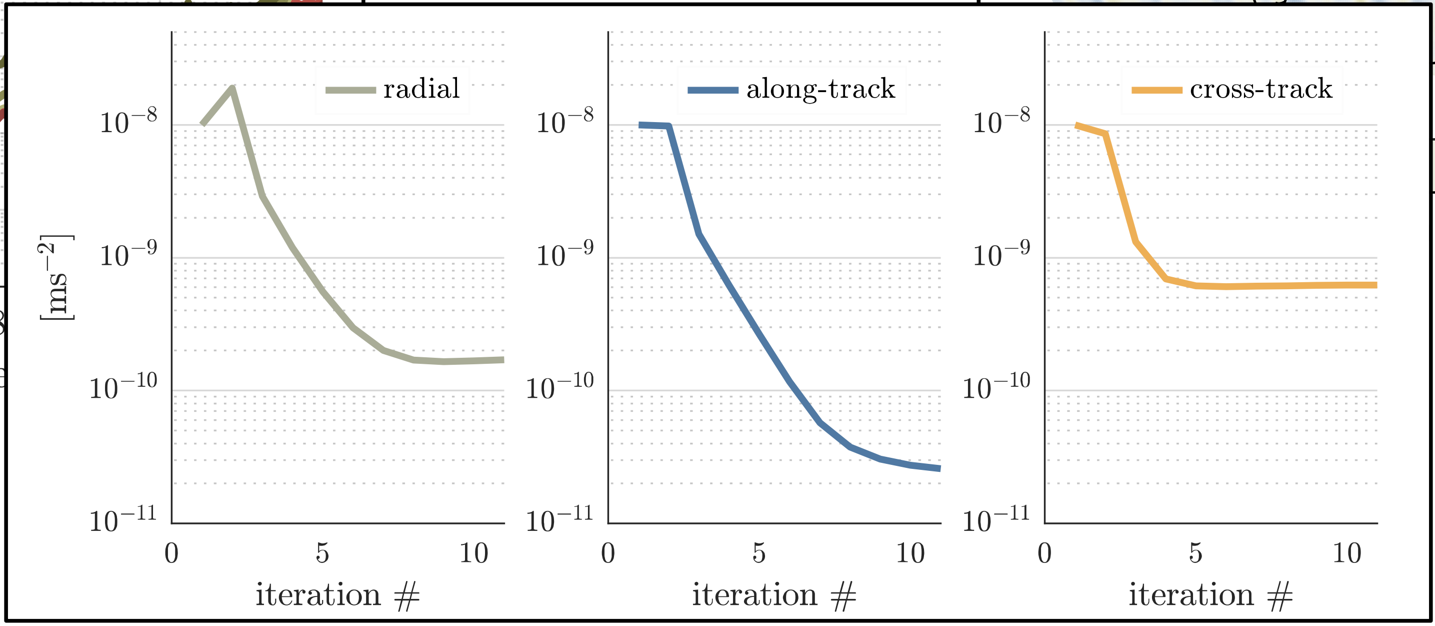


	ms^{-2}
radial	5.6×10^{-10}
along-track	2.6×10^{-10}





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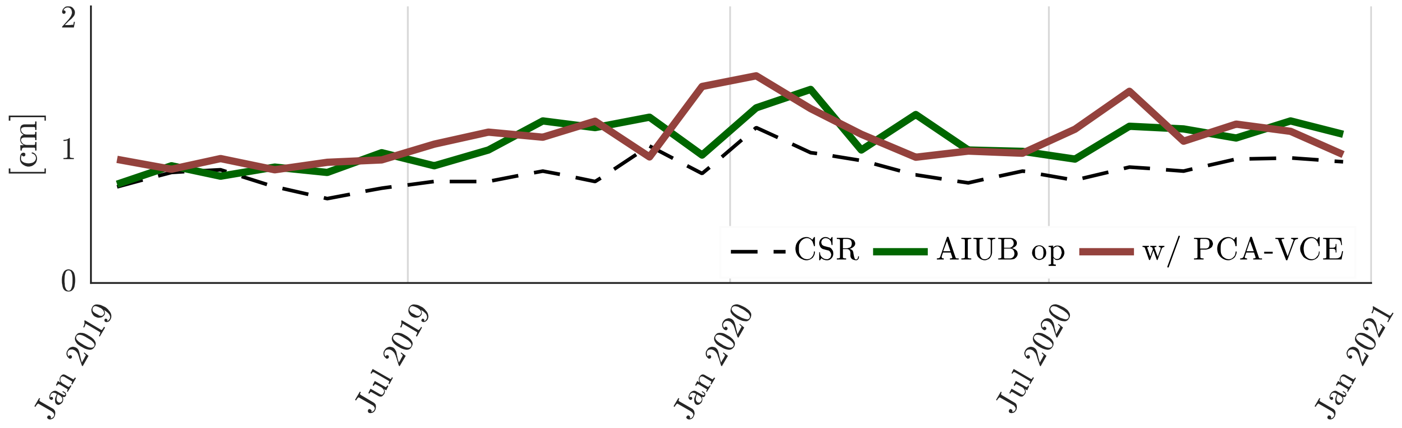
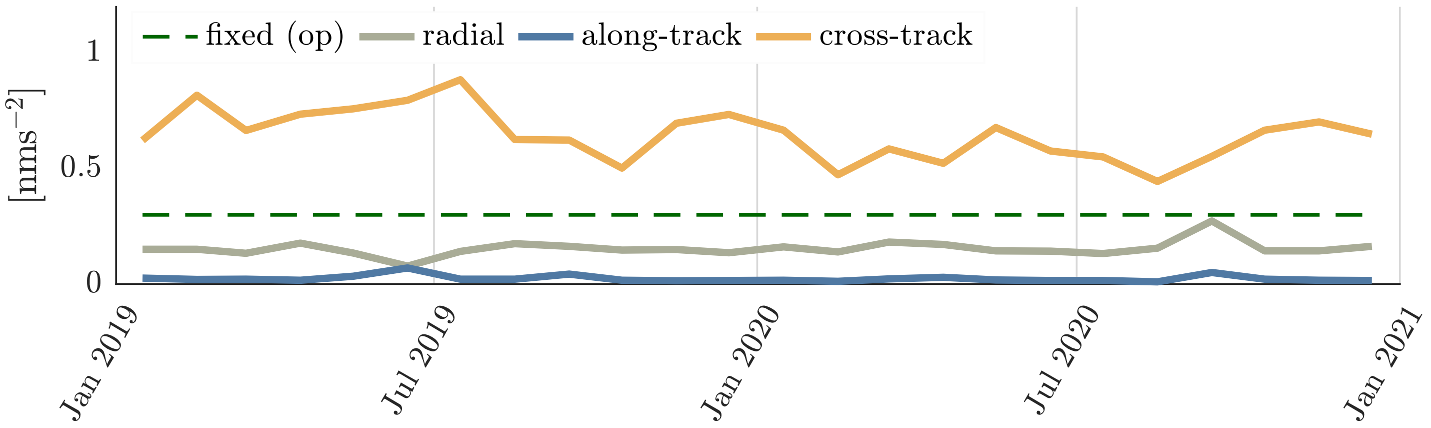
Conclusion

- observation-based approach
- computed together with the solution
- provides a good solution (if PCAs sample correctly)
- improvement...

RMS over the oceans



- computational efficiency?
- observation-based – outliers
- improvement...



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